

Quadratic Graphs

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

(a) Complete the table of values for $y = x^2 - 2x$

x	-1	0	1	2	3
y	3	0	-1	0	3

Substitute each value of x into $y = x^2 - 2x$.

$$x = -1: (-1)^2 - 2 \times (-1) = 1 + 2 = 3$$

$$x = 1: 1^2 - 2 \times 1 = 1 - 2 = -1$$

Answer: 3, -1

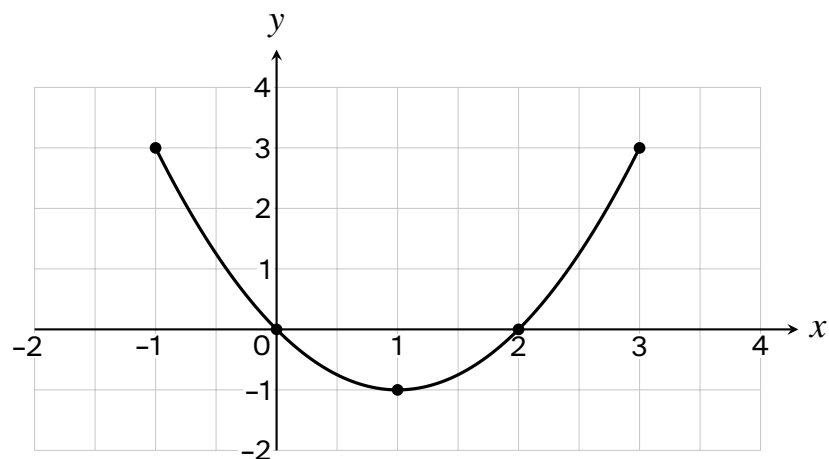
Tip: Quadratic tables do not go up by the same amount each time, so don't just continue a pattern. Work out each value.

Marks: 2 marks for both correct. 1 mark for one correct.

(b) Draw the graph of $y = x^2 - 2x$ for values of x from -1 to 3

Plot the five points: (-1, 3), (0, 0), (1, -1), (2, 0), (3, 3).

Join them with a smooth U-shaped curve.



Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

2 (5 marks)

(a) Complete the table of values for $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	6	1	-2	-3	-2	1	6

Substitute each value of x into $y = x^2 - 3$.

$$x = -2: (-2)^2 - 3 = 4 - 3 = 1$$

$$x = 0: 0^2 - 3 = -3$$

$$x = 1: 1^2 - 3 = 1 - 3 = -2$$

$$x = 3: 3^2 - 3 = 9 - 3 = 6$$

Check: the table is symmetrical (6, 1, -2, -3, -2, 1, 6) ✓

Answer: 1, -3, -2, 6

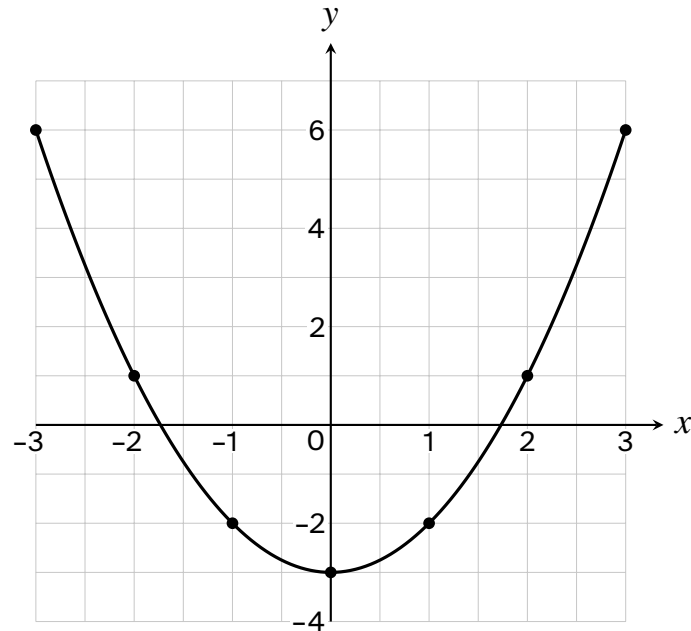
Tip: A negative number squared is positive: $(-2)^2 = +4$. On a calculator, put brackets round the negative number.

Marks: 2 marks for all four correct. 1 mark for two or three correct.

- (b) Draw the graph of $y = x^2 - 3$ for values of x from -3 to 3

Plot the seven points from the table.

Join them with one smooth curve (not straight lines).



Tip: Draw one smooth U-shaped curve in pencil. Don't join the points with a ruler, and don't flatten the bottom of the curve.

Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a smooth curve through all 7 points.

- (c) Write down the coordinates of the turning point of the curve.

The turning point is the lowest point of the curve.

It is at $x = 0$, where $y = -3$.

Answer: $(0, -3)$

3 (1 mark)

Priya is completing a table of values for $y = x^2 - 5$

For $x = -3$ she writes $y = -14$

What mistake has Priya made?

When $x = -3$: $(-3)^2 = (-3) \times (-3) = +9$

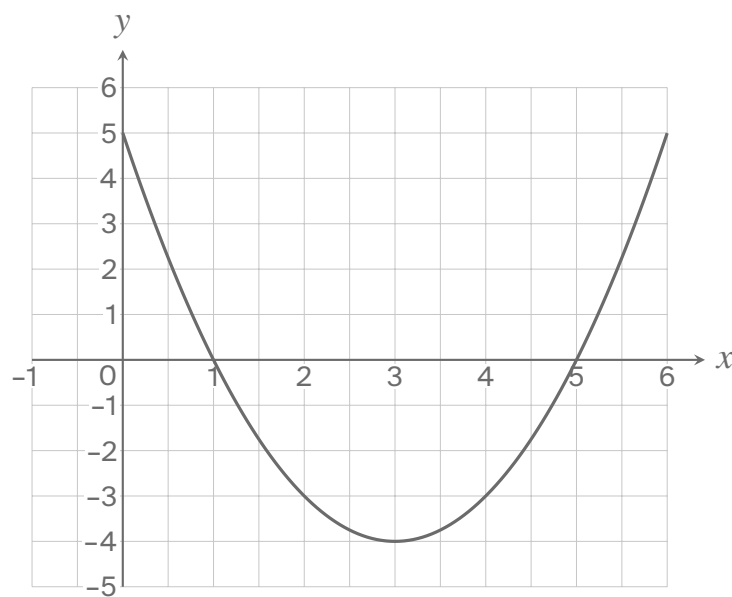
So $y = 9 - 5 = 4$, not -14 .

Priya has worked out $(-3)^2$ as -9 instead of $+9$.

Answer: $(-3)^2 = +9$, so $y = 4$

4 (5 marks)

Here is the graph of $y = x^2 - 6x + 5$ for values of x from 0 to 6



(a) Use the graph to solve $x^2 - 6x + 5 = 0$

The solutions are the x -values where the curve crosses the x -axis (where $y = 0$).

The curve crosses the x -axis at $x = 1$ and $x = 5$.

Answer: $x = 1$ and $x = 5$

Tip: Give the solutions as values of x ($x = 1$ and $x = 5$), not as coordinates, and not the y -intercept.

- (b) Write down the coordinates of the point where the curve crosses the y -axis.

The curve crosses the y -axis where $x = 0$.

$$y = 0^2 - 6 \times 0 + 5 = 5$$

Answer: (0, 5)

- (c) Write down the coordinates of the turning point of the curve.

The turning point is the lowest point of the curve.

It is halfway between the solutions 1 and 5, at $x = 3$.

$$y = 3^2 - 6 \times 3 + 5 = 9 - 18 + 5 = -4$$

Answer: (3, -4)

- (d) Use the graph to solve $x^2 - 6x + 5 = -3$

Draw the horizontal line $y = -3$ on the graph.

Read the x -values where the line meets the curve.

The line meets the curve at $x = 2$ and $x = 4$.

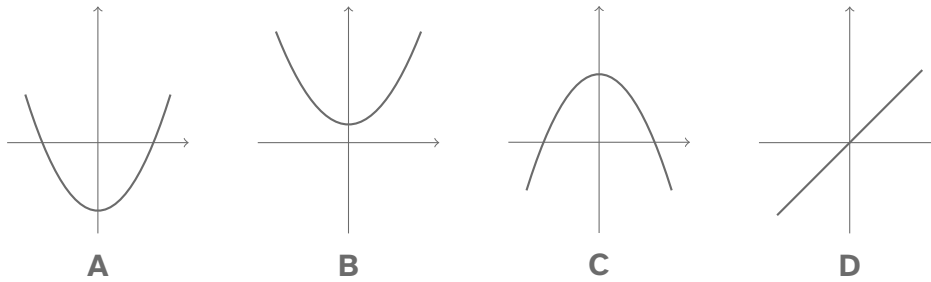
$$\text{Check: } 2^2 - 6 \times 2 + 5 = 4 - 12 + 5 = -3 \quad \checkmark$$

Answer: $x = 2$ and $x = 4$

Tip: Solving " $= -3$ " means finding where the curve is at height -3 . Draw the line $y = -3$; do not read where the curve crosses the x -axis.

Marks: 1 mark for drawing $y = -3$ or for one correct solution. 1 mark for both solutions.

5 (1 mark)



Circle the sketch of a graph of the form $y = x^2 + c$ where $c > 0$.

Options: A, B, C, D

$y = x^2 + c$ is a U-shaped curve.

When $x = 0$, $y = c$, so it crosses the y -axis at c .

$c > 0$, so the turning point is above the x -axis, on the y -axis.

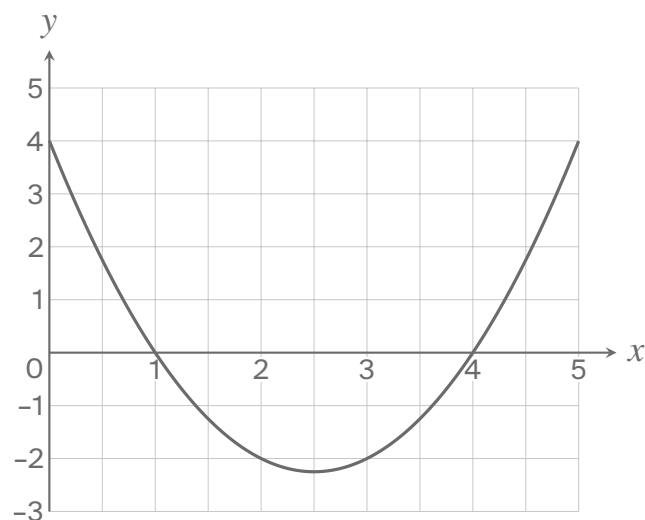
A crosses below the x -axis, C is $\cap$ -shaped and D is a straight line.

So the answer is B.

Answer: B

6 (3 marks)

Here is the graph of $y = x^2 - 5x + 4$ for values of x from 0 to 5



- (a) Write down the solutions of $x^2 - 5x + 4 = 0$

The solutions are the x -values where the curve crosses the x -axis (where $y = 0$).

Read them from the graph: $x = 1$ and $x = 4$.

Answer: $x = 1$ and $x = 4$

- (b) Write down the equation of the line of symmetry of the curve.

The line of symmetry is halfway between the solutions.

$$(1 + 4) \div 2 = 2.5$$

It is a vertical line, so its equation is $x = 2.5$

Answer: $x = 2.5$

Tip: A vertical line has an equation $x = \dots$, not $y = \dots$

- (c) Work out the minimum value of y .

The minimum is at the turning point, on the line of symmetry $x = 2.5$

$$y = 2.5^2 - 5 \times 2.5 + 4$$

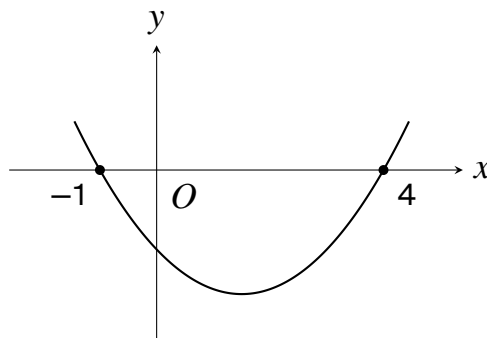
$$= 6.25 - 12.5 + 4 = -2.25$$

Answer: -2.25

7 (2 marks)

Here is a sketch of a quadratic curve.

It crosses the x -axis at -1 and 4



Not drawn accurately

- (a) Circle the equation of the curve.

Options: $y = (x - 1)(x + 4)$, $y = (x + 1)(x - 4)$, $y = (x - 1)(x - 4)$

The curve crosses the x -axis at $x = -1$ and $x = 4$, so $y = 0$ there.

Check $y = (x + 1)(x - 4)$: $x = -1$ gives $0 \times (-5) = 0$ ✓ $x = 4$ gives $5 \times 0 = 0$ ✓

The signs in the brackets are the opposite of the roots.

Answer: $y = (x + 1)(x - 4)$

Tip: A root at $x = -1$ comes from the bracket $(x + 1)$, because $-1 + 1 = 0$. The sign in the bracket is the opposite of the root.

- (b) Write down the coordinates of the point where the curve crosses the y -axis.

The curve crosses the y -axis where $x = 0$.

$$y = (0 + 1)(0 - 4) = 1 \times (-4) = -4$$

Answer: $(0, -4)$

Tip: The sketch is not drawn to scale, so work the value out from the equation. Don't try to read it off the sketch.
