

nth Term of Linear Sequences

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

The n th term of a sequence is $5n - 2$

(a) Work out the 1st term.

Substitute $n = 1$ into $5n - 2$.

$$5 \times 1 - 2 = 3$$

Answer: 3

(b) Work out the 10th term.

Substitute $n = 10$ into $5n - 2$.

$$5 \times 10 - 2 = 50 - 2 = 48$$

Answer: 48

(c) Is 100 a term in the sequence? Tick a box.
Give a reason for your answer.

If 100 is a term, then $5n - 2 = 100$.

Add 2: $5n = 102$

Divide by 5: $n = 20.4$

n must be a whole number (a position in the sequence), so 100 is not a term.

(Also: the 20th term is 98 and the 21st term is 103.)

Answer: No, because $5n - 2 = 100$ gives $n = 20.4$

2 (2 marks)

A linear sequence starts 8 14 20 26 ...

Work out an expression for the n th term of the sequence.

The terms go up by 6 each time.

So the n th term starts with $6n$.

Write out the $6n$ sequence: 6, 12, 18, 24

Each term of our sequence is 2 more than this.

So the n th term is $6n + 2$.

Check with $n = 2$: $6 \times 2 + 2 = 14$ ✓

Answer: $6n + 2$

Tip: The difference goes in front of n . "Add 6" gives $6n$, not $n + 6$.

Marks: 2 marks for $6n + 2$. 1 mark for $6n$ (the number in front of n correct).

3 (2 marks)

A linear sequence starts 9 13 17 21 ...

Work out an expression for the n th term of the sequence.

The terms go up by 4 each time.

So the n th term starts with $4n$.

Write out the $4n$ sequence: 4, 8, 12, 16

Each term of our sequence is 5 more than this.

So the n th term is $4n + 5$.

Check with $n = 2$: $4 \times 2 + 5 = 13$ ✓

Answer: $4n + 5$

Marks: 2 marks for $4n + 5$. 1 mark for $4n$ (the number in front of n correct).

4 (2 marks)

A linear sequence starts 20 16 12 8 ...

Work out an expression for the n th term of the sequence.

The terms go down by 4 each time, so the difference is -4 .

So the n th term starts with $-4n$.

Write out the $-4n$ sequence: $-4, -8, -12, -16$

Each term of our sequence is 24 more than this.

So the n th term is $-4n + 24$, which is the same as $24 - 4n$.

Check with $n = 2$: $-4 \times 2 + 24 = 16$ ✓

Answer: $24 - 4n$

Tip: The sequence goes **down**, so the number in front of n is negative ($-4n$). Writing $n - 4$ is a common wrong answer.

Marks: 2 marks for $24 - 4n$. 1 mark for $-4n$ (the number in front of n correct).

5 (1 mark)

Circle the expression for the n th term of the sequence 2 7 12 17 ...

Options: $n + 5$, $5n + 2$, $5n - 3$, $2n + 5$

The terms go up by 5, so the n th term starts with $5n$.

The $5n$ sequence is 5, 10, 15, 20.

Each term is 3 less than this, so it is $5n - 3$.

Check: $n = 1$ gives $5 - 3 = 2$ ✓

Answer: $5n - 3$

6 (3 marks)

Here is part of a linear sequence. The 1st and 2nd terms are missing.

____ ____ 19 26 33 ...

Work out the 40th term of the sequence.

19 is the 3rd term, 26 is the 4th term and 33 is the 5th term.

The difference is $26 - 19 = 7$, so the n th term starts with $7n$.

The 3rd term: $7 \times 3 = 21$, but the 3rd term is 19.

$19 - 21 = -2$, so subtract 2.

The n th term is $7n - 2$.

Check the 5th term: $7 \times 5 - 2 = 33$ ✓

40th term: $7 \times 40 - 2 = 280 - 2 = 278$

Answer: 278

Tip: 19 is the **3rd** term, not the 1st term. Writing $7n + 12$ (as if 19 were the 1st term) is a common mistake.

Marks: 1 mark for the difference of 7. 1 mark for $7n - 2$ (or $33 + 35 \times 7$). 1 mark for 278.

7 (3 marks)

Sequence A has n th term $5n + 2$

Sequence B starts 4 7 10 13 16 ...

Work out the smallest number greater than 100 that is in both sequences.

Sequence A: 7, 12, 17, 22, 27, 32, 37, ...

Sequence B goes up by 3: 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, ...

The numbers in both sequences are 7, 22, 37, ...

These go up by 15 each time: 7, 22, 37, 52, 67, 82, 97, 112, ...

The smallest one greater than 100 is 112.

Check: $5 \times 22 + 2 = 112$, and $112 - 4 = 108$ is a multiple of 3, so 112 is in sequence B ✓

Answer: 112

Tip: 97 is in both sequences but it is not greater than 100. Check that your answer meets every condition.

Marks: 1 mark for listing terms of both sequences. 1 mark for finding common numbers, e.g. 7, 22, 37. 1 mark for 112.

8 (3 marks)

Patterns are made from sticks.

Pattern number 1 uses 4 sticks, pattern number 2 uses 7 sticks and pattern number 3 uses 10 sticks.

- (a) Write down an expression for the number of sticks in pattern number n .

The numbers of sticks are 4, 7, 10, ...

They go up by 3 each time, so the n th term starts with $3n$.

The $3n$ sequence is 3, 6, 9.

Each term is 1 more than this, so it is $3n + 1$.

Answer: $3n + 1$

Marks: 2 marks for $3n + 1$. 1 mark for $3n$.

- (b) Can a pattern be made with exactly 150 sticks? Tick a box.
Give a reason for your answer.

Solve $3n + 1 = 150$.

Subtract 1: $3n = 149$

Divide by 3: $n = 49.66 \dots$

This is not a whole number, so no pattern has exactly 150 sticks.

Answer: No, $(150 - 1) \div 3$ is not a whole number

9 (1 mark)

Ellie is finding the n th term of 3 9 15 21

She writes $3n + 6$

What mistake has Ellie made?

The terms go up by 6, so the n th term should start with $6n$, not $3n$.

Ellie has used the first term (3) instead of the difference (6).

The correct n th term is $6n - 3$.

Check: $6 \times 1 - 3 = 3$ ✓

Answer: She used the first term (3) instead of the common difference (6); it should be $6n - 3$.

10 (2 marks)

The n th term of a sequence is $7n + 3$

Work out the first term of the sequence that is greater than 200

We need $7n + 3 > 200$.

Subtract 3: $7n > 197$

Divide by 7: $n > 28.1 \dots$

n must be a whole number, so try $n = 28$ and $n = 29$.

$n = 28$: $7 \times 28 + 3 = 199$ (not greater than 200)

$n = 29$: $7 \times 29 + 3 = 206$

Answer: 206

Tip: This question asks for the **term** (206), not the value of n (29). Read carefully what the question wants.

Marks: 1 mark for $7n + 3 > 200$ or trying values of n . 1 mark for 206.

11 (2 marks)

The n th term of a sequence is $40 - 3n$

How many terms of the sequence are positive?

The terms are 37, 34, 31, ... (going down by 3).

A term is positive when $40 - 3n > 0$, so $3n < 40$.

$40 \div 3 = 13.3 \dots$, so n can be 1, 2, 3, ..., 13.

Check: $n = 13$ gives $40 - 39 = 1$ (positive) and $n = 14$ gives $40 - 42 = -2$ (negative).

So 13 terms are positive.

Answer: 13

Tip: The answer is the number of terms (13), not the last positive term (1). Check the terms either side to be sure.

Marks: 1 mark for $40 - 3n > 0$, $40 \div 3$ or finding the 13th and 14th terms. 1 mark for 13.

12 (4 marks)

A linear sequence has

- 1st term = 7
- 1st term + 2nd term = 25

(a) Work out an expression for the n th term of the sequence.

2nd term: $25 - 7 = 18$

Common difference: $18 - 7 = 11$

So the n th term starts with $11n$.

When $n = 1$, $11n = 11$, but the 1st term is 7

$7 - 11 = -4$

n th term = $11n - 4$

Check: $n = 2$ gives $22 - 4 = 18$, which is correct.

Answer: $11n - 4$

Tip: 25 is the **sum** of the first two terms, not the 2nd term. Subtract to find the 2nd term:
 $25 - 7 = 18$.

Marks: 1 mark for 18 (the 2nd term). 1 mark for $11n$. 1 mark for $11n - 4$.

(b) Which term of the sequence is equal to 150?

Solve $11n - 4 = 150$

$$11n = 154$$

$$n = 154 \div 11 = 14$$

So 150 is the 14th term.

Answer: the 14th term

Tip: The answer is the **position** n , not the value of the term.
