

Direct and Inverse Proportion

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

k is a constant.

Circle the equation where P is inversely proportional to Q .

Options: $P = kQ$, $P = Q - k$, $P = \frac{k}{Q}$, $P = kQ^2$

Inversely proportional means that when Q is multiplied by a number, P is divided by the same number.

The equation for this is $P = \frac{k}{Q}$

($P = kQ$ is direct proportion. $P = Q - k$ is not proportion at all.)

Answer: $P = \frac{k}{Q}$

Tip: Inverse proportion is $P = \frac{k}{Q}$, with Q on the bottom. A subtraction like $Q - k$ is a common wrong choice.

2 (1 mark)

X is inversely proportional to Y .

Tick one box.

Inverse proportion: $X = \frac{k}{Y}$

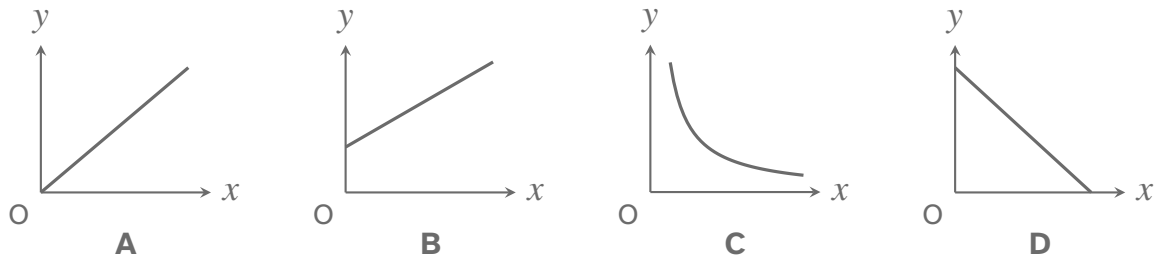
This is the same as $X = k \times \frac{1}{Y}$

So X is directly proportional to $\frac{1}{Y}$

Answer: X is directly proportional to $\frac{1}{Y}$

3 (1 mark)

Here are four graphs.



In one of these graphs, y is directly proportional to x .
Circle the letter of this graph.

Options: A, B, C, D

A direct proportion graph is a straight line through the origin (0, 0).

B is a straight line but does not go through the origin.

C is a curve (inverse proportion).

D slopes down.

Only A is a straight line through the origin.

Answer: A

4 (1 mark)

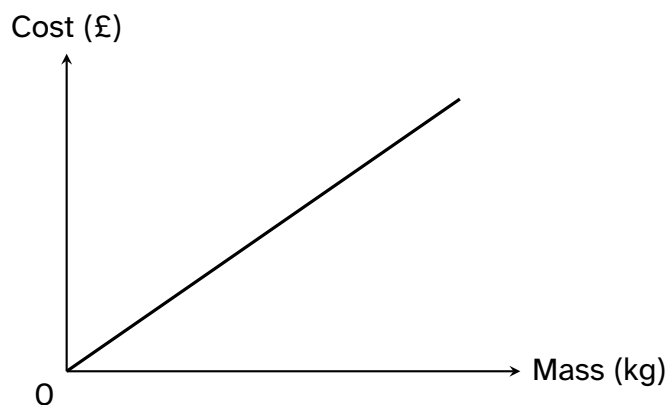
The cost of some apples is directly proportional to their mass.

On the axes, sketch a graph to show this relationship.

Direct proportion: if the mass doubles, the cost doubles.

0 kg costs £0, so the graph starts at the origin.

The graph is a straight line going up from the origin.



Answer: Straight line through the origin

Tip: Use a ruler, and make sure your line starts exactly at (0, 0).

5 (2 marks)

y is inversely proportional to x .

Complete the table.

Inverse proportion: $x \times y$ is always the same.

From the table: $1 \times 40 = 40$

Check: $4 \times 10 = 40$ ✓

When $x = 2$: $y = 40 \div 2 = 20$

When $x = 8$: $y = 40 \div 8 = 5$

x	1	2	4	8
y	40	20	10	5

Answer: 20 and 5

Tip: In inverse proportion, as x goes up, y goes down. But y never reaches 0, and it does not go down by the same amount each time.

Marks: 1 mark for each correct value.

6 (2 marks)

5 kg of apples cost £7.40

Work out the cost of 8 kg of the same apples.

Find the cost of 1 kg first:

$$7.40 \div 5 = 1.48$$

$$\text{Cost of 8 kg: } 8 \times 1.48 = 11.84$$

Answer: £11.84

Marks: 1 mark for $7.40 \div 5$ (1.48). 1 mark for 11.84.

7 (2 marks)

V is directly proportional to H .

When $H = 6$, $V = 21$

Work out the value of V when $H = 10$

Direct proportion: $V = kH$

Use $H = 6$ and $V = 21$: $21 = 6k$

$$k = 21 \div 6 = 3.5$$

So $V = 3.5H$

When $H = 10$: $V = 3.5 \times 10 = 35$

Answer: $V = 35$

Marks: 1 mark for $21 \div 6$ or $\frac{10}{6}$. 1 mark for 35.

8 (2 marks)

It takes 6 workers 10 days to build a wall.

How many days would it take 4 workers to build the same wall?

Assume all workers work at the same rate.

Fewer workers will take **more** days (inverse proportion).

Find how long 1 worker would take:

$$6 \times 10 = 60 \text{ days}$$

4 workers take a quarter of that time:

$$60 \div 4 = 15 \text{ days}$$

Answer: 15 days

Tip: Check your answer makes sense. Fewer workers must take longer, so the answer must be more than 10 days.

Marks: 1 mark for 60. 1 mark for 15.

9 (4 marks)

A charity packs food parcels.

$$t = \frac{150}{v}$$

- t is the time, in minutes, to pack all the parcels.
- v is the number of volunteers.

- (a) Work out the time taken when there are 4 volunteers.
Give your answer in minutes and seconds.

Put $v = 4$ into the formula:

$$t = \frac{150}{4} = 37.5 \text{ minutes}$$

$$0.5 \text{ minutes} = 0.5 \times 60 = 30 \text{ seconds}$$

Answer: 37 minutes 30 seconds

Tip: 37.5 minutes is 37 minutes 30 seconds, not 37 minutes 50 seconds. Multiply the decimal part by 60.

Marks: 1 mark for $150 \div 4$ ($= 37.5$). 1 mark for 37 minutes 30 seconds.

- (b) The parcels must be packed in **less** than 20 minutes.
Work out the smallest number of volunteers needed.

Put $t = 20$ into the formula: $20 = \frac{150}{v}$

$$v = 150 \div 20 = 7.5$$

7.5 volunteers would take exactly 20 minutes, but you cannot have half a volunteer.

7 volunteers take $150 \div 7 = 21.4 \dots$ minutes, which is too long.

8 volunteers take $150 \div 8 = 18.75$ minutes, which is less than 20 minutes.

Answer: 8

Tip: The number of volunteers must be a whole number. Rounding 7.5 **down** to 7 gives a time that is too long, so round up.

Marks: 1 mark for $150 \div 20 (= 7.5)$. 1 mark for 8.

10 (1 mark)

y is directly proportional to x .

When $x = 4$, $y = 10$

Sam says,

‘When $x = 8$, $y = 14$, because I add 4 to both numbers.’

Is Sam correct?

Tick a box.

Give a reason for your answer.

x has doubled, from 4 to 8.

y is directly proportional to x , so y must also double.

$y = 10 \times 2 = 20$, not 14.

Sam has added 4 instead of multiplying, so he is not correct.

Answer: No: x doubles, so y doubles to 20

Tip: Direct proportion means you **multiply** both quantities by the same number. Adding the same number to both is a common mistake.

11 (4 marks)

On a treadmill, the speed is directly proportional to the level number.

At level 10 the speed is 12 km/h.

Jo runs on the treadmill at level 8 for 25 minutes.

How far does Jo run?

The speed is in direct proportion to the level.

Level 1 gives $12 \div 10 = 1.2$ km/h.

Level 8 gives $1.2 \times 8 = 9.6$ km/h.

25 minutes is $\frac{25}{60}$ of an hour.

Distance = speed $\times$ time = $9.6 \times \frac{25}{60}$

$9.6 \div 60 = 0.16$ km each minute, and $0.16 \times 25 = 4$

Answer: 4 km

Tip: The speed is in km per **hour**, but Jo runs for 25 **minutes**. $9.6 \times 25 = 240$ km is far too big. Change 25 minutes into hours, or find the distance for 1 minute first.

Marks: 1 mark for $12 \div 10$ or $\frac{6}{5}$. 1 mark for 9.6 km/h. 1 mark for $9.6 \times \frac{25}{60}$ (or $9.6 \div 60 \times 25$). 1 mark for 4.

12 (3 marks)

A group of 6 people on a camping trip has enough drinking water for 8 days.

2 of the people leave the group.

- (a) Assume that each person drinks the same amount of water each day.
How many days will the water last for the 4 people who are left?

Fewer people means the water lasts **longer**.

For 1 person the water would last: $6 \times 8 = 48$ days

For 4 people: $48 \div 4 = 12$ days

Answer: 12 days

Tip: Fewer people drink the water, so it must last more than 8 days. $8 \div 6 \times 4$ gives an answer that is too small.

Marks: 1 mark for $6 \times 8 (= 48)$. 1 mark for 12.

- (b) In fact, the 2 people who left each drank **less** water each day than the other 4 people.

What does this mean about the number of days the water will last for the 4 people?

Tick one box.

In part (a) we assumed all 6 people drank the same amount.

In fact, the 4 people who are left drink **more** each day than that average amount.

So the water is used up faster.

It will last fewer days than 12.

Answer: It is less than the answer to (a)

Tip: Think about who is left. The 4 people who stay are the ones who drink more, so the water runs out sooner.
