

Factorising Quadratics

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Factorise $x^2 + 6x + 8$

Find two numbers that multiply to make +8 and add to make +6.

Factor pairs of 8: 1×8 , 2×4

$$2 + 4 = 6 \quad \checkmark$$

So the numbers are 2 and 4.

$$\text{Check by expanding: } (x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8 \quad \checkmark$$

Answer: $(x + 2)(x + 4)$

Tip: The answer must be two brackets multiplied together. Do not try to collect x^2 , $6x$ and 8: they are not like terms.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 8 or add to 6. 1 mark for $(x + 2)(x + 4)$.

2 (2 marks)

Factorise $x^2 + 9x + 14$

Find two numbers that multiply to make 14 and add to make 9.

Factor pairs of 14: 1×14 , 2×7

$$2 + 7 = 9 \quad \checkmark$$

So the numbers are 2 and 7.

Answer: $(x + 2)(x + 7)$

Tip: The numbers in the brackets are not the 9 and the 14 from the question. $(x + 9)(x + 14)$ would expand to $x^2 + 23x + 126$.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 14 or add to 9. 1 mark for $(x + 2)(x + 7)$.

3 (1 mark)

Circle the expression that is equivalent to $x^2 - 2x - 15$

Options: $(x - 5)(x + 3)$, $(x + 5)(x - 3)$, $(x - 15)(x + 1)$, $(x - 5)(x - 3)$

Find two numbers that multiply to make -15 and add to make -2 .

$$-5 \times 3 = -15 \text{ and } -5 + 3 = -2 \quad \checkmark$$

$$\text{Check: } (x - 5)(x + 3) = x^2 + 3x - 5x - 15 = x^2 - 2x - 15 \quad \checkmark$$

Answer: $(x - 5)(x + 3)$

4 (2 marks)

Factorise $x^2 + 3x - 10$

Find two numbers that multiply to make -10 and add to make $+3$.

The product is negative, so one number is positive and one is negative.

$$5 \times (-2) = -10 \text{ and } 5 + (-2) = 3 \quad \checkmark$$

So the numbers are +5 and -2 .

Answer: $(x + 5)(x - 2)$

Tip: When the last number is negative, the two numbers have different signs. The bigger one takes the sign of the x term.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -10 or add to 3. 1 mark for $(x + 5)(x - 2)$.

5 (2 marks)

Factorise $x^2 - 9x + 20$

Find two numbers that multiply to make +20 and add to make -9 .

The product is positive but the sum is negative, so both numbers are negative.

$$(-4) \times (-5) = 20 \text{ and } -4 + (-5) = -9 \checkmark$$

Answer: $(x - 4)(x - 5)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 20 or add to -9 . 1 mark for $(x - 4)(x - 5)$.

6 (1 mark)

Factorise $x^2 - 16$

$$x^2 - 16 = x^2 - 4^2$$

This is the difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

$$\text{So } x^2 - 4^2 = (x + 4)(x - 4)$$

Answer: $(x + 4)(x - 4)$

Tip: $(x - 4)^2$ is wrong: it expands to $x^2 - 8x + 16$. One bracket must have + and the other $-$.

7 (3 marks)

(a) Factorise $x^2 - 5x$

There is no number term, so take out the common factor.

Both terms have a common factor of x .

$$x^2 \div x = x \text{ and } -5x \div x = -5$$

Answer: $x(x - 5)$

(b) Factorise $x^2 - 5x + 6$

Find two numbers that multiply to make +6 and add to make -5 .

Both numbers must be negative.

$$(-2) \times (-3) = 6 \text{ and } -2 + (-3) = -5 \quad \checkmark$$

Answer: $(x - 2)(x - 3)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 6 or add to -5 . 1 mark for $(x - 2)(x - 3)$.

8 (3 marks)

(a) Factorise $x^2 + x - 12$

$x^2 + x - 12$ has $1x$ in the middle.

Find two numbers that multiply to make -12 and add to make $+1$.

$$4 \times (-3) = -12 \text{ and } 4 + (-3) = 1 \quad \checkmark$$

So the numbers are $+4$ and -3 .

Answer: $(x + 4)(x - 3)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -12 or add to 1 . 1 mark for $(x + 4)(x - 3)$.

- (b) Write down the two solutions of $(x + 4)(x - 3) = 0$

If two things multiply to give 0, one of them must be 0.

$$x + 4 = 0 \text{ gives } x = -4$$

$$x - 3 = 0 \text{ gives } x = 3$$

Answer: $x = -4$ and $x = 3$

Tip: Take care with the signs: the bracket $(x + 4)$ gives the solution $x = -4$, not $x = 4$.

9 (2 marks)

Jake factorises $x^2 - 8x + 12$

He writes $(x + 2)(x + 6)$

- (a) Explain the mistake Jake has made.

$$\text{Jake's brackets expand to } (x + 2)(x + 6) = x^2 + 8x + 12.$$

This has $+8x$, but the question has $-8x$.

The two numbers must multiply to $+12$ *and* add to -8 , so both must be negative.

Answer: His brackets expand to $x^2 + 8x + 12$; the numbers should be -2 and -6 .

Tip: To explain a mistake, say exactly what is wrong: his numbers add to $+8$, but they need to add to -8 .

- (b) Factorise $x^2 - 8x + 12$ correctly.

$$(-2) \times (-6) = +12$$

$$-2 + (-6) = -8 \quad \checkmark$$

Answer: $(x - 2)(x - 6)$

10 (2 marks)

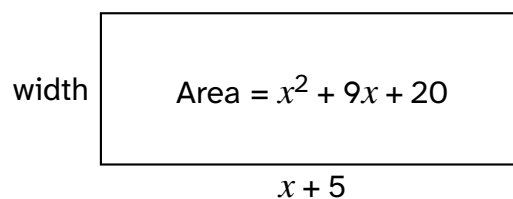
Tick **one** box for each statement.

	True	False
$x^2 - 25 = (x - 5)(x + 5)$	✓	
$x^2 + 9 = (x + 3)(x + 3)$		✓
$x^2 - 1 = (x - 1)(x + 1)$	✓	
$x^2 - 36 = (x - 6)^2$		✓
$x^2 - 25 = x^2 - 5^2 = (x - 5)(x + 5)$ ✓		
$(x + 3)(x + 3) = x^2 + 6x + 9$, not $x^2 + 9$		
$x^2 - 1 = x^2 - 1^2 = (x - 1)(x + 1)$ ✓		
$(x - 6)^2 = x^2 - 12x + 36$, not $x^2 - 36$		
$(x^2 - 36$ is $(x - 6)(x + 6)$.)		

Marks: 2 marks for all four correct. 1 mark for two or three correct.

11 (2 marks)

The area of this rectangle is $(x^2 + 9x + 20) \text{ cm}^2$



Not drawn accurately

Work out an expression for the width of the rectangle.

Area = length $\times$ width

Factorise the area: $x^2 + 9x + 20 = (x + 5)(x + 4)$

(because $5 \times 4 = 20$ and $5 + 4 = 9$)

The length is $x + 5$, so the width is $x + 4$.

Answer: $x + 4$ cm

Marks: 1 mark for $(x + 5)(x + 4)$. 1 mark for $x + 4$.

12 (2 marks)

Factorise $x^2 + 14x + 49$

Find two numbers that multiply to make 49 and add to make 14.

$$7 \times 7 = 49 \text{ and } 7 + 7 = 14 \quad \checkmark$$

So $(x + 7)(x + 7)$, which can be written as $(x + 7)^2$.

Answer: $(x + 7)^2$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 49 or add to 14. 1 mark for $(x + 7)^2$.

13 (2 marks)

a and b are integers.

$$x^2 + bx + 12 \equiv (x + 2)(x + a)$$

Work out the values of a and b .

The two numbers in the brackets multiply to make the number term, 12.

$$2 \times a = 12, \text{ so } a = 6$$

The two numbers add to make the x coefficient, b .

$$b = 2 + 6 = 8$$

Answer: $a = 6, b = 8$

Marks: 1 mark for $a = 6$. 1 mark for $b = 8$ (or 2+ your value of a).

14 (2 marks)

Use factorising to work out the value of $51^2 - 49^2$

Do **not** use a calculator.

$51^2 - 49^2$ is a difference of two squares.

Use $a^2 - b^2 = (a + b)(a - b)$ with $a = 51$ and $b = 49$:

$$(51 + 49)(51 - 49) = 100 \times 2$$

$$= 200$$

Answer: 200

Tip: Do not work out 51^2 and 49^2 separately. The question says use factorising, and $(a + b)(a - b)$ gives easy numbers.

Marks: 1 mark for $(51 + 49)(51 - 49)$. 1 mark for 200.
