

Congruence

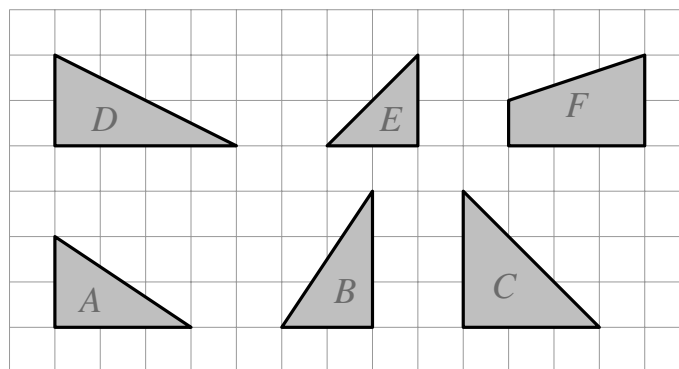
Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here are six shapes on a grid.



Which **two** shapes are congruent?

Congruent shapes are exactly the same shape and size.

They can be turned round or flipped over.

A is a right-angled triangle with shorter sides 3 squares and 2 squares.

B is also a right-angled triangle with shorter sides 2 squares and 3 squares.

So *A* and *B* are congruent (*B* is *A* turned round).

Answer: *A* and *B*

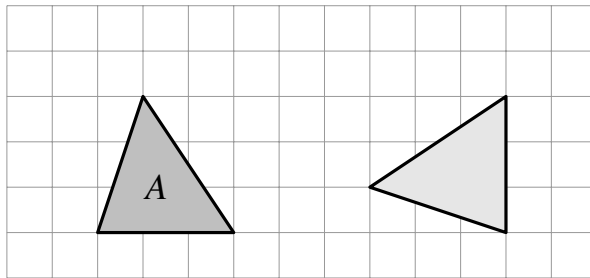
2 (1 mark)

On the grid, draw a triangle congruent to triangle *A* but in a different position.

The copy must be exactly the same shape and size as *A*.

A has a base of 3 squares, and its top vertex is 1 square right of the left end and 3 squares up.

You can move it, turn it or flip it. For example:

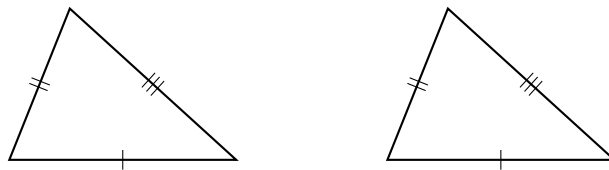


Answer: Any triangle the same shape and size as A

Tip: Count the squares for every side, including how far across the top vertex is. Do not just draw a triangle that “looks similar”.

3 (1 mark)

Here are two triangles.



Not drawn accurately

The marks show which sides are equal.

Circle the condition that shows these triangles are congruent.

Options: SSS, SAS, ASA, RHS

The marks show that all three pairs of sides are equal.

Side, Side, Side is the condition SSS.

Answer: SSS

4 (1 mark)

Triangles P and Q both have angles of 55° and 75°

In both triangles, the side between these two angles is 8 cm long.

Circle the reason why triangles P and Q are congruent.

Options: SSS, SAS, ASA, RHS

The triangles have two pairs of equal angles (55° and 75°).

The equal sides (8 cm) are between these two angles in both triangles.

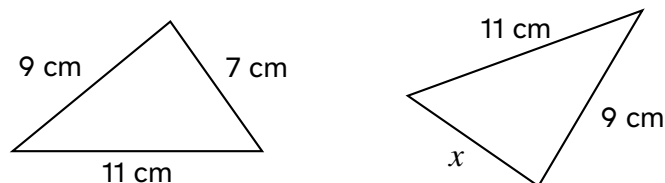
Angle, Side, Angle: so the reason is ASA.

Answer: ASA

Tip: SAS needs two sides with the angle between them. Here there is only one side, with an angle at each end of it, so it is ASA.

5 (1 mark)

Here are two triangles.



Not drawn accurately

These two triangles are congruent.

Write down the value of x .

Congruent triangles have the same three side lengths.

The first triangle has sides 7, 9 and 11 cm.

The second has 9 and 11 cm, so the missing side x must be 7 cm.

Answer: 7 cm

6 (2 marks)

Triangles ABC and DEF are congruent.

$AB = DE = 7\text{ cm}$, $BC = EF = 9\text{ cm}$ and $AC = 11\text{ cm}$

Angle $ABC = 48^\circ$

- (a) Write down the size of angle DEF .

The letters match in order: $A \rightarrow D$, $B \rightarrow E$, $C \rightarrow F$.

So angle DEF matches angle ABC .

Matching angles in congruent triangles are equal.

Answer: 48°

- (b) Write down the length of DF .

DF matches AC .

$AC = 11\text{ cm}$

Answer: 11 cm

7 (2 marks)

- (a) Triangle P has sides 5 cm and 7 cm with an angle of 40° between them.
Triangle Q has sides 7 cm and 5 cm with an angle of 40° between them.
Are the triangles congruent? Tick a box.

Give a reason for your answer.

Both triangles have sides 5 cm and 7 cm .

The 40° angle is **between** these two sides in both triangles.

Two sides and the angle between them is the condition SAS.

Answer: Yes, SAS

- (b) Triangles R and S are both right-angled.
Each has a hypotenuse of 13 cm and another side of 5 cm.
Are the triangles congruent? Tick a box.
Give a reason for your answer.

Both have a right angle.

Both have a hypotenuse of 13 cm and one other side of 5 cm.

Right angle, Hypotenuse, Side is the condition RHS.

(By Pythagoras, the third side of each is 12 cm.)

Answer: Yes, RHS

8 (2 marks)

Which **two** of the triangles B , C , D and E are congruent to triangle A ?

Triangle	Information
A	sides 6 cm, 8 cm, 10 cm
B	sides 8 cm, 10 cm, 6 cm
C	sides 6 cm, 8 cm, angle between them 60°
D	sides 6 cm and 8 cm with a right angle between them
E	angles 37° , 53° , 90°

$$6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

So by Pythagoras, A is right-angled, with 90° between the 6 cm and 8 cm sides.

B has the same three sides in a different order, so it is congruent (SSS).

D has sides 6 cm and 8 cm with 90° between them, just like A , so it is congruent (SAS).

C has 60° between those sides, not 90° , so it is not congruent.

E gives only angles and no lengths, so it could be any size.

Answer: B and D

Marks: 1 mark for each correct triangle.

9 (3 marks)

Here is some information about three pairs of triangles.

For each pair, tick the condition that proves the triangles are congruent.

If there is not enough information to be sure, tick **Not enough**.

	SAS	ASA	RHS	Not enough
Both triangles have a right angle, a hypotenuse of 10 cm and another side of 6 cm			✓	
Both triangles have sides of 4 cm and 9 cm with an angle of 35° between them	✓			
Both triangles have one side of 5 cm and one side of 12 cm				✓

Row 1: right angle, equal hypotenuses and one other pair of equal sides.

So the condition is RHS.

Row 2: two pairs of equal sides with the equal angle **between** them.

So the condition is SAS.

Row 3: only two sides are known. The third sides could be different lengths (the angle between the sides could be anything).

So there is not enough information.

Answer: RHS, SAS, Not enough

Tip: Two pairs of equal sides are not enough on their own. You also need the angle between them (SAS), the third side (SSS) or a right angle with the hypotenuse (RHS).

Marks: 3 marks for all three correct. 2 marks for two correct. 1 mark for one correct.

10 (1 mark)

Sam says,

‘If two triangles have the same three angles, they must be congruent.’

Explain why Sam is wrong.

Two triangles can have the same three angles but be different sizes.

For example, one could be an enlargement of the other.

So equal angles are not enough to show congruence.

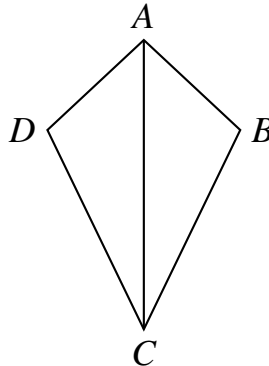
Answer: The triangles could be different sizes

11 (3 marks)

$ABCD$ is a kite.

$AB = AD$ and $CB = CD$

The diagonal AC splits the kite into two triangles.



Not drawn accurately

For each statement, tick the correct box.

	Must be true	Could be true	Must be false
Triangle ABC is congruent to triangle ADC	✓		
Triangle ABC is isosceles		✓	
The perimeter of triangle ABC is greater than the perimeter of triangle ADC			✓

Triangles ABC and ADC : $AB = AD$, $CB = CD$ and AC is a side of both triangles.

So all three pairs of sides are equal (SSS).

Statement 1: the triangles are congruent, so this **must be true**.

Statement 2: triangle ABC has sides AB , BC and AC . We are not told any two of these are equal, but two of them could be, so this **could be true**.

Statement 3: congruent triangles have the same side lengths, so they have the same perimeter. This **must be false**.

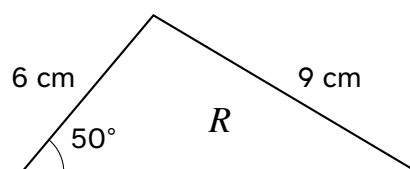
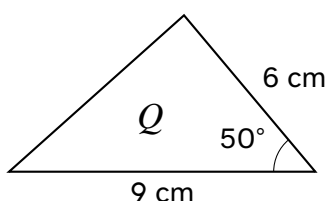
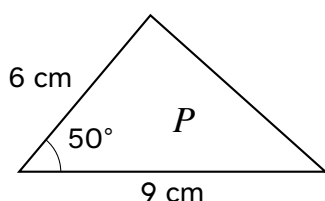
Answer: Must be true; Could be true; Must be false

Tip: Congruent shapes are exactly the same shape and size, so their perimeters and areas are equal too. Use “could be true” when the information does not decide it either way.

Marks: 1 mark for each correct row.

12 (2 marks)

Here are three triangles.



Not drawn accurately

- (a) Triangle Q is congruent to triangle P .

Circle the condition that shows this.

Options: SSS, SAS, ASA, RHS

Both triangles have sides 6 cm and 9 cm.

In both, the 50° angle is **between** the 6 cm and 9 cm sides.

Q is P flipped over.

Side, Angle, Side: the condition is SAS.

Answer: SAS

- (b) Kim says,

‘Triangle R is also congruent to triangle P .

It has sides of 6 cm and 9 cm and an angle of 50° too.’

Explain why Kim is wrong.

In triangle P the 50° angle is between the 6 cm and 9 cm sides.

In triangle R the 50° angle is **not** between the 6 cm and 9 cm sides.

So the SAS condition does not apply, and the triangles are different shapes.

Answer: In R the 50° angle is not between the 6 cm and 9 cm sides

Tip: For SAS the angle must be the **included** angle: the one between the two given sides.
