

Averages Problems

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Four numbers have a mean of 9
Three of the numbers are equal.
The other number is 18
Work out the value of each of the three equal numbers.

Mean = total $\div$ number of values, so total = mean $\times$ number of values.

Total of the four numbers = $4 \times 9 = 36$

Total of the three equal numbers: $36 - 18 = 18$

Each of the three equal numbers = $18 \div 3 = 6$

Check: $(6 + 6 + 6 + 18) \div 4 = 36 \div 4 = 9$ ✓

Answer: 6

Tip: Multiply the mean by how many numbers there are to get the total. Dividing $(9 \div 4)$ is a common mistake.

Marks: 1 mark for 4×9 or 36. 1 mark for $36 - 18$ or 18. 1 mark for 6.

2 (3 marks)

The mean of three numbers is 8
The mean of two other numbers is 13
Work out the mean of all five numbers.

Total of the three numbers: $3 \times 8 = 24$

Total of the two numbers: $2 \times 13 = 26$

Total of all five numbers: $24 + 26 = 50$

Mean of all five = $50 \div 5 = 10$

Answer: 10

Marks: 1 mark for 24 or 26. 1 mark for $50 \div 5$. 1 mark for 10.

3 (3 marks)

A class has 12 boys and 18 girls.

The mean test score of the boys is 64

The mean test score of the girls is 70

Work out the mean test score of the whole class.

Total score of the boys: $12 \times 64 = 768$

Total score of the girls: $18 \times 70 = 1260$

Total score of the class: $768 + 1260 = 2028$

There are $12 + 18 = 30$ students.

Mean = $2028 \div 30 = 67.6$

Answer: 67.6

Tip: $(64 + 70) \div 2 = 67$ is wrong. There are more girls, so the class mean is closer to the girls' mean of 70.

Marks: 1 mark for 768 or 1260. 1 mark for dividing the total by 30. 1 mark for 67.6.

4 (2 marks)

Here are the salaries of five people who work in a small company.

£18 000 £19 000 £20 000 £21 000 £120 000

- (a) Which average would best represent a typical salary?
Circle your answer.

Options: mean, median, mode

There is no mode, because every salary is different.

One salary (£120 000) is much bigger than the rest.

The median (£20 000) is not affected by this one very large salary.

Answer: median

- (b) Give a reason why the mean is not a good average to use.

£120 000 is an outlier: it is much larger than the other salaries.

It pulls the mean up to $198\,000 \div 5 = £39\,600$.

Four of the five people earn much less than this, so the mean is not typical.

Answer: Distorted by the outlier £120 000

Tip: Your reason must be about the **mean**, e.g. “the £120 000 makes the mean too high”. Just saying what the median is does not get the mark.

5 (3 marks)

Each number in a list is increased by 5

Tick **one** box for each statement.

	True	False	Cannot tell
The mean is increased by 5	✓		
The range is increased by 5		✓	
The median is now greater than the mean			✓

Mean: adding 5 to every number makes the total bigger, and the mean goes up by 5. True.

Range: the largest and smallest numbers both go up by 5.

So the difference between them stays the same. The range does not change. False.

Median and mean: both go up by 5.

We do not know the numbers, so we do not know which was bigger to start with.
Cannot tell.

Answer: True, False, Cannot tell

Marks: 1 mark for each correct row.

6 (3 marks)

Four positive whole numbers have

- a mode of 3
- a median of 4
- a mean of 5

Work out the four numbers.

Mean 5, so the total = $4 \times 5 = 20$

Mode 3, so there are (at least) two 3s.

The median is 4, which is more than 3, so the two 3s must be the smallest numbers.

In order the numbers are 3, 3, a , b .

The median is halfway between the 2nd and 3rd numbers: $(3 + a) \div 2 = 4$

So $3 + a = 8$, which gives $a = 5$

$b = 20 - 3 - 3 - 5 = 9$

Check: 3, 3, 5, 9 has mode 3, median 4 and mean $20 \div 4 = 5$ ✓

Answer: 3, 3, 5, 9

Tip: Write the numbers in order as you go. Adding a number to fix the mode or mean can move the median, so always check all three at the end.

Marks: 1 mark for the total 20. 1 mark for 3, 3 and 5. 1 mark for all four numbers correct.

7 (3 marks)

Here are four cards.

4	7	9	12
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Another card is added.

The mean of the numbers on the five cards is 1 more than the mean of the numbers on the four cards.

Work out the number on the new card.

You must show your working.

Mean of the four cards: $(4 + 7 + 9 + 12) \div 4 = 32 \div 4 = 8$

Mean of the five cards: $8 + 1 = 9$

Total of the five cards: $5 \times 9 = 45$

New card: $45 - 32 = 13$

Check: $(4 + 7 + 9 + 12 + 13) \div 5 = 45 \div 5 = 9 \quad \checkmark$

Answer: 13

Tip: Work with totals: total = mean $\times$ number of cards. Remember there are **five** cards after the new one is added.

Marks: 1 mark for the mean 8. 1 mark for 5×9 or 45. 1 mark for 13.

8 (3 marks)

Tom has six cards. Each card has a number on it.

The numbers on four of the cards are 9 4 12 5

For the six numbers

the mode is 4

the mean is 7

Work out the numbers on the other two cards.

Total of all six numbers = $6 \times 7 = 42$

Total of the four known numbers: $9 + 4 + 12 + 5 = 30$

So the other two numbers add up to $42 - 30 = 12$

The mode is 4, but there is only one 4 so far.

So one of the other cards must be 4.

The last card is $12 - 4 = 8$

Check: 4, 4, 5, 8, 9, 12 has mode 4 and total 42, so the mean is $42 \div 6 = 7$ ✓

Answer: 4 and 8

Tip: Both conditions must work. Use the mode to fix one card, then use the total to find the other. Check the mode is still 4 at the end.

Marks: 1 mark for 6×7 or 42. 1 mark for finding the other two add up to 12, or for one card being 4. 1 mark for 4 and 8.

9 (3 marks)

The mean of these four expressions is 7

$$x \quad x + 3 \quad 2x \quad 2x + 1$$

Work out the value of x .

Add the four expressions: $x + (x + 3) + 2x + (2x + 1) = 6x + 4$

The mean is 7, so the total is $4 \times 7 = 28$

$$6x + 4 = 28$$

Subtract 4 from both sides: $6x = 24$

Divide both sides by 6: $x = 4$

Answer: 4

Tip: Remember to include the first x when you add up. Set the total equal to $4 \times 7 = 28$, not to 7.

Marks: 1 mark for $6x + 4$. 1 mark for setting the total equal to 28. 1 mark for $x = 4$.

10 (4 marks)

Priya has done four science tests. Each test was out of 40 marks.
Her mean percentage for the four tests is 60%
She wants her mean percentage for five tests to be 64%
Work out the mark, out of 40, that Priya needs in the fifth test.

Total of the four percentages: $4 \times 60 = 240$

Total needed for five tests: $5 \times 64 = 320$

Percentage needed in the fifth test: $320 - 240 = 80\%$

80% of 40 = $0.8 \times 40 = 32$

(Or work in marks: four tests total $0.6 \times 160 = 96$ marks, five tests need $0.64 \times 200 = 128$ marks, so $128 - 96 = 32$.)

Answer: 32 out of 40

Tip: Keep percentages and marks separate. Work in percentages all the way through, then change 80% back into a mark out of 40 at the end.

Marks: 1 mark for 240 or 96 marks. 1 mark for 320 or 128 marks. 1 mark for 80%. 1 mark for 32.
