

Coordinates and Midpoints

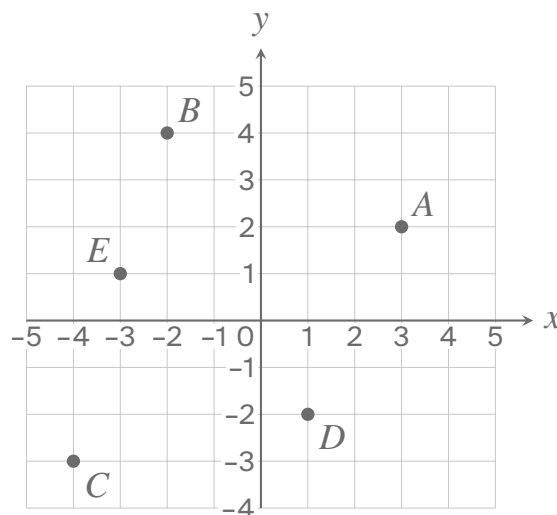
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Points *A*, *B*, *C* and *D* are plotted on the grid. (Point *E* is shown plotted.)



(a) Write down the coordinates of *B*.

Start at the origin (0, 0).

Go 2 to the left, so $x = -2$.

Then go 4 up, so $y = 4$.

The x -coordinate (across) comes first.

Answer: $(-2, 4)$

Tip: Always write the x -coordinate (across) first, then the y -coordinate (up or down).

Writing $(4, -2)$ is a common mistake.

- (b) Write down the coordinates of C .

From the origin, go 4 to the left, so $x = -4$.

Then go 3 down, so $y = -3$.

Answer: $(-4, -3)$

- (c) Plot the point E at $(-3, 1)$ on the grid.

From the origin, go 3 to the left, then 1 up.

Point E is shown on the grid above.

Answer: Point at $(-3, 1)$

- (d) Write down the coordinates of the midpoint of AD .

A is $(3, 2)$ and D is $(1, -2)$.

Add the x -coordinates and halve: $\frac{3+1}{2} = 2$

Add the y -coordinates and halve: $\frac{2+(-2)}{2} = 0$

(Or on the grid: the point halfway along the line from A to D .)

Answer: $(2, 0)$

Tip: Read D carefully: it is **below** the x -axis, so its y -coordinate is -2 .

2 (1 mark)

Circle the coordinates of a point on the y -axis.

Options: $(3, 0)$, $(0, 3)$, $(3, 3)$, $(-3, 3)$

The y -axis is the vertical line through the origin.

Every point on the y -axis has x -coordinate 0.

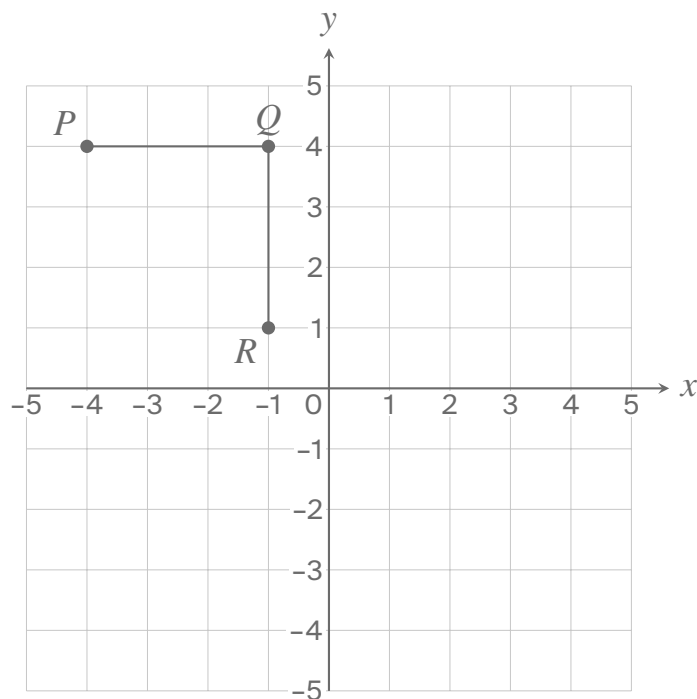
The only option with $x = 0$ is $(0, 3)$.

Answer: $(0, 3)$

Tip: Points on the y -axis look like $(0, \dots)$. Points on the x -axis look like $(\dots, 0)$.

3 (1 mark)

P , Q and R are three vertices of a square $PQRS$.



$PQRS$ is a square.

Work out the coordinates of S .

To go from Q to R you go 3 down.

S is the same move from P : 3 down from $(-4, 4)$.

$$S = (-4, 4 - 3) = (-4, 1)$$

Answer: $(-4, 1)$

4 (2 marks)

A is the point $(2, 3)$.

B is the point $(8, 7)$.

Work out the coordinates of the midpoint of AB .

The midpoint is halfway between the two points.

Add the x -coordinates and halve: $\frac{2 + 8}{2} = \frac{10}{2} = 5$

Add the y -coordinates and halve: $\frac{3 + 7}{2} = \frac{10}{2} = 5$

Answer: $(5, 5)$

Tip: Add the coordinates, then halve. Subtracting them (to get $(6, 4)$) or halving the differences (to get $(3, 2)$) is a common mistake.

Marks: 1 mark for adding and halving the x or the y coordinates. 1 mark for $(5, 5)$.

5 (2 marks)

C is the point $(-4, 5)$.

D is the point $(6, -1)$.

Work out the coordinates of the midpoint of CD .

Add the x -coordinates and halve: $\frac{-4 + 6}{2} = \frac{2}{2} = 1$

Add the y -coordinates and halve: $\frac{5 + (-1)}{2} = \frac{4}{2} = 2$

Answer: $(1, 2)$

Marks: 1 mark for adding and halving the x or the y coordinates. 1 mark for $(1, 2)$.

6 (2 marks)

M is the midpoint of PQ .

P is the point $(1, 4)$ and M is the point $(5, 7)$.

Work out the coordinates of Q .

M is halfway, so the step from M to Q is the same as the step from P to M .

From P to M : x goes up by $5 - 1 = 4$ and y goes up by $7 - 4 = 3$.

Do the same step again from M :

$$Q = (5 + 4, 7 + 3) = (9, 10)$$

Check: the midpoint of $(1, 4)$ and $(9, 10)$ is $(5, 7)$ ✓

Answer: $(9, 10)$

Tip: $(5, 7)$ is the midpoint, not an end. Don't just find the midpoint of P and M (that gives $(3, 5.5)$).

Marks: 1 mark for finding the step +4 across and +3 up. 1 mark for $(9, 10)$.

7 (1 mark)

Ali plots the point $(4, 1)$.

He goes up 4 and then across 1.

Is he correct? Tick a box.

Give a reason for your answer.

In $(4, 1)$ the x -coordinate is 4 and the y -coordinate is 1.

So you go 4 across first, then 1 up.

Ali has done it the wrong way round. He has plotted $(1, 4)$.

Answer: No, x comes first: across 4 then up 1.

8 (2 marks)

Three vertices of a rectangle are $(1, 1)$, $(6, 1)$ and $(6, 4)$.

Work out the coordinates of the fourth vertex.

A quick sketch helps.

$(1, 1)$ to $(6, 1)$ is the bottom side.

$(6, 1)$ to $(6, 4)$ is the right-hand side.

The fourth vertex is above $(1, 1)$ and level with $(6, 4)$.

So it has $x = 1$ and $y = 4$.

Answer: $(1, 4)$

Marks: 2 marks for $(1, 4)$. 1 mark for one coordinate correct.

9 (2 marks)

A is the point $(-2, 3)$ and B is the point $(7, 3)$.

Work out the length of AB .

Both points have $y = 3$, so AB is a horizontal line.

The length is the difference between the x -coordinates.

$$7 - (-2) = 7 + 2 = 9$$

Answer: 9 units

Marks: 1 mark for $7 - (-2)$ or counting across. 1 mark for 9.

10 (2 marks)

A triangle has vertices at $(0, 0)$ and $(6, 0)$.

The area of the triangle is 12 square units.

Work out one possible pair of coordinates of the other vertex.

The base goes from $(0, 0)$ to $(6, 0)$, so it has length 6.

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\frac{1}{2} \times 6 \times h = 12$$

$$3h = 12, \text{ so } h = 4$$

The other vertex must be 4 units above or below the x -axis.

Any point with $y = 4$ (or $y = -4$) works, for example $(2, 4)$.

Answer: e.g. $(2, 4)$ (any point with y -coordinate 4 or -4)

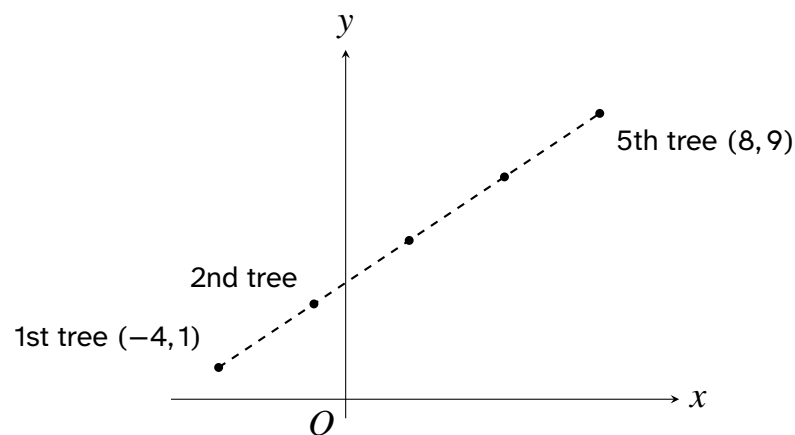
Marks: 1 mark for finding the height, 4. 1 mark for a correct point.

11 (3 marks)

A gardener plants five trees in a straight line.

The trees are equally spaced.

On a plan, the first tree is at $(-4, 1)$ and the fifth tree is at $(8, 9)$.



Not drawn accurately

Work out the coordinates of the **second** tree.

Find the step from the first tree to the fifth tree:

$$x \text{ goes up by } 8 - (-4) = 12$$

$$y \text{ goes up by } 9 - 1 = 8$$

There are 5 trees, so there are 4 equal gaps between them.

$$\text{One gap: } 12 \div 4 = 3 \text{ across and } 8 \div 4 = 2 \text{ up.}$$

Add one gap to the first tree:

$$(-4 + 3, 1 + 2) = (-1, 3)$$

Answer: $(-1, 3)$

Tip: Five trees in a row have only **four** gaps between them. Dividing by 5 is the most common mistake here. Check: adding the gap four times to $(-4, 1)$ should take you to $(8, 9)$.

Marks: 1 mark for the total step (12 across and 8 up). 1 mark for dividing by 4 (3 across and 2 up). 1 mark for $(-1, 3)$.

12 (3 marks)

A triangle is drawn using the lines

$$y = x$$

$$x = -3$$

$$y = 5$$

Work out the coordinates of the three vertices of the triangle.

$x = -3$ is a vertical line. $y = 5$ is a horizontal line.

$y = x$ is the diagonal line through $(0, 0)$, $(1, 1)$, $(2, 2)$, ...

Each vertex is where two of the lines meet.

$y = x$ meets $x = -3$ at $(-3, -3)$.

$y = x$ meets $y = 5$ at $(5, 5)$.

$x = -3$ meets $y = 5$ at $(-3, 5)$.

Answer: $(-3, -3)$, $(5, 5)$, $(-3, 5)$

Tip: $x = -3$ is a **vertical** line (every point has $x = -3$). $y = 5$ is a **horizontal** line. Mixing these up is very common. A quick sketch helps.

Marks: 1 mark for each correct vertex.
