

# Prime Factorisation, HCF and LCM

## Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

**1** (3 marks)

Here is an incomplete factor tree for 132

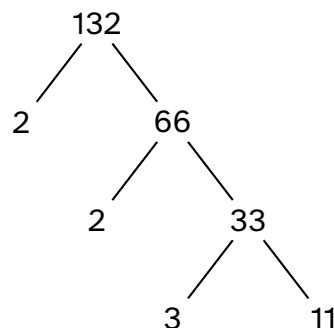
(a) Complete the factor tree for 132

Split 66 into two factors:  $66 = 2 \times 33$

2 is prime, so that branch stops.

33 is not prime, so split it:  $33 = 3 \times 11$

3 and 11 are both prime, so the tree is finished.



Other splits are also correct, e.g.  $66 = 3 \times 22$  then  $22 = 2 \times 11$

**Answer:** 2, 33, 3, 11 (or another correct split)

*Marks:* 2 marks for all four correct. 1 mark for one level correct (2 and 33, or 3 and 11 following from your split).

(b) Write 132 as a product of its prime factors.

Multiply together the primes at the ends of the branches: 2, 2, 3 and 11.

$$132 = 2 \times 2 \times 3 \times 11$$

In index form:  $2^2 \times 3 \times 11$

**Answer:**  $2^2 \times 3 \times 11$

**Tip:** “Product” means multiply. Write  $2 \times 2 \times 3 \times 11$ , not a list (2, 2, 3, 11) or a sum.

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**2** (2 marks)

Write 280 as a product of prime factors.

Give your answer in index form.

Keep dividing by prime numbers, starting with 2:

$$280 = 2 \times 140$$

$$140 = 2 \times 70$$

$$70 = 2 \times 35$$

$$35 = 5 \times 7$$

$$\text{So } 280 = 2 \times 2 \times 2 \times 5 \times 7$$

In index form:  $2^3 \times 5 \times 7$

**Answer:**  $2^3 \times 5 \times 7$

*Marks:* 1 mark for a correct method, e.g. a factor tree with at least two correct steps. 1 mark for  $2^3 \times 5 \times 7$ .

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**3** (1 mark)

Circle the product of prime factors of 100

Options:  $4 \times 25$ ,  $2 \times 5 \times 10$ ,  $2^2 \times 5^2$ ,  $2 \times 50$

All the options multiply to 100, but the numbers must all be prime.

4, 25, 10 and 50 are not prime.

$2^2 \times 5^2 = 2 \times 2 \times 5 \times 5$  uses only the primes 2 and 5.

**Answer:**  $2^2 \times 5^2$

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**4** (4 marks)

- (a) Work out the highest common factor (HCF) of 36 and 90

Write each number as a product of primes:

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

Take the primes that are in **both**: one 2 and two 3s.

$$\text{HCF} = 2 \times 3 \times 3 = 18$$

$$\text{Check: } 36 \div 18 = 2 \text{ and } 90 \div 18 = 5 \quad \checkmark$$

**Answer:** 18

**Tip:** The HCF divides into both numbers, so it cannot be bigger than the smaller number. A common factor like 2, 3, 6 or 9 is not enough: you need the **highest**.

*Marks:* 1 mark for a correct method, e.g. prime factors of both numbers or lists of factors.  
1 mark for 18.

- (b) Work out the lowest common multiple (LCM) of 14 and 21

List the multiples of 21: 21, 42, 63, ...

Check each one: is it in the 14 times table?

21 is not, but  $42 = 14 \times 3$  is.

Or use primes:  $14 = 2 \times 7$  and  $21 = 3 \times 7$

$$\text{LCM} = 2 \times 3 \times 7 = 42$$

**Answer:** 42

**Tip:** The LCM is a multiple of both numbers, so it cannot be smaller than the bigger number. Do not mix it up with the HCF.

*Marks:* 1 mark for listing multiples of both numbers, or for their prime factors. 1 mark for 42.

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**5** (2 marks)

$$A = 2^3 \times 5 \qquad B = 2^2 \times 3 \times 5^2$$

- (a) Work out the HCF of  $A$  and  $B$ .  
Give your answer in index form.

For the HCF, use only the primes that are in both  $A$  and  $B$ .

2 is in both: the lower power is  $2^2$ .

5 is in both: the lower power is 5 (that is,  $5^1$ ).

3 is only in  $B$ , so leave it out.

$$\text{HCF} = 2^2 \times 5$$

**Answer:**  $2^2 \times 5$  (= 20)

- (b) Work out the LCM of  $A$  and  $B$ .  
Give your answer in index form.

For the LCM, use every prime that appears in either number.

Use the higher power of each one:

$$2: 2^3$$

$$3: 3$$

$$5: 5^2$$

$$\text{LCM} = 2^3 \times 3 \times 5^2$$

**Answer:**  $2^3 \times 3 \times 5^2$  (= 600)

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**6** (3 marks)

A café oven bakes scones and muffins.

A tray of scones comes out of the oven every 25 minutes.

A tray of muffins comes out of the oven every 40 minutes.

At 7:30 am a tray of scones and a tray of muffins come out together.

The oven is switched off at 4 pm.

How many more times will a tray of scones and a tray of muffins come out of the oven together before 4 pm?

Scones and muffins come out together after a number of minutes that is a multiple of 25 **and** of 40.

So find the LCM of 25 and 40.

Multiples of 40: 40, 80, 120, 160, 200, ...

200 is the first one that is also a multiple of 25 ( $25 \times 8 = 200$ ).

The LCM is 200 minutes = 3 hours 20 minutes.

Times together: 7:30 am  $\rightarrow$  10:50 am  $\rightarrow$  2:10 pm  $\rightarrow$  5:30 pm

5:30 pm is after 4 pm, so only 10:50 am and 2:10 pm count.

**Answer:** 2 times (10:50 am and 2:10 pm)

**Tip:** Things that happen together again are LCM questions. Change the LCM from minutes into hours and minutes before adding it on: 200 minutes is 3 hours 20 minutes, not 2 hours.

**Marks:** 1 mark for 200 minutes (the LCM of 25 and 40). 1 mark for 10:50 am or 2:10 pm. 1 mark for 2.

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**7** (2 marks)

Lucy has 36 lollipops and 48 toffees.

She wants to put them into bags so that each bag has the same number of lollipops and the same number of toffees.

Work out the largest number of identical bags she can make with no sweets left over.

The number of bags must divide exactly into 36 **and** into 48.

So find the HCF of 36 and 48.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

The highest factor in both lists is 12.

Each bag has  $36 \div 12 = 3$  lollipops and  $48 \div 12 = 4$  toffees.

**Answer:** 12 bags

**Tip:** Sharing things into equal groups with none left over is an HCF question.

*Marks:* 1 mark for a method to find the HCF, e.g. lists of factors of 36 and 48. 1 mark for 12.

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**8** (1 mark)

Kai says,

‘120 written as a product of its prime factors is  $2^2 \times 3 \times 5$ ’

Is Kai correct? Tick a box.

Give a reason for your answer.

All of 2, 3 and 5 are prime, so check that the product really is 120.

$2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60$ , not 120.

Kai has one 2 missing:  $120 = 2 \times 60$

The correct answer is  $120 = 2^3 \times 3 \times 5$

**Answer:** No,  $2^2 \times 3 \times 5 = 60$  ( $120 = 2^3 \times 3 \times 5$ )

**Tip:** Always multiply your primes back together to check you get the number you started with.

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**9** (2 marks)

$$720 = 2^a \times 3^b \times 5$$

Work out the value of  $a$  and the value of  $b$ .

Write 720 as a product of primes:

$$720 = 2 \times 360$$

$$360 = 2 \times 180$$

$$180 = 2 \times 90$$

$$90 = 2 \times 45$$

$$45 = 3 \times 15$$

$$15 = 3 \times 5$$

$$\text{So } 720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^4 \times 3^2 \times 5$$

Compare with  $2^a \times 3^b \times 5$ :  $a = 4$  and  $b = 2$

**Answer:**  $a = 4$ ,  $b = 2$

*Marks:* 1 mark for  $a = 4$ . 1 mark for  $b = 2$ .

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**10** (3 marks)

$$756 = 2^2 \times 3^3 \times 7$$

Work out the highest common factor (HCF) of 756 and 1080

First write 1080 as a product of primes:

$$1080 = 2 \times 540 = 2 \times 2 \times 270 = 2 \times 2 \times 2 \times 135$$

$$135 = 3 \times 45 = 3 \times 3 \times 15 = 3 \times 3 \times 3 \times 5$$

$$\text{So } 1080 = 2^3 \times 3^3 \times 5$$

For the HCF, use the primes that are in both numbers, with the lower power.

2:  $2^2$  and  $2^3$ , so use  $2^2$

3:  $3^3$  and  $3^3$ , so use  $3^3$

7 is only in 756 and 5 is only in 1080, so leave them out.

$$\text{HCF} = 2^2 \times 3^3 = 4 \times 27 = 108$$

**Answer:** 108

**Tip:** For the HCF, use the **lower** power of each prime that is in both. For the LCM, use the **higher** power of every prime.

*Marks:* 1 mark for  $1080 = 2^3 \times 3^3 \times 5$ . 1 mark for  $2^2 \times 3^3$ . 1 mark for 108.

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**11** (4 marks)

$N$  is a whole number.

$N$  is a multiple of both 6 and 15

$N$  is also a factor of both 180 and 450

Work out **all** the possible values of  $N$ .

$N$  is a multiple of 6 and of 15, so it is a multiple of their LCM.

$$6 = 2 \times 3 \text{ and } 15 = 3 \times 5, \text{ so } \text{LCM} = 2 \times 3 \times 5 = 30$$

So  $N$  is one of 30, 60, 90, 120, ...

$N$  divides exactly into 180 and into 450, so it is a factor of their HCF.

$$180 = 2^2 \times 3^2 \times 5 \text{ and } 450 = 2 \times 3^2 \times 5^2$$

$$\text{HCF} = 2 \times 3^2 \times 5 = 90$$

Which multiples of 30 are factors of 90?

$$90 \div 30 = 3 \quad \checkmark \quad 90 \div 60 = 1.5 \text{ (not a factor)} \quad 90 \div 90 = 1 \quad \checkmark$$

Multiples of 30 bigger than 90 cannot be factors of 90.

So  $N = 30$  or  $N = 90$

**Answer:** 30 and 90

**Tip:** Read each condition carefully: a number that is a **multiple** of both 6 and 15 must be a multiple of their LCM, and a number that is a **factor** of both 180 and 450 must be a factor of their HCF.

**Marks:** 1 mark for 30 (the LCM of 6 and 15). 1 mark for 90 (the HCF of 180 and 450). 1 mark for one correct value of  $N$ . 1 mark for both 30 and 90 and no others.