

Relative Frequency and Expected Outcomes

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Imogen flips a plastic bottle 60 times.

It lands upright 9 times.

(a) Write down the relative frequency of the bottle landing upright.

$$\text{Relative frequency} = \frac{\text{number of times upright}}{\text{number of flips}}$$

$$= \frac{9}{60}$$

As a decimal: $9 \div 60 = 0.15$

Answer: $\frac{9}{60}$ (0.15)

Tip: A relative frequency is a fraction or decimal, like $\frac{9}{60}$. Writing just 9, or 9:60, gets no marks.

(b) Imogen says,

“The probability that the bottle lands upright is exactly 0.15”

Give a reason why she might be wrong.

0.15 is only an **estimate** of the probability.

It comes from just 60 flips.

If she flipped the bottle again, or many more times, she would probably get a different relative frequency.

Answer: It is only an estimate from 60 flips; more flips could give a different value

2 (2 marks)

The probability that a spinner lands on red is 0.3
The spinner is spun 150 times.
Work out the expected number of times it lands on red.

Expected number = probability $\times$ number of spins

$$0.3 \times 150 = 45$$

Answer: 45

Marks: 1 mark for 0.3×150 . 1 mark for 45.

3 (2 marks)

The probability that a biased coin lands on heads is 0.65
The coin is thrown 300 times.
Work out an estimate for the number of times the coin lands on tails.

The question asks about **tails**.

$$P(\text{tails}) = 1 - 0.65 = 0.35$$

$$\text{Expected number of tails} = 0.35 \times 300 = 105$$

Answer: 105

Tip: $0.65 \times 300 = 195$ is the number of **heads**. Read the question: it asks for tails.

Marks: 1 mark for 0.35×300 (or $300 - 0.65 \times 300$). 1 mark for 105.

4 (2 marks)

Jo rolls a dice 50 times.

Here are her results.

Score	1	2	3	4	5	6
Frequency	6	9	8	7	10	10

- (a) Work out the relative frequency of rolling a 5

Jo rolled a 5 ten times out of 50 rolls.

$$\text{Relative frequency} = \frac{10}{50} = 0.2$$

Answer: $\frac{10}{50} = 0.2$

- (b) Jo says the dice is biased because she rolled more 5s and 6s than 1s.

Is she correct? Tick a box.

Give a reason for your answer.

For a fair dice, each score would come up about $50 \div 6 \approx 8$ times.

All of Jo's frequencies (6 to 10) are close to 8.

50 rolls is not enough to show the dice is biased.

Answer: No, not enough rolls / results close to 8 each

5 (3 marks)

A garden centre sowed 200 tomato seeds.
The table shows what happened.

Result	Frequency
Did not grow	30
Grew, but the plant died	20
Grew into a healthy plant	150

- (a) Write down the relative frequency that a seed grew into a healthy plant.

150 of the 200 seeds grew into a healthy plant.

$$\text{Relative frequency} = \frac{150}{200} = 0.75$$

Answer: $\frac{150}{200}$ (0.75)

- (b) Next year the garden centre wants 600 healthy plants.
Using the results in the table, work out how many seeds it should sow.

About 0.75 of the seeds sown grow into healthy plants.

$$\text{So } 0.75 \times (\text{number of seeds}) = 600$$

$$\text{Number of seeds} = 600 \div 0.75 = 800$$

$$\text{Check: } 0.75 \times 800 = 600 \quad \checkmark$$

Answer: 800

Tip: $0.75 \times 600 = 450$ is the wrong way round. You know the number of healthy plants, so **divide** by 0.75.

Marks: 1 mark for $600 \div 0.75$ (or $600 \div \frac{150}{200}$). 1 mark for 800.

6 (2 marks)

There are 50 counters in a bag.

Harry takes a counter at random, records its colour and puts it back.

He does this 40 times.

He gets a red counter 12 times.

Work out an estimate of the number of red counters in the bag.

$$\text{Relative frequency of red} = \frac{12}{40} = 0.3$$

Use this as an estimate of the probability of red.

$$\text{Estimated red counters} = 0.3 \times 50 = 15$$

Answer: 15

Tip: Do not stop at 0.3. The question asks for a **number** of counters, so multiply by the 50 counters in the bag.

Marks: 1 mark for $\frac{12}{40} \times 50$. 1 mark for 15.

7 (3 marks)

Four groups each spin the same spinner.

The table shows how many times it lands on blue.

Group	Number of spins	Number of blues
1	30	11
2	30	8
3	60	19
4	80	22

- (a) Use all of the results to work out an estimate of the probability that the spinner lands on blue.

$$\text{Total number of blues: } 11 + 8 + 19 + 22 = 60$$

$$\text{Total number of spins: } 30 + 30 + 60 + 80 = 200$$

$$\text{Estimate} = \frac{60}{200} = 0.3$$

$$\text{Answer: } \frac{60}{200} \text{ (0.3)}$$

Tip: Add up the blues and add up the spins. Do not take the mean of the four separate relative frequencies.

Marks: 1 mark for 60 or 200. 1 mark for $\frac{60}{200}$ or an equivalent.

- (b) The spinner will be spun another 500 times.
Use your answer to part (a) to work out an estimate for the number of times it lands on blue.

Expected number = probability $\times$ number of spins

$$0.3 \times 500 = 150$$

$$\text{Answer: } 150$$

8 (3 marks)

A bus company records whether its buses arrive early, on time or late.

Out of 160 buses, 24 arrived early.

The relative frequency of a bus arriving late is 0.35

Work out the relative frequency of a bus arriving on time.

$$\text{Relative frequency of early: } \frac{24}{160} = 0.15$$

The relative frequencies of early, on time and late add up to 1.

$$\text{On time: } 1 - 0.15 - 0.35 = 0.5$$

(Or: late = $0.35 \times 160 = 56$ buses, on time = $160 - 24 - 56 = 80$ buses, and $\frac{80}{160} = 0.5$)

Answer: 0.5 (or $\frac{80}{160}$)

Tip: 24 is a number of buses, not a relative frequency. Change it to $\frac{24}{160}$ before you subtract from 1.

Marks: 1 mark for $\frac{24}{160}$ (0.15) or 56 late buses. 1 mark for $1 - 0.15 - 0.35$ or 80 buses on time. 1 mark for 0.5.

9 (2 marks)

Mia rolls a dice 100 times, in five sets of 20 rolls.
The table shows the number of sixes in each set.

Set	1st	2nd	3rd	4th	5th
Number of sixes	4	2	5	3	6

Work out the best estimate for the probability of rolling a six.
Give a reason for your answer.

The best estimate uses all of the rolls.

Total sixes: $4 + 2 + 5 + 3 + 6 = 20$

Total rolls: $5 \times 20 = 100$

$$\frac{20}{100} = 0.2$$

It uses all 100 rolls, which is the most trials.

Answer: $\frac{20}{100}$ (0.2), as it uses all the rolls

Tip: Do not pick just one set, like $\frac{6}{20}$. The more rolls you use, the more reliable the estimate.

Marks: 1 mark for $\frac{20}{100}$. 1 mark for the reason: it uses all the rolls (the most trials).

10 (3 marks)

Emma and Tom each test some bulbs from the same factory to see if they are faulty. The table shows their results.

	Emma	Tom
Number of bulbs tested	40	250
Number of faulty bulbs	4	20

(a) Tom says,

“My results show a smaller proportion of faulty bulbs than Emma’s results.”

Is Tom correct? Tick a box.

You must show your working.

Compare the relative frequencies, not the numbers of faulty bulbs.

Emma: $\frac{4}{40} = 0.1$

Tom: $\frac{20}{250} = 0.08$

0.08 is smaller than 0.1

So Tom is correct.

Answer: Yes. Tom $\frac{20}{250} = 0.08$, Emma $\frac{4}{40} = 0.1$, and $0.08 < 0.1$

Tip: Tom found more faulty bulbs (20) than Emma (4), but he also tested many more bulbs. Compare the fractions (or percentages), not the counts.

Marks: 1 mark for $\frac{4}{40}$ and $\frac{20}{250}$ (or 0.1 and 0.08, or 10% and 8%). 1 mark for Yes with both values.

- (b) The factory makes 5000 bulbs each day.

Emma says,

“Using my results, about 500 bulbs each day will be faulty.”

Tom says,

“Using my results, about 400 bulbs each day will be faulty.”

Whose estimate is more reliable?

Give a reason for your answer.

Emma: $\frac{4}{40} \times 5000 = 500$

Tom: $\frac{20}{250} \times 5000 = 400$

Both worked it out correctly.

The more bulbs tested, the more reliable the estimate.

Tom tested 250 bulbs and Emma only tested 40.

Answer: Tom, because he tested more bulbs

Tip: The reason must be about the number of bulbs tested (the number of trials), not about which estimate is bigger or smaller.

11 (3 marks)

Nina rolls a dice 120 times.

The relative frequency of rolling a six is 0.15

She then rolls the dice 80 more times.

After all 200 rolls, the relative frequency of rolling a six is 0.19

How many sixes did Nina roll in the last 80 rolls?

Sixes in the first 120 rolls: $0.15 \times 120 = 18$

Sixes in all 200 rolls: $0.19 \times 200 = 38$

Sixes in the last 80 rolls: $38 - 18 = 20$

Answer: 20

Tip: Use 0.19 with all 200 rolls, not with the last 80 only. Then take away the sixes she already had.

Marks: 1 mark for 18. 1 mark for 38. 1 mark for 20.
