

# Averages from Grouped Frequency Tables

## Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

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Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

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**1** (5 marks)

The table shows information about the lengths of 25 phone calls.

| Length ( $t$ minutes) | Frequency | Midpoint |
|-----------------------|-----------|----------|
| $0 < t \leq 4$        | 6         | 2        |
| $4 < t \leq 8$        | 9         | 6        |
| $8 < t \leq 12$       | 7         | 10       |
| $12 < t \leq 16$      | 3         | 14       |

(a) Complete the midpoint column in the table.

The midpoint is halfway between the two ends of the class.

Add the two ends and divide by 2.

$$(0 + 4) \div 2 = 2$$

$$(4 + 8) \div 2 = 6$$

$$(8 + 12) \div 2 = 10$$

$$(12 + 16) \div 2 = 14$$

**Answer:** 2, 6, 10, 14

- (b) Write down the modal class.

The modal class is the class with the highest frequency.

The highest frequency is 9.

**Answer:**  $4 < t \leq 8$

- (c) Work out an estimate of the mean length of a phone call.

Multiply each midpoint by its frequency:

$$2 \times 6 = 12, \quad 6 \times 9 = 54, \quad 10 \times 7 = 70, \quad 14 \times 3 = 42$$

Add these up:  $12 + 54 + 70 + 42 = 178$

Total frequency (number of calls) =  $6 + 9 + 7 + 3 = 25$

$$\text{Mean} \approx 178 \div 25 = 7.12$$

**Answer:** 7.12 minutes

**Tip:** Divide by the total frequency (25 calls), not by the number of classes (4). This is the most common mistake in exams.

*Marks:* 1 mark for at least three midpoint  $\times$  frequency values. 1 mark for dividing the total (178) by 25. 1 mark for 7.12.

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**2** (7 marks)

The table shows the distances that 40 members of a running club ran last Sunday.

| Distance ( $d$ km) | Frequency |
|--------------------|-----------|
| $0 < d \leq 5$     | 6         |
| $5 < d \leq 10$    | 15        |
| $10 < d \leq 15$   | 12        |
| $15 < d \leq 20$   | 7         |

(a) Kim says,

‘The modal class is 15.’

Kim is wrong.

Write down the correct modal class.

The modal class is the class with the highest frequency.

The highest frequency is 15.

15 is the frequency, not the class. The class is  $5 < d \leq 10$ .

**Answer:**  $5 < d \leq 10$

**Tip:** The modal class is a class interval, such as  $5 < d \leq 10$ . Do not write down the frequency.

(b) Find the class interval that contains the median.

You must show your working.

There are  $6 + 15 + 12 + 7 = 40$  values.

The median is halfway between the 20th and 21st values.

Add up the frequencies as you go down the table (running totals):

First class: 6

First two classes:  $6 + 15 = 21$

So the 20th and 21st values are both in the second class.

**Answer:**  $5 < d \leq 10$

**Tip:** The median class is **not** simply the middle row of the table. Use running totals to find where the middle value is.

*Marks:* 1 mark for the running totals (6, 21) or for finding the position of the median. 1 mark for the correct class.

- (c) Work out an estimate of the mean distance.

The midpoints are 2.5, 7.5, 12.5 and 17.5.

Multiply each midpoint by its frequency:

$$2.5 \times 6 = 15, \quad 7.5 \times 15 = 112.5, \quad 12.5 \times 12 = 150, \quad 17.5 \times 7 = 122.5$$

$$\text{Add these up: } 15 + 112.5 + 150 + 122.5 = 400$$

$$\text{Divide by the total frequency: } 400 \div 40 = 10$$

**Answer:** 10 km

**Tip:** Use the midpoints (2.5, 7.5, ...), not the class ends (5, 10, ...). Divide by 40 (the number of runners), not by 4.

*Marks:* 1 mark for at least three midpoint  $\times$  frequency values. 1 mark for  $400 \div 40$ . 1 mark for 10.

- (d) Why is your answer to part (c) an estimate?

We do not know the exact distance each member ran.

We only know which class each distance is in.

So we used the midpoint of each class instead of the real values.

**Answer:** Exact distances unknown / midpoints used

**Tip:** Saying “because the answer was rounded” does not get the mark. The reason is that the exact values are not known.

**3** (3 marks)

The table shows information about the heights of some plants.

| Height ( $h$ cm)   | Frequency |
|--------------------|-----------|
| $140 < h \leq 150$ | 6         |
| $150 < h \leq 160$ | 11        |
| $160 < h \leq 170$ | 9         |
| $170 < h \leq 180$ | 4         |

- (a) Could the range of the heights be 45 cm? Tick a box.  
Give a reason for your answer.

The shortest plant is more than 140 cm.

The tallest plant is at most 180 cm.

So the range must be less than  $180 - 140 = 40$  cm.

45 cm is more than this, so the range cannot be 45 cm.

**Answer:** No, the range must be less than 40 cm

- (b) What percentage of the plants are taller than 160 cm?

Taller than 160 cm means the last two classes.

Number taller than 160 cm:  $9 + 4 = 13$

Total number of plants:  $6 + 11 + 9 + 4 = 30$

Percentage =  $\frac{13}{30} \times 100 = 43.33 \dots$

**Answer:** 43.3%

**Tip:**  $160 < h \leq 170$  means more than 160 cm, so do not include the  $150 < h \leq 160$  class.

*Marks:* 1 mark for  $\frac{13}{30}$ . 1 mark for 43.3.

**4** (3 marks)

The table shows the masses of 40 parcels.

| Mass ( $m$ kg)   | Frequency |
|------------------|-----------|
| $0 < m \leq 4$   | 7         |
| $4 < m \leq 8$   | 15        |
| $8 < m \leq 12$  | 13        |
| $12 < m \leq 16$ | 5         |

Work out an estimate of the mean mass.

Give your answer to 1 decimal place.

The midpoints are 2, 6, 10 and 14.

Multiply each midpoint by its frequency:

$$2 \times 7 = 14, \quad 6 \times 15 = 90, \quad 10 \times 13 = 130, \quad 14 \times 5 = 70$$

$$\text{Add these up: } 14 + 90 + 130 + 70 = 304$$

There are 40 parcels.

$$\text{Mean} \approx 304 \div 40 = 7.6$$

**Answer:** 7.6 kg

*Marks:* 1 mark for at least three midpoint  $\times$  frequency values. 1 mark for  $304 \div 40$ . 1 mark for 7.6.

**5** (1 mark)

The table shows the times, in seconds, some people took to complete a puzzle.

| Time ( $t$ seconds) | Frequency |
|---------------------|-----------|
| $10 < t \leq 20$    | 4         |
| $20 < t \leq 30$    | 9         |
| $30 < t \leq 40$    | 7         |

Tom works out an estimate of the mean as 41.5 seconds.

Without working out the mean, explain why Tom must be wrong.

Look at the last class: every time is 40 seconds or less.

A mean can never be bigger than the largest value.

So the mean cannot be more than 40 seconds.

**Answer:** All times are 40 or less

**6** (3 marks)

The table shows information about the lengths,  $l$  cm, of 15 fish caught in a river.

| Length ( $l$ cm) | Frequency |
|------------------|-----------|
| $10 < l \leq 20$ | 4         |
| $20 < l \leq 30$ | 8         |
| $30 < l \leq 40$ | 3         |

Tick **one** box for each statement.

|                                                    | True | False |
|----------------------------------------------------|------|-------|
| The shortest fish could be exactly 10 cm long      |      | ✓     |
| At least one fish is longer than 30 cm             | ✓    |       |
| The median length is in the class $20 < l \leq 30$ | ✓    |       |
| An estimate of the mean length is 30 cm            |      | ✓     |

$10 < l$  means every fish is **more than** 10 cm. So a fish cannot be exactly 10 cm. Statement 1 is false.

There are 3 fish in the class  $30 < l \leq 40$ , so they are all longer than 30 cm. Statement 2 is true.

There are  $4 + 8 + 3 = 15$  fish, so the median is the 8th value.

Running totals: 4, then  $4 + 8 = 12$ . The 8th value is in  $20 < l \leq 30$ . Statement 3 is true.

Estimate of mean =  $(15 \times 4 + 25 \times 8 + 35 \times 3) \div 15 = 365 \div 15 = 24.3$  cm (1 d.p.). Statement 4 is false.

**Answer:** False, True, True, False

**Tip:** Read the inequality signs in the class carefully:  $10 < l$  does not include 10, but  $l \leq 20$  does include 20.

*Marks:* 3 marks for all four correct. 2 marks for three correct. 1 mark for two correct.

**7** (4 marks)

The table shows how long 30 patients waited at a health clinic one morning.

| Waiting time, $t$<br>(minutes) | Number of<br>patients | Midpoint |
|--------------------------------|-----------------------|----------|
| $0 \leq t < 10$                | 9                     | 5        |
| $10 \leq t < 20$               | 12                    | 15       |
| $20 \leq t < 30$               | 6                     | 25       |
| $30 \leq t < 40$               | 3                     | 35       |

- (a) Work out an estimate of the mean waiting time.  
You may use the midpoint column to help you.

The midpoints are 5, 15, 25 and 35.

Multiply each midpoint by its frequency:

$$5 \times 9 = 45, \quad 15 \times 12 = 180, \quad 25 \times 6 = 150, \quad 35 \times 3 = 105$$

$$\text{Add these up: } 45 + 180 + 150 + 105 = 480$$

$$\text{Total number of patients: } 9 + 12 + 6 + 3 = 30$$

$$\text{Mean} \approx 480 \div 30 = 16$$

**Answer:** 16 minutes

**Tip:** Divide by the total number of patients (30), not by the number of classes (4).

*Marks:* 1 mark for at least three midpoint  $\times$  frequency values. 1 mark for  $480 \div 30$ . 1 mark for 16.

- (b) Later that day, 10 more patients are seen.  
Each of them waits less than 10 minutes.  
Without doing any more calculations, explain what happens to the estimate of the mean waiting time.

The 10 new patients all go in the class  $0 \leq t < 10$ , with midpoint 5 minutes.

5 minutes is less than the estimated mean of 16 minutes.

Adding values below the mean makes the mean smaller.

(Check:  $(480 + 50) \div 40 = 13.25$  minutes.)

**Answer:** It decreases, because the new values (midpoint 5) are below the mean of 16

**Tip:** Say whether the mean goes up or down **and** give a reason that compares the new values with the mean.

**8** (4 marks)

A café starts using a new ordering system.

Before the change, the mean time customers waited for their food was 8 minutes.

After the change, the café records the waiting times of 50 customers.

| Waiting time, $t$<br>(minutes) | Frequency  |
|--------------------------------|------------|
| $0 \leq t < 4$                 | 18         |
| $4 \leq t < 6$                 | 20         |
| $6 \leq t < 10$                | 9          |
| $10 \leq t < 16$               | 3          |
|                                | Total = 50 |

The café wanted the mean waiting time to fall by at least 40%.

Use an estimate of the mean to decide whether the café met its target.

You must show your working.

The classes are different widths, so work out each midpoint carefully.

Midpoints: 2, 5, 8, 13

Multiply each midpoint by its frequency:

$$2 \times 18 = 36, \quad 5 \times 20 = 100, \quad 8 \times 9 = 72, \quad 13 \times 3 = 39$$

$$\text{Add these up: } 36 + 100 + 72 + 39 = 247$$

$$\text{Mean} \approx 247 \div 50 = 4.94 \text{ minutes}$$

$$\text{A 40\% fall from 8 minutes: } 8 \times 0.6 = 4.8 \text{ minutes}$$

4.94 is more than 4.8, so the target was **not** met.

(Or: percentage fall =  $\frac{8 - 4.94}{8} \times 100 = 38.25\%$ , which is less than 40%.)

**Answer:** Not met: estimated mean 4.94 minutes, but the target is 4.8 minutes (a fall of only 38.25%)

**Tip:** With unequal classes the midpoints are not evenly spaced: the midpoint of  $10 \leq t < 16$  is 13. A fall of 40% leaves 60% of the old mean.

*Marks:* 1 mark for at least three midpoint  $\times$  frequency values. 1 mark for  $247 \div 50$  (4.94). 1 mark for 4.8 or 38.25%. 1 mark for the correct conclusion with figures.

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