

Identities and Algebraic Arguments

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

$5(x + 2) \equiv 5x + 10$ is an identity.

Tick **one** box.

Tick

It is true for all values of x

✓

It is true for some values of x

It is true for no values of x

The $\equiv$ sign means “is identical to”.

Expanding $5(x + 2)$ always gives $5x + 10$, whatever x is.

So an identity is true for **all** values of x .

Tip: An equation (like $5x = 10$) is true for only some values of x . An identity is true for every value.

2 (3 marks)

Tick **one** box for each row.

	Identity	Equation
$2(x + 4) = 2x + 8$	✓	
$2x + 4 = 10$		✓
$5x - x = 4x$	✓	
$x^2 = 9$		✓

$2(x + 4)$ expands to $2x + 8$, so it is true for all x . Identity

$2x + 4 = 10$ is only true when $x = 3$. Equation

$5x - x$ simplifies to $4x$, so it is true for all x . Identity

$x^2 = 9$ is only true when $x = 3$ or $x = -3$. Equation

Marks: 3 marks for all four correct. 2 marks for three correct. 1 mark for two correct.

3 (2 marks)

Show that $4(2x + 3) - 2(x - 1) \equiv 2(3x + 7)$

Expand the left-hand side:

$$4(2x + 3) = 8x + 12$$

$$-2(x - 1) = -2x + 2$$

So the left side is $8x + 12 - 2x + 2 = 6x + 14$.

Expand the right-hand side: $2(3x + 7) = 6x + 14$

Both sides are $6x + 14$, so the identity is true.

Tip: $-2 \times (-1) = +2$. Multiplying the bracket by -2 changes the sign of **both** terms inside.

Marks: 1 mark for expanding the left side correctly. 1 mark for showing both sides are $6x + 14$.

4 (2 marks)

Here is an identity.

$$12x - 30 \equiv a(bx - 5)$$

Work out the values of a and b .

Expand the right-hand side: $a(bx - 5) = abx - 5a$

So $12x - 30 \equiv abx - 5a$

Compare the numbers: $-5a = -30$, so $a = 6$

Compare the x -terms: $ab = 12$

With $a = 6$: $6b = 12$, so $b = 2$

Check: $6(2x - 5) = 12x - 30$ ✓

Answer: $a = 6$, $b = 2$

Tip: Start with the number terms: $-5a$ must equal -30 . Then use your value of a to match the x -terms.

Marks: 1 mark for $a = 6$. 1 mark for $b = 2$ (follow-through from your a).

5 (3 marks)

Here is an identity.

$$a(x + 2) + 5(x - b) \equiv 8x - 4$$

Work out the values of a and b .

Expand both brackets:

$$a(x + 2) = ax + 2a$$

$$5(x - b) = 5x - 5b$$

Add: $ax + 5x + 2a - 5b$

Compare the x -terms: $a + 5 = 8$, so $a = 3$

Compare the numbers: $2a - 5b = -4$

With $a = 3$: $6 - 5b = -4$

So $5b = 10$ and $b = 2$

Answer: $a = 3, b = 2$

Tip: The x -terms are ax and $5x$, so the coefficient of x is $a + 5$, not a . Find a first, then use it to find b .

Marks: 1 mark for expanding both brackets. 1 mark for $a = 3$. 1 mark for $b = 2$.

6 (3 marks)

Show that $(x + 3)^2 - (x - 3)^2 \equiv 12x$

$$(x + 3)^2 = (x + 3)(x + 3) = x^2 + 6x + 9$$

$$(x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9$$

Subtract: $(x^2 + 6x + 9) - (x^2 - 6x + 9)$

$$= x^2 + 6x + 9 - x^2 + 6x - 9$$

$$= 12x, \text{ as required.}$$

Tip: $(x + 3)^2$ is not $x^2 + 9$. Write it as $(x + 3)(x + 3)$ and multiply out all four parts.

Marks: 1 mark for one correct expansion. 1 mark for both. 1 mark for a complete argument.

7 (4 marks)

n is an integer.

(a) Show that the sum of n and the next two integers is always a multiple of 3

The next two integers are $n + 1$ and $n + 2$.

$$\text{Sum: } n + (n + 1) + (n + 2) = 3n + 3$$

$$\text{Factorise: } 3n + 3 = 3(n + 1)$$

$n + 1$ is an integer, so $3(n + 1)$ is 3 times an integer.

So the sum is always a multiple of 3.

Tip: Trying a few numbers does not prove it is **always** true. Use algebra, and finish with a sentence.

Marks: 1 mark for $3n + 3$. 1 mark for $3(n + 1)$ with a conclusion.

- (b) Show that the sum of the integer before n and the integer after n is always even.

The integer before n is $n - 1$.

The integer after n is $n + 1$.

Sum: $(n - 1) + (n + 1) = 2n$

$2n$ is 2 times an integer, so it is a multiple of 2.

So the sum is always even.

Marks: 1 mark for $n - 1$ and $n + 1$ (or their sum). 1 mark for $2n$ with a conclusion.

8 (2 marks)

Kim thinks of a whole number.

She doubles it and then adds 1

Show that the answer must be an odd number.

Call Kim's number n .

Double it: $2n$

Add 1: $2n + 1$

$2n$ is always even, because it is a multiple of 2.

An even number +1 is always odd.

So the answer must be odd.

Marks: 1 mark for $2n + 1$ (or for saying that doubling always gives an even number). 1 mark for a full explanation.

9 (1 mark)

Priya says,

‘When you add two square numbers, the answer is never a square number.’

Is Priya correct?

Show working to support your answer.

Square numbers are 1, 4, 9, 16, 25, 36, ...

Try adding pairs of them.

$$9 + 16 = 25$$

25 is a square number (5^2).

So one example shows that Priya is wrong.

Answer: No. For example, $9 + 16 = 25$, which is a square number.

Tip: Checking a few pairs that don't work (like $1 + 4 = 5$) does not prove Priya is right. One example that works against her (a counter-example) is enough to show “No”.

10 (1 mark)

Leo says,

$$(a + b)^2 \equiv a^2 + b^2$$

Is Leo correct? Tick a box.

Give a reason for your answer.

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

This is not the same as $a^2 + b^2$.

For example, try $a = 1$ and $b = 2$:

$$(1 + 2)^2 = 3^2 = 9$$

$$1^2 + 2^2 = 1 + 4 = 5$$

9 is not 5, so it is not an identity.

Answer: No. For example, $a = 1$, $b = 2$ gives $(a + b)^2 = 9$ but $a^2 + b^2 = 5$.

11 (3 marks)

Here is the term-to-term rule for a sequence.

Multiply the previous term by 3 and then add 2

The first term of the sequence is n , where n is a whole number.

- (a) Show that the third term is $9n + 8$

2nd term: multiply n by 3 and add 2: $3n + 2$

3rd term: multiply $3n + 2$ by 3 and add 2:

$$3(3n + 2) + 2 = 9n + 6 + 2$$

$$= 9n + 8, \text{ as required.}$$

Tip: Multiply the **whole** of $3n + 2$ by 3: $3(3n + 2) = 9n + 6$. Writing $9n + 2$ is a common slip.

Marks: 1 mark for the 2nd term $3n + 2$. 1 mark for $3(3n + 2) + 2 = 9n + 8$.

- (b) Show that the difference between the third term and the first term is always a multiple of 8

$$\text{Third term} - \text{first term} = (9n + 8) - n$$

$$= 8n + 8$$

$$= 8(n + 1), \text{ which is 8 times a whole number.}$$

So the difference is always a multiple of 8.