

Pythagoras

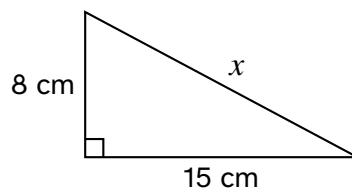
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Here is a right-angled triangle.



Not drawn accurately

Work out the length x .

Pythagoras' theorem: $a^2 + b^2 = c^2$, where c is the hypotenuse (the longest side, opposite the right angle).

x is the hypotenuse, so add the squares of the other two sides.

$$x^2 = 8^2 + 15^2$$

$$x^2 = 64 + 225 = 289$$

$$x = \sqrt{289} = 17$$

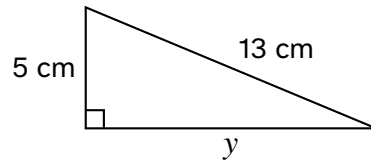
Answer: 17 cm

Tip: 8^2 means $8 \times 8 = 64$, not $8 \times 2 = 16$.

Marks: 1 mark for $8^2 + 15^2$. 1 mark for 17.

2 (3 marks)

Here is a right-angled triangle.



Not drawn accurately

Work out the length y .

The hypotenuse is 13 cm.

y is one of the shorter sides.

To find a shorter side, subtract the squares.

$$y^2 = 13^2 - 5^2$$

$$y^2 = 169 - 25 = 144$$

$$y = \sqrt{144} = 12$$

Answer: 12 cm

Tip: Finding the longest side: add the squares. Finding a shorter side: subtract. Check your answer is shorter than the hypotenuse.

Marks: 1 mark for $13^2 - 5^2$. 1 mark for $\sqrt{144}$. 1 mark for 12.

3 (2 marks)

A right-angled triangle has shorter sides 7 cm and 4 cm.

Work out the length of the hypotenuse.

Give your answer to 1 decimal place.

Call the hypotenuse h .

$$h^2 = 7^2 + 4^2 = 49 + 16 = 65$$

$$h = \sqrt{65} = 8.062 \dots$$

To 1 decimal place, $h = 8.1$ cm

Answer: 8.1 cm

Tip: Remember the last step: square root. 65 is h^2 , not h .

Marks: 1 mark for $7^2 + 4^2$. 1 mark for 8.1.

4 (2 marks)

A rectangle is 8 cm long and 5 cm wide.

Work out the length of a diagonal.

Give your answer to 1 decimal place.

The diagonal splits the rectangle into two right-angled triangles.

The diagonal is the hypotenuse, with shorter sides 8 cm and 5 cm.

$$d^2 = 8^2 + 5^2 = 64 + 25 = 89$$

$$d = \sqrt{89} = 9.433 \dots = 9.4 \text{ cm (1 d.p.)}$$

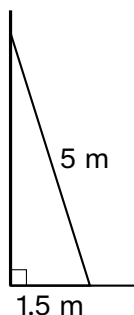
Answer: 9.4 cm

Marks: 1 mark for $8^2 + 5^2$. 1 mark for 9.4.

5 (3 marks)

A ladder 5 m long leans against a vertical wall.

The foot of the ladder is on horizontal ground, 1.5 m from the wall.



Not drawn accurately

How far up the wall does the ladder reach?

Give your answer to 1 decimal place.

The ladder, the wall and the ground make a right-angled triangle.

The ladder (5 m) is the hypotenuse.

Call the height up the wall h .

h is a shorter side, so subtract: $h^2 = 5^2 - 1.5^2$

$$h^2 = 25 - 2.25 = 22.75$$

$$h = \sqrt{22.75} = 4.769 \dots = 4.8 \text{ m (1 d.p.)}$$

Answer: 4.8 m

Marks: 1 mark for $5^2 - 1.5^2$. 1 mark for the square root. 1 mark for 4.8.

6 (3 marks)

Work out the distance between the points $(-1, 2)$ and $(4, 9)$.

Give your answer to 1 decimal place.

Sketch a right-angled triangle with the line between the points as the hypotenuse.

Across: $4 - (-1) = 5$

Up: $9 - 2 = 7$

$$\text{Distance}^2 = 5^2 + 7^2 = 25 + 49 = 74$$

$$\text{Distance} = \sqrt{74} = 8.602 \dots = 8.6 \text{ (1 d.p.)}$$

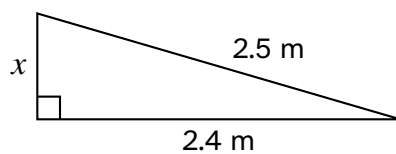
Answer: 8.6

Tip: Take care with the negative coordinate: from -1 to 4 is 5 across, not 3 .

Marks: 1 mark for 5 and 7 . 1 mark for $5^2 + 7^2$. 1 mark for 8.6 .

7 (2 marks)

Here is a right-angled triangle.



Not drawn accurately

Use Pythagoras' theorem to show that $x = 0.7$

The hypotenuse is 2.5 m, so x is a shorter side: subtract.

$$x^2 = 2.5^2 - 2.4^2$$

$$x^2 = 6.25 - 5.76$$

$$x^2 = 0.49$$

$$x = \sqrt{0.49} = 0.7$$

Answer: $\sqrt{6.25 - 5.76} = \sqrt{0.49} = 0.7$

Tip: In a “show that” question write every step on a new line. Do not write $0.49 = 0.7$: it is $\sqrt{0.49} = 0.7$.

Marks: 1 mark for 2.5^2 and 2.4^2 (6.25 and 5.76). 1 mark for $\sqrt{0.49} = 0.7$ with no errors (or checking $0.7^2 + 2.4^2 = 6.25 = 2.5^2$).

8 (2 marks)

A triangle has sides 9 cm, 12 cm and 15 cm.

Is the triangle right-angled? Tick a box.

You must show your working.

If the triangle is right-angled, the longest side is the hypotenuse and $a^2 + b^2 = c^2$.

$$9^2 + 12^2 = 81 + 144 = 225$$

$$15^2 = 225$$

These are equal, so Pythagoras' theorem works and the triangle is right-angled.

Answer: Yes, $81 + 144 = 225$

Marks: 1 mark for $9^2 + 12^2$ and 15^2 . 1 mark for Yes with the correct working.

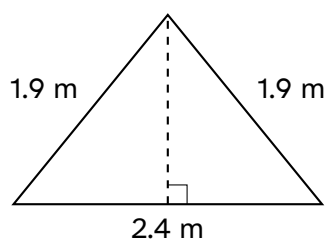
9 (4 marks)

The front of a tent is an isosceles triangle.

The width of the tent is 2.4 m and each sloping side is 1.9 m.

Asha says,

‘The tent is more than 1.5 m tall in the middle.’



Not drawn accurately

Use Pythagoras' theorem to show that Asha is wrong.

The height of an isosceles triangle cuts the base in half.

So each half of the base is $2.4 \div 2 = 1.2$ m.

This makes a right-angled triangle with hypotenuse 1.9 m and one shorter side 1.2 m.

The height h is a shorter side, so subtract the squares.

$$h^2 = 1.9^2 - 1.2^2 = 3.61 - 1.44 = 2.17$$

$$h = \sqrt{2.17} = 1.473 \dots \text{ m}$$

1.473 ... m is less than 1.5 m, so the tent is **not** more than 1.5 m tall. Asha is wrong.

(Or: $1.5^2 = 2.25$, and 2.17 is less than 2.25.)

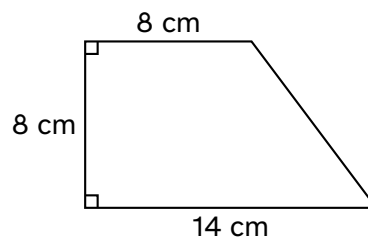
Answer: $h = \sqrt{2.17} = 1.47$ m, which is less than 1.5 m

Tip: Use half the width (1.2 m), not the whole width. The height is a shorter side, so subtract the squares. Finish with a sentence that compares with 1.5 m.

Marks: 1 mark for 1.2. 1 mark for $1.9^2 - 1.2^2$. 1 mark for $\sqrt{2.17}$ or 1.47.... 1 mark for a conclusion that the height is less than 1.5 m (or $2.17 < 2.25$).

10 (4 marks)

Here is a trapezium.



Not drawn accurately

Work out the perimeter of the trapezium.

Draw a vertical line from the top right corner down to the base.

This makes a rectangle and a right-angled triangle.

The triangle's base is $14 - 8 = 6$ cm and its height is 8 cm.

$$\text{Slant side}^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$\text{Slant side} = \sqrt{100} = 10 \text{ cm}$$

$$\text{Perimeter} = 14 + 8 + 8 + 10 = 40 \text{ cm}$$

Answer: 40 cm

Marks: 1 mark for 6. 1 mark for $6^2 + 8^2$. 1 mark for 10. 1 mark for 40.