

Sample Space Diagrams

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Spinner *A* is numbered 1, 2 and 3

Spinner *B* is numbered 1, 2, 3 and 4

Both spinners are spun and the numbers are added to give a total.

The rows of the table show spinner *A*.

(a) Complete the table to show all the possible totals.

Each box is the row number (spinner *A*) plus the column number (spinner *B*).

For example, row 2, column 3: $2 + 3 = 5$

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7

Answer: See table

Marks: 2 marks for a fully correct table. 1 mark for at least 6 of the 8 missing totals correct.

(b) Work out the probability that the total is 5

There are $3 \times 4 = 12$ totals in the table, all equally likely.

The total is 5 in 3 of them.

Answer: $\frac{3}{12}$

- (c) Work out the probability that the total is greater than 5

Totals greater than 5 are 6, 6 and 7

That is 3 of the 12 totals.

Answer: $\frac{3}{12}$

2 (3 marks)

A fair coin is thrown and a fair six-sided dice is rolled.

- (a) List all the possible outcomes.

Work systematically: take heads with each number, then tails with each number.

H1, H2, H3, H4, H5, H6

T1, T2, T3, T4, T5, T6

That is $2 \times 6 = 12$ outcomes.

Answer: 12 outcomes: H1 ... T6

Tip: Keep one item fixed and go through all the options for the other. Then you will not miss or repeat any.

Marks: 2 marks for all 12 with no extras. 1 mark for at least 6 correct.

- (b) Work out the probability of getting a head and an even number.

Head and an even number: H2, H4, H6

That is 3 of the 12 outcomes.

Answer: $\frac{3}{12}$

3 (2 marks)

Ben has two bags of cards.

Bag 1 has cards labelled A , B and C .

Bag 2 has cards labelled 1 and 2

He takes one card from each bag.

Is the probability of getting A and 2 equal to $\frac{1}{5}$? Tick a box.

You must show your working.

List the outcomes: $A1$, $A2$, $B1$, $B2$, $C1$, $C2$

There are $3 \times 2 = 6$ equally likely outcomes.

Only one of them is A and 2

So $P(A \text{ and } 2) = \frac{1}{6}$, not $\frac{1}{5}$.

Answer: No, it is $\frac{1}{6}$

Tip: There are 5 cards, but there are 6 possible **pairs**. The bottom of the fraction is the number of outcomes.

Marks: 1 mark for 6 outcomes. 1 mark for No with $\frac{1}{6}$.

4 (3 marks)

Two fair six-sided dice are rolled and the scores are added.

(a) Complete the sample space diagram to show all the possible totals.

Each box is the row number plus the column number.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Answer: See table

- (b) Which total is most likely?
Give the probability of getting this total.

There are $6 \times 6 = 36$ totals in the table.

The total 7 appears most often: along the diagonal, 6 times.

$$P(7) = \frac{6}{36}$$

Answer: 7, $\frac{6}{36}$

Marks: 1 mark for 7. 1 mark for $\frac{6}{36}$ or an equivalent.

5 (5 marks)

Lena has two bags of number tiles.

Bag *A* has tiles numbered 2, 3 and 5

Bag *B* has tiles numbered 1, 4, 6 and 7

She takes one tile at random from each bag and multiplies the two numbers to get a score.

The rows of the table show bag *A*.

- (a) Complete the table to show all the possible scores.

Each box is the row number times the column number.

For example, row 5, column 7: $5 \times 7 = 35$

Bag *B*

$\times$	1	4	6	7
2	2	8	12	14
3	3	12	18	21
5	5	20	30	35

Answer: See table

Marks: 2 marks for a fully correct table. 1 mark for at least 7 correct.

- (b) Work out the probability that the score is an odd number.

There are $3 \times 4 = 12$ scores in the table.

Odd scores: 3, 21, 5 and 35

That is 4 of the 12 scores.

Answer: $\frac{4}{12}$ (or $\frac{1}{3}$)

Tip: A product is only odd when **both** numbers are odd, so the 2 row has no odd scores.

- (c) Lena plays the game 60 times.

Estimate the number of times the score is an odd number.

Expected number = probability $\times$ number of games

$$\frac{4}{12} \times 60$$

$$60 \div 12 = 5$$

$$5 \times 4 = 20$$

Answer: 20

Marks: 1 mark for $\frac{4}{12} \times 60$. 1 mark for 20 (or a correct answer from your probability in part (b)).

6 (5 marks)

In a game, a fair coin is thrown and a fair four-sided spinner numbered 1, 2, 3 and 4 is spun.

If the coin shows Heads, the score is the spinner number multiplied by 2

If the coin shows Tails, the score is the spinner number add 3

- (a) Complete the table to show all the possible scores.

Heads row: double the spinner number: 2, 4, 6, 8

Tails row: add 3 to the spinner number: 4, 5, 6, 7

Spinner

	1	2	3	4
Heads	2	4	6	8
Tails	4	5	6	7

Answer: See table

Marks: 2 marks for a fully correct table. 1 mark for one row correct.

- (b) Write down the probability that the score is 6

There are $2 \times 4 = 8$ scores in the table.

The score 6 appears twice (Heads with 3, Tails with 3).

Answer: $\frac{2}{8}$ (or $\frac{1}{4}$)

- (c) Kai says,
“The score is just as likely to be odd as even.”

Is Kai correct?

Tick a box.

You must show your working.

Even scores: 2, 4, 6, 8, 4, 6: that is 6 of the 8.

Odd scores: 5 and 7: that is 2 of the 8.

$$P(\text{even}) = \frac{6}{8} \text{ and } P(\text{odd}) = \frac{2}{8}$$

An even score is more likely, so Kai is not correct.

Answer: No: $P(\text{even}) = \frac{6}{8}$, $P(\text{odd}) = \frac{2}{8}$

Tip: Count the scores in your table. The Heads row is all even, so even scores come up more often.

Marks: 1 mark for 6 even scores or 2 odd scores (or both probabilities). 1 mark for No with correct working.

7 (5 marks)

Two fair four-sided dice are each numbered 1, 2, 3 and 4

Both dice are rolled.

The score is the larger of the two numbers. If the numbers are the same, the score is that number.

The rows of the table show the first dice.

- (a) Complete the table to show all the possible scores.

Each box is the larger of the row number and the column number.

When the two numbers are the same, the score is that number.

Second dice

	1	2	3	4
1	1	2	3	4
2	2	2	3	4
3	3	3	3	4
4	4	4	4	4

Answer: See table

Marks: 2 marks for a fully correct table. 1 mark for at least 10 correct.

- (b) Work out the probability that the score is 4

There are $4 \times 4 = 16$ scores in the table.

The score 4 is in the whole bottom row and the whole right-hand column.

That is $4 + 4 - 1 = 7$ scores (the 4 in the corner is only counted once).

Answer: $\frac{7}{16}$

Tip: Count the boxes in the table rather than working it out in your head. The corner box is in the row and the column, so count it once.

- (c) Mia says,

“The scores 1, 2, 3 and 4 are all equally likely.”

Is Mia correct?

Give a reason for your answer.

From the table, the score 1 appears only once: $P(1) = \frac{1}{16}$

The score 4 appears 7 times: $P(4) = \frac{7}{16}$

These are not equal, so Mia is not correct.

Answer: No: for example $P(1) = \frac{1}{16}$ but $P(4) = \frac{7}{16}$

Marks: 1 mark for two different probabilities (or counts) from the table. 1 mark for No with a correct reason.

8 (3 marks)

Here are four number cards.

2	5	6	9
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Two of the cards are picked at random and put side by side to make a two-digit number.

Work out the probability that the number is even **and** greater than 50

List every two-digit number, one tens digit at a time:

25, 26, 29

52, 56, 59

62, 65, 69

92, 95, 96

There are 12 numbers, all equally likely.

Even and greater than 50: 52, 56, 62, 92 and 96

That is 5 of the 12 numbers.

Answer: $\frac{5}{12}$

Tip: 25 and 52 are different numbers, so list both. A card cannot be used twice, so 55 is not possible.

Marks: 1 mark for a systematic list of the 12 numbers. 1 mark for 5 numbers that are even and greater than 50. 1 mark for $\frac{5}{12}$.
