

Probabilities Summing to 1

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The probability that it rains tomorrow is 0.35

Write down the probability that it does **not** rain tomorrow.

It either rains or it does not, so the two probabilities add up to 1.

$$P(\text{not rain}) = 1 - 0.35$$

Answer: 0.65

2 (1 mark)

The probability that a biased dice lands on 6 is $\frac{2}{9}$

Circle the probability that it does **not** land on 6

Options: $\frac{2}{9}$, $\frac{7}{9}$, $\frac{9}{2}$, $\frac{1}{9}$

$$P(\text{not 6}) = 1 - P(6)$$

Write 1 as $\frac{9}{9}$:

$$\frac{9}{9} - \frac{2}{9} = \frac{7}{9}$$

Answer: $\frac{7}{9}$

3 (3 marks)

A spinner can land on red, blue, green or yellow.
The table shows some of the probabilities.

Colour	Red	Blue	Green	Yellow
Probability	0.3	0.15	0.4	

- (a) Work out the probability that the spinner lands on yellow.

The four probabilities must add up to 1.

Add the ones you know: $0.3 + 0.15 + 0.4 = 0.85$

Yellow: $1 - 0.85 = 0.15$

Answer: 0.15

Tip: Line up the decimal points when you add: $0.15 + 0.4 = 0.55$, not 0.19.

Marks: 1 mark for $1 - (0.3 + 0.15 + 0.4)$. 1 mark for 0.15.

- (b) Work out the probability that the spinner lands on red **or** green.

Red and green cannot both happen, so add their probabilities.

$0.3 + 0.4 = 0.7$

Answer: 0.7

4 (1 mark)

A spinner has ten sections, each labelled A , B , C or D .
Here are the probabilities that it lands on each letter.

Letter	A	B	C	D
Probability	0.1	0.5	0.2	0.3

Give **one** reason why there must be a mistake in the table.

Add all the probabilities:

$$0.1 + 0.5 + 0.2 + 0.3 = 1.1$$

The probabilities of all the outcomes should add up to exactly 1.

Answer: The probabilities add up to 1.1, not 1

Tip: Take care with the adding: $0.1 + 0.5 + 0.2 + 0.3 = 1.1$, not 0.11. Say clearly that the total should be 1.

5 (2 marks)

There are 60 counters in a bag.
The probability that a counter taken at random is red is 0.4
How many of the counters are **not** red?

$$P(\text{not red}) = 1 - 0.4 = 0.6$$

$$\text{Number not red} = 0.6 \times 60$$

$$= 36$$

(Or: red = $0.4 \times 60 = 24$, so not red = $60 - 24 = 36$.)

Answer: 36

Marks: 1 mark for 0.6×60 or $60 - 24$. 1 mark for 36.

6 (2 marks)

Four teams, A , B , C and D , play in a competition.

The probability that A wins is 0.3

The probability that B wins is 0.25

C and D are equally likely to win.

Work out the probability that D wins.

The four probabilities add up to 1.

C and D together: $1 - 0.3 - 0.25 = 0.45$

C and D are equally likely, so share 0.45 equally.

$0.45 \div 2 = 0.225$

Answer: 0.225

Marks: 1 mark for 0.45. 1 mark for 0.225.

7 (5 marks)

A vending machine sells only water, cola, lemonade and juice.

The table shows the probability that the next drink it sells is each type.

- (a) The probability that the drink is lemonade is 0.1 more than the probability that it is juice.

Complete the table.

Lemonade and juice together: $1 - 0.35 - 0.25 = 0.4$

Lemonade is 0.1 more than juice.

Take away the extra 0.1: $0.4 - 0.1 = 0.3$

Share equally: juice = $0.3 \div 2 = 0.15$

Lemonade = $0.15 + 0.1 = 0.25$

Drink	Water	Cola	Lemonade	Juice
Probability	0.35	0.25	0.25	0.15

Answer: Lemonade 0.25, juice 0.15

Tip: Check both facts: $0.25 + 0.15 = 0.4$ **and** $0.25 - 0.15 = 0.1$. Two numbers that just add to 0.4 are not enough.

Marks: 1 mark for $1 - (0.35 + 0.25)$ or 0.4. 1 mark for both values correct.

- (b) One day the machine sells 40 lemonades.

Work out an estimate for the number of waters it sells that day.

$P(\text{lemonade}) = 0.25$, so 0.25 of all the drinks sold is 40

Total drinks sold = $40 \div 0.25 = 160$

Water: $0.35 \times 160 = 56$

Answer: 56

Tip: You know the number of lemonades, so **divide** by 0.25 to find the total first. 0.35×40 uses the wrong total.

Marks: 1 mark for $40 \div 0.25$. 1 mark for 160. 1 mark for 56.

8 (3 marks)

A bag contains red, blue, green and yellow beads.

A bead is picked at random.

The probability of picking green is twice the probability of picking blue.

Colour	Red	Blue	Green	Yellow
Probability	0.2	x	$2x$	0.35

Work out the probability of picking green.

The probabilities add up to 1:

$$0.2 + x + 2x + 0.35 = 1$$

$$\text{Collect terms: } 3x + 0.55 = 1$$

$$\text{Subtract 0.55: } 3x = 0.45$$

$$\text{Divide by 3: } x = 0.15$$

$$P(\text{green}) = 2x = 2 \times 0.15 = 0.3$$

Answer: 0.3

Tip: x is the probability of **blue**. Remember to double it to get green.

Marks: 1 mark for $1 - 0.2 - 0.35$ (0.45). 1 mark for $0.45 \div 3$. 1 mark for 0.3.

9 (3 marks)

Sam puts 20 counters in an empty bag.

Each counter is red, blue, green or yellow, and there is at least one of each colour.

A counter will be taken from the bag at random.

Complete the table to show how many counters of each colour Sam could use so that
the probability of red is 0.3

the probability of blue is twice the probability of green

there are more yellow counters than green counters.

Red: $0.3 \times 20 = 6$ counters.

That leaves $20 - 6 = 14$ counters for blue, green and yellow.

Blue must be twice green. Try green = 2, so blue = 4.

Yellow: $14 - 2 - 4 = 8$, which is more than 2 green ✓

Colour	Red	Blue	Green	Yellow
Number of counters	6	4	2	8

Other correct answers: green 1, blue 2, yellow 11 or green 3, blue 6, yellow 5.

Answer: e.g. Red 6, Blue 4, Green 2, Yellow 8

Tip: Check your table at the end: the numbers must add up to 20, and every condition must be true.

Marks: 1 mark for 6 red. 1 mark for blue = $2 \times$ green. 1 mark for a fully correct table that adds up to 20.

10 (4 marks)

There are 1200 students at a college.

A student is chosen at random.

- (a) The probability that the student is taking a GCSE resit is 0.09
How many of the students are taking a GCSE resit?

Expected number = probability $\times$ number of students

$$0.09 \times 1200 = 108$$

Answer: 108

Marks: 1 mark for 0.09×1200 . 1 mark for 108.

- (b) The probability that the student is studying
A-levels is 0.72
Core Maths is 0.35
Show that some students are studying A-levels **and** Core Maths.

Add the two probabilities: $0.72 + 0.35 = 1.07$

If nobody did both, the total could not be more than 1.

1.07 is more than 1, so the two groups must overlap.

So some students study both.

Another way, using numbers of students:

$$\text{A-levels: } 0.72 \times 1200 = 864$$

$$\text{Core Maths: } 0.35 \times 1200 = 420$$

$864 + 420 = 1284$, which is more than the 1200 students.

Answer: $0.72 + 0.35 = 1.07 > 1$

Tip: Do not stop at 1.07. Explain what it means: it is more than 1, so some students must be in both groups.

Marks: 1 mark for $0.72 + 0.35$ (or 864 and 420). 1 mark for 1.07 (or 1284) with a correct conclusion.

11 (4 marks)

A florist has a bucket of roses, tulips and lilies.

The table shows the number of each type of flower.

Flower	Rose	Tulip	Lily
Number	$2x$	$x + 6$	x

There are 54 flowers in the bucket.

A flower is taken at random.

Work out the probability that it is **not** a lily.

Add up the table and set it equal to 54:

$$2x + x + 6 + x = 54$$

$$4x + 6 = 54$$

$$4x = 48, \text{ so } x = 12$$

$$\text{Lilies: } x = 12$$

$$\text{Not a lily: } 54 - 12 = 42 \text{ flowers}$$

$$P(\text{not a lily}) = \frac{42}{54}$$

Answer: $\frac{42}{54}$ (or $\frac{7}{9}$)

Tip: $x = 12$ is not the answer. Use it to find the number of flowers, then write the probability as a fraction out of 54.

Marks: 1 mark for $2x + x + 6 + x = 54$. 1 mark for $x = 12$. 1 mark for 42 (or 12 lilies). 1 mark for $\frac{42}{54}$ or an equivalent.
