

Arcs and Sectors

Solutions

For AQA GCSE Mathematics (8300) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

A sector of a circle has an angle of 90°

What fraction of the whole circle is the sector?

Circle your answer.

Options: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$

A whole circle is 360° .

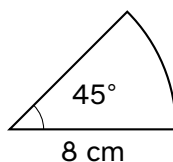
The fraction is $\frac{90}{360}$

Simplify: $\frac{90}{360} = \frac{1}{4}$

Answer: $\frac{1}{4}$

2 (2 marks)

The diagram shows a sector of a circle with radius 8 cm and angle 45°



Not drawn accurately

Work out the area of the sector.

Give your answer to 1 decimal place.

Area of a sector = $\frac{\text{angle}}{360} \times \pi r^2$

Here the angle is 45° and the radius is $r = 8$.

$$\text{Area} = \frac{45}{360} \times \pi \times 8^2$$

$$= 25.13 \dots$$

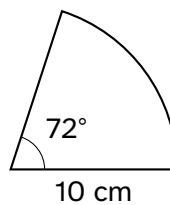
Round to 1 decimal place: 25.1

Answer: 25.1 cm^2

Marks: 1 mark for $\frac{45}{360} \times \pi \times 8^2$. 1 mark for 25.1 (or 25.13...).

3 (2 marks)

A sector has radius 10 cm and angle 72°



Not drawn accurately

Work out the arc length of the sector.

Give your answer to 1 decimal place.

$$\text{Arc length} = \frac{\text{angle}}{360} \times 2\pi r$$

Here the angle is 72° and $r = 10$.

$$\text{Arc length} = \frac{72}{360} \times 2 \times \pi \times 10$$

$$= 12.56 \dots$$

Round to 1 decimal place: 12.6

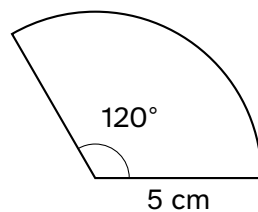
Answer: 12.6 cm

Tip: Arc length uses the circumference, $2\pi r$. Area uses πr^2 . Using the wrong one is a very common mistake.

Marks: 1 mark for $\frac{72}{360} \times 2 \times \pi \times 10$. 1 mark for 12.6 (or 12.56...).

4 (3 marks)

A sector has radius 5 cm and angle 120°



Not drawn accurately

Work out the perimeter of the sector.

Give your answer to 1 decimal place.

The perimeter is the arc plus the two straight sides (two radii).

$$\text{Arc length} = \frac{120}{360} \times 2 \times \pi \times 5$$

$$= 10.47 \dots$$

$$\text{Add the two radii: } 10.47 \dots + 5 + 5$$

$$= 20.47 \dots$$

Round to 1 decimal place: 20.5

Answer: 20.5 cm

Tip: Do not forget the two straight edges. Many students give just the arc length (10.5) as the perimeter.

Marks: 1 mark for working out the arc length. 1 mark for adding 10 (the two radii). 1 mark for 20.5.

5 (3 marks)

A circular pizza has diameter 30 cm.
It is cut into 8 equal slices.

Work out the area of one slice.
Give your answer to 1 decimal place.

The diameter is 30 cm, so the radius is $30 \div 2 = 15$ cm.

$$\begin{aligned}\text{Area of the whole pizza} &= \pi \times 15^2 \\ &= 706.8 \dots \text{ cm}^2\end{aligned}$$

There are 8 equal slices, so divide by 8:

$$706.8 \dots \div 8 = 88.35 \dots$$

Round to 1 decimal place: 88.4

Answer: 88.4 cm²

Tip: The formula πr^2 needs the radius, so halve the diameter first. Using 30 instead of 15 is a common mistake.

Marks: 1 mark for using radius 15. 1 mark for dividing the circle's area by 8. 1 mark for 88.4.

6 (2 marks)

Work out the area of a sector with radius 6 cm and angle 60°
Give your answer in terms of π .

$$60^\circ \text{ is } \frac{60}{360} = \frac{1}{6} \text{ of a circle.}$$

$$\text{Area of the whole circle} = \pi \times 6^2 = 36\pi$$

$$\text{Sector area} = \frac{1}{6} \times 36\pi$$

$$= 6\pi$$

Answer: $6\pi \text{ cm}^2$

Tip: “In terms of π ” means leave π as a symbol in your answer. Do not change it to a decimal, and do not leave it out.

Marks: 1 mark for $\frac{60}{360} \times \pi \times 6^2$ or $\frac{1}{6} \times 36\pi$. 1 mark for 6π .

7 (3 marks)

The minute hand of a clock is 12 cm long.

How far does the tip of the minute hand travel in 20 minutes?

Give your answer to 1 decimal place.

The minute hand goes all the way round in 60 minutes.

20 minutes is $\frac{20}{60} = \frac{1}{3}$ of a turn (which is 120°).

The tip moves along an arc of a circle with radius 12 cm.

$$\text{Circumference} = 2 \times \pi \times 12 = 75.39 \dots$$

$$\text{Arc length} = \frac{1}{3} \times 75.39 \dots = 25.13 \dots$$

Round to 1 decimal place: 25.1

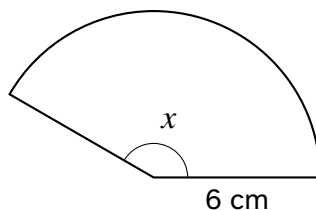
Answer: 25.1 cm

Marks: 1 mark for $\frac{1}{3}$ or 120° . 1 mark for $2 \times \pi \times 12$. 1 mark for 25.1.

8 (3 marks)

Here is a sector of a circle with radius 6 cm.

The area of the sector is $15\pi \text{ cm}^2$



Not drawn accurately

Work out the size of angle x .

Write the sector area formula with the angle x :

$$\frac{x}{360} \times \pi \times 6^2 = 15\pi$$

$$\pi \times 6^2 = 36\pi, \text{ so } \frac{x}{360} \times 36\pi = 15\pi$$

$$\text{Divide both sides by } 36\pi: \frac{x}{360} = \frac{15}{36}$$

$$\text{Multiply both sides by } 360: x = \frac{15}{36} \times 360$$

$$x = 150$$

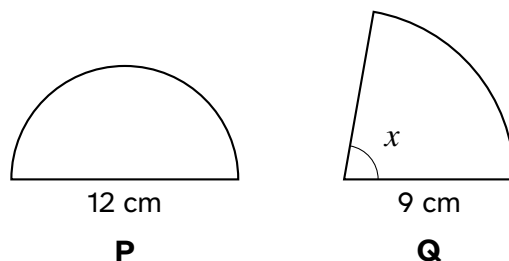
Answer: 150°

Marks: 1 mark for $\frac{x}{360} \times \pi \times 6^2$ (or 36π seen). 1 mark for $\frac{15}{36} \times 360$. 1 mark for 150.

9 (4 marks)

Shape P is a semicircle with diameter 12 cm.

Shape Q is a sector of a circle with radius 9 cm and angle x .



Not drawn accurately

Shape P and shape Q have the same area.

Work out the size of angle x .

You must show your working.

Shape P is half of a circle with diameter 12 cm, so its radius is 6 cm.

$$\text{Area of P} = \frac{1}{2} \times \pi \times 6^2 = 18\pi \text{ cm}^2$$

Shape Q is a sector with radius 9 cm and angle x .

$$\text{Area of Q} = \frac{x}{360} \times \pi \times 9^2 = \frac{x}{360} \times 81\pi$$

$$\text{The areas are equal: } \frac{x}{360} \times 81\pi = 18\pi$$

$$\text{Divide both sides by } 81\pi: \frac{x}{360} = \frac{18}{81}$$

$$x = \frac{18}{81} \times 360 = 80$$

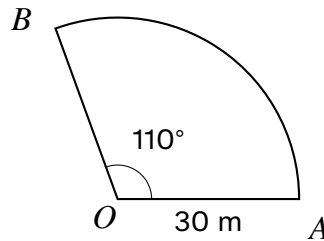
Answer: 80°

Tip: The 12 cm is the **diameter** of P. Halve it to get the radius before you use πr^2 , and remember P is only half a circle.

Marks: 1 mark for the area of P (18π or 56.5...). 1 mark for $\frac{x}{360} \times \pi \times 9^2$. 1 mark for putting the two areas equal. 1 mark for 80.

10 (4 marks)

A path in a park is in the shape of a sector of a circle, centre O .
The radius is 30 m and angle $AOB = 110^\circ$
The path goes from O to A , along the arc to B , then back to O .



Not drawn accurately

Esi walks once around the path at an average speed of 1.2 m/s.
How long does it take her?
Give your answer to the nearest second.

The path is OA , then the arc AB , then BO .

$$\text{Arc } AB = \frac{110}{360} \times 2 \times \pi \times 30$$

$$= 57.59 \dots \text{ m}$$

$$\text{Total length} = 30 + 57.59 \dots + 30 = 117.59 \dots \text{ m}$$

$$\text{Time} = \text{distance} \div \text{speed}$$

$$= 117.59 \dots \div 1.2 = 97.99 \dots \text{ s}$$

To the nearest second: 98 s

Answer: 98 seconds

Tip: The path goes along both straight sides as well as the arc. Do not forget to add $30 + 30$.

Marks: 1 mark for $\frac{110}{360} \times 2 \times \pi \times 30$. 1 mark for adding the two 30 m radii. 1 mark for dividing the total length by 1.2. 1 mark for 98.