

Function Machines

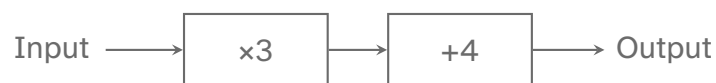
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Here is a number machine.



(a) Work out the output when the input is 5

Go through the machine in order, left to right.

$$5 \times 3 = 15$$

$$15 + 4 = 19$$

Answer: 19

(b) Work out the output when the input is -2

$$-2 \times 3 = -6$$

$$-6 + 4 = -2$$

Answer: -2

(c) Work out the input when the output is 31

Work backwards from the output.

Undo each operation, starting with the last one.

$$\text{Undo } +4: 31 - 4 = 27$$

$$\text{Undo } \times 3: 27 \div 3 = 9$$

Check: $9 \times 3 + 4 = 31$ ✓

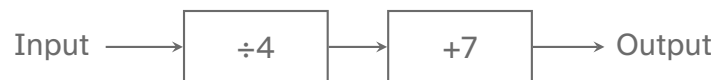
Answer: 9

Tip: Do not put 31 into the machine as the input. Start at the output and use the inverse operations in reverse order.

Marks: 1 mark for $31 - 4$ then $\div 3$. 1 mark for 9.

2 (3 marks)

Here is a number machine.



- (a) Work out the output when the input is 16

$$16 \div 4 = 4$$

$$4 + 7 = 11$$

Answer: 11

- (b) Work out the input when the output is 3

Work backwards from the output.

$$\text{Undo } +7: 3 - 7 = -4$$

$$\text{Undo } \div 4: -4 \times 4 = -16$$

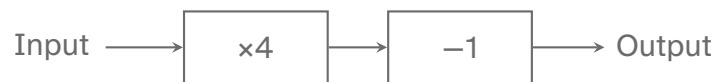
$$\text{Check: } -16 \div 4 = -4 \text{ and } -4 + 7 = 3 \text{ ✓}$$

Answer: -16

Marks: 1 mark for $3 - 7$ then $\times 4$. 1 mark for -16.

3 (2 marks)

Here is a number machine.



Complete the table for this number machine.

Input	1	2	3	6
Output	3	7	11	23

Input 2: $2 \times 4 = 8$, then $8 - 1 = 7$

Input 3: $3 \times 4 = 12$, then $12 - 1 = 11$

For the output 23, work backwards:

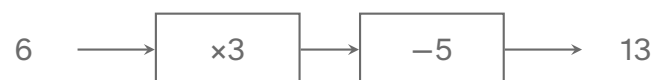
Undo -1 : $23 + 1 = 24$

Undo $\times 4$: $24 \div 4 = 6$

Answer: Outputs 7 and 11. The missing input is 6.

Marks: 1 mark for 7 and 11. 1 mark for 6.

4 (1 mark)



Complete the number machine.

Work backwards from the output, 13.

Undo -5 : $13 + 5 = 18$

So the first box turns 6 into 18.

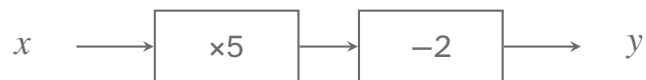
$18 \div 6 = 3$, so the first box is $\times 3$.

Answer: $\times 3$

Tip: Write the operation sign as well as the number: $\times 3$, not just 3.

5 (2 marks)

(a) Write down the output y in terms of x .

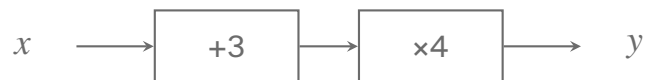


Multiply x by 5: $5x$

Then subtract 2: $5x - 2$

Answer: $y = 5x - 2$

(b) Write down the output y in terms of x .



Add 3 first: $x + 3$

Then multiply **all** of $x + 3$ by 4.

So you need a bracket: $4(x + 3)$

Expanded, this is $4x + 12$.

Answer: $y = 4(x + 3)$ (or $4x + 12$)

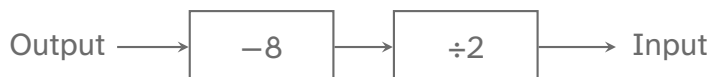
Tip: $x + 3 \times 4$, $x + 12$ and $4x + 3$ are all wrong. The bracket shows that the whole of $x + 3$ is multiplied by 4.

6 (2 marks)

Here is a number machine.



Here is the inverse machine.



Complete the **inverse** machine, which takes the output back to the input.

The inverse machine undoes the operations in reverse order.

The last operation was $+8$, so undo it first with -8 .

Then undo $\times 2$ with $\div 2$.

Answer: Output $\rightarrow -8 \rightarrow \div 2 \rightarrow$ Input

Tip: Changing each operation is not enough. You must also reverse the order, so -8 comes first.

Marks: 1 mark for -8 in the first box. 1 mark for $\div 2$ in the second box.

7 (1 mark)

Here is a number machine.



Kofi says that the formula for this machine is $y = \frac{x+6}{2}$

Explain why Kofi is wrong.

The machine divides by 2 **first**: $x \div 2 = \frac{x}{2}$

Then it adds 6: $y = \frac{x}{2} + 6$

In Kofi's formula, $\frac{x+6}{2}$, the 6 is added **before** dividing by 2.

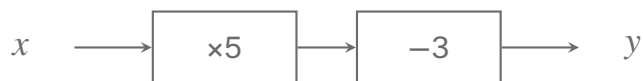
These are different. For example, $x = 10$ gives $y = 11$ from the machine, but $\frac{10 + 6}{2} = 8$.

Answer: The machine divides by 2 before adding 6, so $y = \frac{x}{2} + 6$. His formula adds 6 first.

Tip: Say what the machine does first. “The formula is wrong” on its own does not explain anything.

8 (3 marks)

(a) Complete this number machine so that $y = 5x - 3$

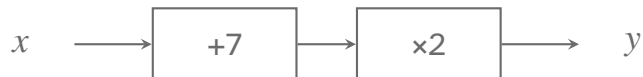


$5x - 3$ means multiply x by 5 first.

Then subtract 3.

Answer: $x \rightarrow \times 5 \rightarrow -3 \rightarrow y$

(b) Complete this number machine so that $y = 2x + 14$



The $\times 2$ comes second, so the output is $2(x + \dots)$.

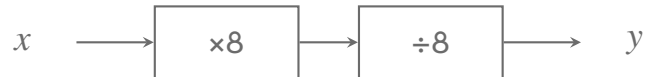
We need $2(x + \dots) = 2x + 14$.

$14 \div 2 = 7$, so the first box is $+7$.

Check: $2(x + 7) = 2x + 14$ ✓

Answer: $+7$

(c) Complete this number machine so that $y = x$



The output must be the same as the input.

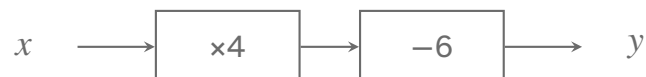
So the second box must undo $\times 8$.

The inverse of $\times 8$ is $\div 8$.

Answer: $\div 8$ (or $\times \frac{1}{8}$)

9 (2 marks)

Here is a number machine.



When $x = 4$, $y = 10$

and when $x = 9$, $y = 30$

Complete the number machine.

The input goes up by $9 - 4 = 5$.

The output goes up by $30 - 10 = 20$.

So each 1 on the input adds $20 \div 5 = 4$ to the output.

This means the machine multiplies by 4.

Input 4: $4 \times 4 = 16$

To get from 16 to 10, subtract 6.

Check with the other pair: $9 \times 4 - 6 = 30$ ✓

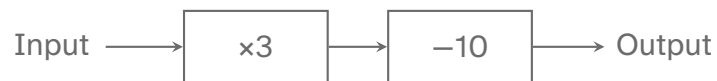
Answer: $\times 4$ then -6

Tip: Your two operations must work for **both** pairs of values. Always check them with the second pair.

Marks: 2 marks for $\times 4$ and -6 . 1 mark for operations that work for just one pair, e.g. $\times 3$ and -2 (which works only for $x = 4$).

10 (2 marks)

Here is a number machine.



Work out the input that gives an output equal to the input.

Call the input x .

The output is $3x - 10$.

Output = input, so $3x - 10 = x$

Subtract x from both sides: $2x - 10 = 0$

Add 10 to both sides: $2x = 10$

Divide both sides by 2: $x = 5$

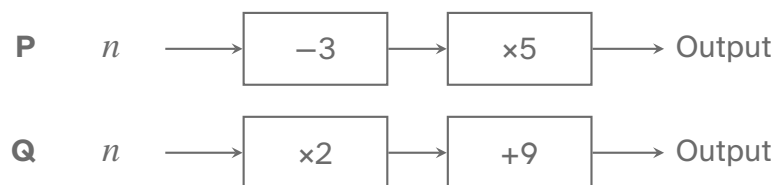
Check: $5 \times 3 - 10 = 5$ ✓

Answer: 5

Marks: 1 mark for $3x - 10 = x$ (or a correct trial). 1 mark for 5.

11 (4 marks)

Here are two number machines, P and Q.



The same number, n , is put into both machines.

The two outputs are equal.

Work out the value of n .

Machine P subtracts 3, then multiplies by 5: $5(n - 3)$

Machine Q gives $2n + 9$.

The outputs are equal, so $5(n - 3) = 2n + 9$

Expand the bracket: $5n - 15 = 2n + 9$

Subtract $2n$ from both sides: $3n - 15 = 9$

Add 15 to both sides: $3n = 24$

Divide both sides by 3: $n = 8$

Check: P gives $5 \times 5 = 25$ and Q gives $16 + 9 = 25$ ✓

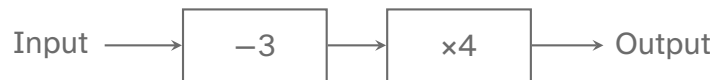
Answer: $n = 8$

Tip: Machine P subtracts 3 **before** multiplying, so its output is $5(n - 3)$, not $5n - 3$.

Marks: 1 mark for $5(n - 3)$ or $2n + 9$. 1 mark for $5(n - 3) = 2n + 9$. 1 mark for $3n = 24$. 1 mark for 8.

12 (4 marks)

Here is a number machine.



Rhian puts a number into the machine.

The output is 10 more than double the number she put in.

Work out the number Rhian put in.

Call Rhian's number x .

The machine subtracts 3, then multiplies by 4: $\text{output} = 4(x - 3)$

Double the number, add 10: $2x + 10$

These are equal, so $4(x - 3) = 2x + 10$

Expand the bracket: $4x - 12 = 2x + 10$

Subtract $2x$ from both sides: $2x - 12 = 10$

Add 12 to both sides: $2x = 22$

Divide both sides by 2: $x = 11$

Check: the machine gives $4 \times 8 = 32$, and $2 \times 11 + 10 = 32$ ✓

Answer: 11

Tip: "10 more than double the number" is $2x + 10$, not $2(x + 10)$. Double first, then add 10.

Marks: 1 mark for $4(x - 3)$ or $2x + 10$. 1 mark for $4(x - 3) = 2x + 10$. 1 mark for $4x - 12 = 2x + 10$ or $2x = 22$. 1 mark for 11.