

Inequalities on a Number Line

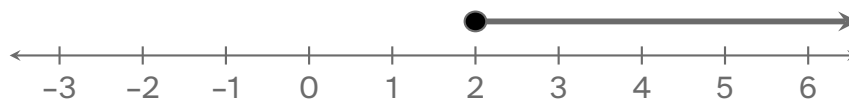
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here is a number line.



Circle the inequality shown.

Options: $x > 2$, $x \geq 2$, $x < 2$, $x \leq 2$

The circle at 2 is filled in, so 2 is included.

The arrow points to the right, so x is greater than 2.

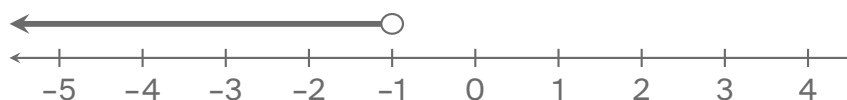
Greater than or equal to 2: $x \geq 2$

Answer: $x \geq 2$

Tip: Filled circle: the number **is** included ($\leq$ or $\geq$). Open circle: the number is **not** included ($<$ or $>$).

2 (1 mark)

Here is a number line.



Circle the inequality shown.

Options: $x > -1$, $x \geq -1$, $x < -1$, $x \leq -1$

The circle at -1 is open, so -1 is not included.

The arrow points to the left, so x is less than -1 .

So $x < -1$

Answer: $x < -1$

3 (3 marks)

- (a) A minibus can carry no more than 16 passengers.

p is the number of passengers on the minibus.

Circle the inequality for p .

Options: $p < 16$, $p \leq 16$, $p > 16$, $p \geq 16$

“No more than 16” means 16 or fewer.

So 16 is allowed, but nothing bigger.

This is $p \leq 16$.

Answer: $p \leq 16$

Tip: “No more than” and “at most” both mean $\leq$. “At least” means $\geq$.

- (b) The temperature inside a freezer is always below -18°C .

t is the temperature, in $^\circ\text{C}$, inside the freezer.

Circle the inequality for t .

Options: $t < -18$, $t \leq -18$, $t > -18$, $t \geq -18$

“Below -18 ” means colder than -18 , so less than -18 .

-18 itself is not below -18 , so it is not included.

This is $t < -18$.

Answer: $t < -18$

- (c) A youth club is open to people aged from 11 to 16 inclusive.

a is the age, in years, of a member.

Circle the inequality for a .

Options: $11 < a < 16$, $11 \leq a < 16$, $11 \leq a \leq 16$, $11 < a \leq 16$

“Inclusive” means both end ages are allowed.

So 11 is included: $11 \leq a$

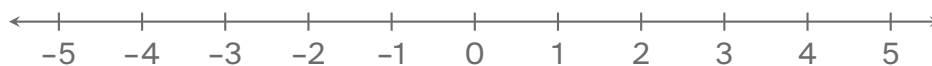
And 16 is included: $a \leq 16$

Put them together: $11 \leq a \leq 16$

Answer: $11 \leq a \leq 16$

4 (1 mark)

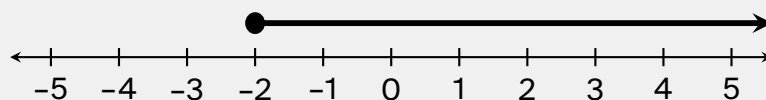
Represent $x \geq -2$ on the number line.



-2 is included, so draw a filled circle at -2.

x is greater than -2, so draw an arrow to the right.

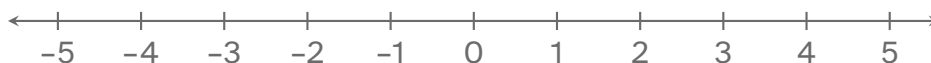
Answer:



Tip: Draw a circle at -2 **and** an arrow. An arrow or a line on its own does not show whether -2 is included.

5 (1 mark)

Represent $-4 < x < 2$ on the number line.

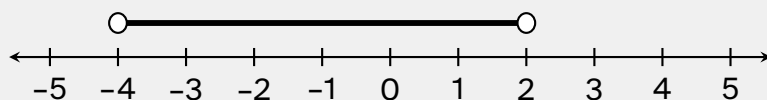


Neither -4 nor 2 is included (both signs are $<$).

So draw open circles at -4 and at 2 .

Join the circles with a line.

Answer:



Tip: Put the circles exactly on -4 and 2 , not just before them. Do not draw arrows at the ends.

6 (2 marks)

Here is a number line.



Write down the inequality represented by the number line.

Open circle at -2 , so -2 is not included: $-2 < x$

Filled circle at 3 , so 3 is included: $x \leq 3$

Put them together, with x in the middle.

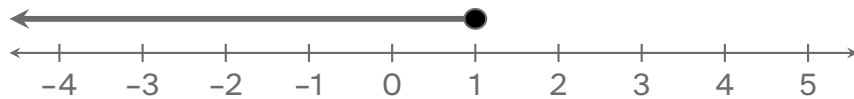
Answer: $-2 < x \leq 3$

Tip: Write the answer as one inequality with x in the middle. Do not just list the integers.

Marks: 2 marks for $-2 < x \leq 3$. 1 mark if one end is correct.

7 (2 marks)

Here is a number line.



- (a) Write down the inequality shown on the number line.

The circle at 1 is filled in, so 1 is included.

The arrow points left, so x is less than 1.

So $x \leq 1$

Answer: $x \leq 1$

- (b) Write down the largest integer that satisfies this inequality.

The circle at 1 is filled in, so 1 is included.

No integer bigger than 1 is allowed.

Answer: 1

8 (1 mark)

Circle the value of x that satisfies $5 < x < 6$

Options: 5, 5.9, 6, 6.1

x must be bigger than 5 and smaller than 6.

x cannot be 5 or 6 themselves.

5.9 is the only option between 5 and 6.

Answer: 5.9

9 (2 marks)

Tick **one** box for each statement.

	True	False
$-4 < -2$	✓	
$0 > -1$	✓	
$-5 \geq -3$		✓
$3 \leq 3$	✓	

On a number line, numbers further to the left are smaller.

-4 is to the left of -2 , so $-4 < -2$ is true.

0 is to the right of -1 , so $0 > -1$ is true.

-5 is to the left of -3 , so -5 is smaller. $-5 \geq -3$ is false.

$\leq$ includes “equal to”, so $3 \leq 3$ is true.

Marks: 2 marks for all four correct. 1 mark for two or three correct.

10 (1 mark)

Write down all the integers that satisfy the inequality $0 < n \leq 4$

$0 < n$ means 0 is not included.

$n \leq 4$ means 4 is included.

So the integers are $1, 2, 3$ and 4 .

Answer: 1, 2, 3, 4

11 (2 marks)

x is an integer.

$$x > -5 \quad \text{and} \quad x \leq -1$$

Write down all the possible values of x .

$x > -5$ means -5 is not included, so x starts at -4 .

$x \leq -1$ means -1 is included.

Together: $-5 < x \leq -1$

So the integers are $-4, -3, -2$ and -1 .

Answer: $-4, -3, -2, -1$

Tip: x must satisfy **both** inequalities. Sketch them on a number line and look for where they overlap.

Marks: 2 marks for all four and no others. 1 mark for three correct and none wrong, or all four with one extra.

12 (1 mark)

The integers $-2, -1, 0, 1$ and 2 are the **only** integers that satisfy an inequality. Circle the inequality.

Options: $-2 < x < 2$, $-3 < x \leq 2$, $-3 \leq x \leq 2$, $-2 < x \leq 3$

Test each option.

$-2 < x < 2$ gives $-1, 0, 1$ only (no -2 or 2).

$-3 < x \leq 2$ gives $-2, -1, 0, 1, 2$. This one works.

$-3 \leq x \leq 2$ also includes -3 .

$-2 < x \leq 3$ leaves out -2 and includes 3 .

Answer: $-3 < x \leq 2$

13 (1 mark)

Zak says,

‘There are 5 integers that satisfy $-1 \leq x < 3$ ’

Is Zak correct? Tick a box.

Give a reason for your answer.

-1 is included, because the sign is $\leq$.

3 is not included, because the sign is $<$.

So the integers are $-1, 0, 1, 2$.

That is only 4 integers, not 5.

Answer: No. The integers are $-1, 0, 1, 2$ (four of them). 3 is not included.

14 (1 mark)

Dan has solved the inequality $3x > 12$ and shown his solution on the number line below.



Give **one** reason why Dan's number line is wrong.

Divide both sides of $3x > 12$ by 3: $x > 4$

The sign is $>$, so 4 is not included.

So the circle at 4 should be open, not filled in.

Answer: The circle at 4 should be open, because $x > 4$ does not include 4.

Tip: Be specific. Say **which** circle is wrong and how: "the circle at 4 should be open".

15 (2 marks)

n is an integer.

$$-6 < 2n \leq 4$$

Work out all the possible values of n .

Divide all three parts by 2:

$$-6 \div 2 = -3 \text{ and } 4 \div 2 = 2$$

$$\text{So } -3 < n \leq 2$$

-3 is not included, 2 is included.

$$\text{So } n = -2, -1, 0, 1, 2$$

Answer: $-2, -1, 0, 1, 2$

Tip: The inequality is about $2n$, not n . Divide by 2 first, then list the values of n .

Marks: 1 mark for $-3 < n \leq 2$. 1 mark for all five values.

16 (3 marks)

Here are two inequalities.

$$-4 \leq a < 2$$

$$1 < b \leq 6$$

a and b are integers.

Work out the smallest possible value of ab

To make ab as small as possible, make it negative and as large in size as possible.

a can be negative, but b is always positive.

The most negative value of a is -4 (-4 is included).

$b \leq 6$, so the largest value of b is 6 .

$$ab = -4 \times 6 = -24$$

Answer: -24

Tip: The smallest value is the most negative one. Do not just use the smallest value of each letter: $-4 \times 2 = -8$, but $-4 \times 6 = -24$ is smaller.

Marks: 1 mark for $a = -4$. 1 mark for $b = 6$. 1 mark for -24 .
