

Mock Set D – Paper 2

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

On the grid, draw the line $3x + 4y = 11$ for values of x from -3 to 5

Work out some points on the line. Substitute values of x into $3x + 4y = 11$:

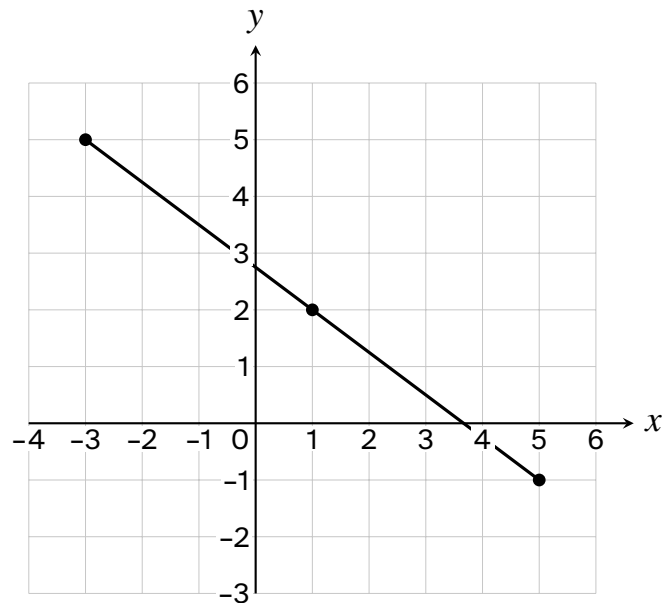
$$x = -3: -9 + 4y = 11, \text{ so } 4y = 20 \text{ and } y = 5$$

$$x = 1: 3 + 4y = 11, \text{ so } 4y = 8 \text{ and } y = 2$$

$$x = 5: 15 + 4y = 11, \text{ so } 4y = -4 \text{ and } y = -1$$

$$(\text{Or rearrange: } y = \frac{11 - 3x}{4}, \text{ a line with gradient } -\frac{3}{4}.)$$

Plot $(-3, 5)$, $(1, 2)$ and $(5, -1)$ and join them with a ruler from $x = -3$ to $x = 5$.



Answer: Ruled straight line from $(-3, 5)$ to $(5, -1)$

Tip: Choose values of x that make $11 - 3x$ divisible by 4, such as $x = -3, 1$ and 5 , so the points have whole-number coordinates. Plot at least three points: if they are not in a straight line, check your working.

Marks: 1 mark for one correct point (or $y = \frac{11-3x}{4}$). 2 marks for at least two correct points plotted, or a correct line that does not cover the whole range. 3 marks for the full line from $x = -3$ to $x = 5$.

2 (4 marks)

Water flows into a tank.

The volume of water in the tank, V litres, t minutes after a tap is turned on is given by

$$V = 4t^3 - t^2 + 5t + 7 \quad 0 \leq t \leq 3$$

- (a) Show that, when the rate of change of V with respect to t is 11 litres per minute, $6t^2 - t - 3 = 0$

The rate of change of V with respect to t is $\frac{dV}{dt}$

$$\frac{dV}{dt} = 12t^2 - 2t + 5$$

Set it equal to 11: $12t^2 - 2t + 5 = 11$

Subtract 11: $12t^2 - 2t - 6 = 0$

Divide every term by 2: $6t^2 - t - 3 = 0$ as required.

Answer: $12t^2 - 2t + 5 = 11$, so $12t^2 - 2t - 6 = 0$, so $6t^2 - t - 3 = 0$

Tip: "Rate of change of V with respect to t " means $\frac{dV}{dt}$. Do not put 11 into the formula for V : 11 is a rate, not a volume.

Marks: 1 mark for $\frac{dV}{dt} = 12t^2 - 2t + 5$. 1 mark for setting it equal to 11 and rearranging to $6t^2 - t - 3 = 0$ with no errors.

- (b) Work out the time when the rate of change of V with respect to t is 11 litres per minute.

Give your answer to 3 significant figures.

Use the quadratic formula with $a = 6$, $b = -1$, $c = -3$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 6 \times (-3)}}{2 \times 6} = \frac{1 \pm \sqrt{73}}{12}$$

$$t = \frac{1 + \sqrt{73}}{12} = 0.7953 \dots \quad \text{or} \quad t = \frac{1 - \sqrt{73}}{12} = -0.6286 \dots$$

t is a time after the tap is turned on, so $t \geq 0$: reject the negative value.

$t = 0.795$ minutes (3 s.f.)

Answer: 0.795 minutes

Tip: Bracket the negative numbers when you substitute: $(-1)^2 = 1$, and $-4 \times 6 \times (-3) = +72$. Time cannot be negative here, so give only the positive solution.

Marks: 1 mark for a correct substitution into the quadratic formula. 1 mark for 0.795 (with the negative value rejected).

3 (4 marks)

Here are the first four terms of a linear sequence.

48.6 46.3 44 41.7

- (a) Work out an expression for the n th term.

The terms go down by 2.3 each time, so the common difference is -2.3

The n th term starts $-2.3n \dots$

When $n = 1$, $-2.3n = -2.3$ and the first term is 48.6

$$48.6 - (-2.3) = 50.9$$

$$n\text{th term} = 50.9 - 2.3n$$

Answer: $50.9 - 2.3n$

Tip: The difference is negative because the terms are going down. The constant is the first term minus the difference: $48.6 - (-2.3) = 50.9$, not $48.6 - 2.3$.

Marks: 1 mark for $-2.3n$. 1 mark for the full expression $50.9 - 2.3n$.

- (b) Show that -20 is not a term of this sequence.

If -20 were a term, then $50.9 - 2.3n = -20$ for a whole number n .

$$2.3n = 50.9 + 20 = 70.9$$

$$n = \frac{70.9}{2.3} = 30.826 \dots$$

n is not a whole number, so -20 is not a term of the sequence.

(Check: the 30th term is $50.9 - 69 = -18.1$ and the 31st term is $50.9 - 71.3 = -20.4$, so the sequence jumps over -20 .)

Answer: $50.9 - 2.3n = -20$ gives $n = 30.8 \dots$, which is not a whole number, so -20 is not a term.

Tip: Set the n th term equal to -20 and solve for n . A position in a sequence must be a whole number. Finish with a sentence that answers the question.

Marks: 1 mark for $50.9 - 2.3n = -20$ (or for the 30th and 31st terms). 1 mark for $n = 30.8 \dots$ and the conclusion that it is not a whole number.

4 (3 marks)

n is a number such that

$$-6 < n \leq 3$$

- (a) Work out the possible values of n^2
Give your answer as an inequality.

n can be negative, zero or positive.

The smallest value of n^2 is when $n = 0$, so $n^2 \geq 0$

The largest values of n^2 come from the end of the interval furthest from 0: n close to -6

n cannot equal -6 , so n^2 cannot equal 36, but it can be as close as we like. So $n^2 < 36$

(The other end gives $3^2 = 9$, which is smaller.)

$$0 \leq n^2 < 36$$

Answer: $0 \leq n^2 < 36$

Tip: Squaring the two ends (36 and 9) does not give the range. Because n can be 0, the least value of n^2 is 0. Keep the signs: -6 is not allowed, so $n^2 < 36$.

Marks: 1 mark for each end of the inequality: $0 \leq n^2$ and $n^2 < 36$.

- (b) Hence, write down the possible values of $20 - n^2$ as an inequality.

Multiply the inequality in (a) by -1 (this reverses the signs): $-36 < -n^2 \leq 0$

Add 20: $-16 < 20 - n^2 \leq 20$

Answer: $-16 < 20 - n^2 \leq 20$

Tip: Subtracting n^2 turns the largest value of n^2 into the smallest value of $20 - n^2$, and the inequality signs swap sides.

5 (4 marks)

- (a) Which matrix represents an enlargement, centre the origin, scale factor $\frac{1}{2}$?

Options: $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$, $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$, $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

An enlargement with scale factor k , centre the origin, multiplies both coordinates by k .

Its matrix is $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, so here it is $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Answer: $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Tip: Check with the unit square: $(1, 0)$ must go to $(\frac{1}{2}, 0)$ and $(0, 1)$ to $(0, \frac{1}{2})$. These images are the two columns of the matrix.

- (b) **S** is a rotation of 90° clockwise about the origin and **T** is a reflection in the x -axis. Work out the matrix for **S** followed by **T**.

Find each matrix from the images of $(1, 0)$ and $(0, 1)$, which form its columns.

Rotation 90° clockwise: $(1, 0) \rightarrow (0, -1)$ and $(0, 1) \rightarrow (1, 0)$, so **S** = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Reflection in the x -axis: $(1, 0) \rightarrow (1, 0)$ and $(0, 1) \rightarrow (0, -1)$, so **T** = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

S followed by **T** is **TS** (the second transformation goes on the left).

$$\mathbf{TS} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 \times 0 + 0 \times (-1) & 1 \times 1 + 0 \times 0 \\ 0 \times 0 + (-1) \times (-1) & 0 \times 1 + (-1) \times 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(This is a reflection in the line $y = x$.)

Answer: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Tip: For “**S** followed by **T**” work out **TS**: the second transformation’s matrix goes on the left. **ST** gives $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, which is wrong.

Marks: 1 mark for the matrix for **S** or **T**. 1 mark for multiplying in the correct order, **TS**. 1 mark for the correct matrix.

6 (3 marks)

A permit code is two different letters (not I, O or Q) followed by three digits. The first digit cannot be 0 and the last digit must be odd. Digits may repeat. How many codes are possible?

Letters: $26 - 3 = 23$ letters can be used.

First letter: 23 choices. Second letter: 22 choices (it must be different).

First digit: 1 to 9, so 9 choices. Middle digit: 10 choices.

Last digit: 1, 3, 5, 7 or 9, so 5 choices.

Multiply the numbers of choices: $23 \times 22 \times 9 \times 10 \times 5 = 227\,700$

Answer: 227700

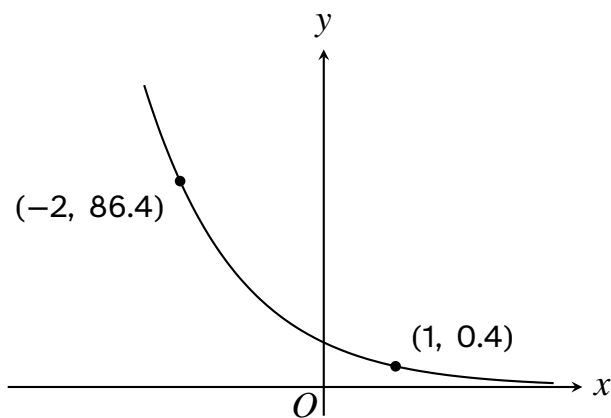
Tip: Work position by position and start with the restrictions. The letters must be different, so the second letter has one fewer choice. The digits can repeat, so the middle digit has all 10 choices.

Marks: 2 marks for $23 \times 22 \times 9 \times 10 \times 5$ (1 mark if at least three of the five numbers are right). 1 mark for 227700.

7 (4 marks)

The sketch shows the graph of $y = a \times b^{-x}$, where $a > 0$ and $b > 0$

The graph goes through $(-2, 86.4)$ and $(1, 0.4)$



Not drawn accurately

Work out the value of a and the value of b .

Substitute each point into $y = a \times b^{-x}$

$$(-2, 86.4): 86.4 = a \times b^{-(-2)} = ab^2 \quad (1)$$

$$(1, 0.4): 0.4 = a \times b^{-1} = \frac{a}{b} \quad (2)$$

$$\text{Divide (1) by (2): } \frac{ab^2}{a \div b} = b^3, \text{ so } b^3 = \frac{86.4}{0.4} = 216$$

$$b = \sqrt[3]{216} = 6$$

$$\text{From (2): } a = 0.4 \times 6 = 2.4$$

Answer: $a = 2.4$, $b = 6$

Tip: Take care with the negative power: when $x = -2$, $b^{-x} = b^2$. Dividing the two equations cancels a .

Marks: 1 mark for one correct equation, such as $ab^2 = 86.4$. 1 mark for eliminating a to get $b^3 = 216$. 1 mark for $b = 6$. 1 mark for $a = 2.4$.

8 (3 marks)

k is an integer. Prove that $(k + 3)^3 - (k - 3)^3$ is a multiple of 18

Row 3 of Pascal's triangle is 1 3 3 1

$$(k + 3)^3 = k^3 + 3 \times k^2 \times 3 + 3 \times k \times 3^2 + 3^3 = k^3 + 9k^2 + 27k + 27$$

$$(k - 3)^3 = k^3 + 3 \times k^2 \times (-3) + 3 \times k \times (-3)^2 + (-3)^3 = k^3 - 9k^2 + 27k - 27$$

Subtract, keeping the second expansion in a bracket:

$$k^3 + 9k^2 + 27k + 27 - (k^3 - 9k^2 + 27k - 27) = k^3 + 9k^2 + 27k + 27 - k^3 + 9k^2 - 27k + 27 = 18k^2 + 54$$

$$\text{Factorise: } 18k^2 + 54 = 18(k^2 + 3)$$

$k^2 + 3$ is an integer, so $18(k^2 + 3)$ is a multiple of 18.

Answer: $18k^2 + 54 = 18(k^2 + 3)$, which is a multiple of 18

Tip: Use Pascal's triangle (1, 3, 3, 1) and bracket the negative number: $(-3)^2 = 9$ and $(-3)^3 = -27$. Put the whole of $(k - 3)^3$ in a bracket before subtracting, then finish with a sentence saying why $18(k^2 + 3)$ is a multiple of 18.

Marks: 1 mark for expanding either cube correctly. 1 mark for the subtraction with the bracket, giving $18k^2 + 54$. 1 mark for $18(k^2 + 3)$ with a conclusion.

9 (3 marks)

Write $\frac{5}{x+6} + \frac{34-11x}{3x^2+16x-12}$ as a single fraction in its simplest form.

Factorise the quadratic denominator: $3x^2 + 16x - 12 = (x + 6)(3x - 2)$

Common denominator: $(x + 6)(3x - 2)$

$$\frac{5}{x+6} = \frac{5(3x-2)}{(x+6)(3x-2)} = \frac{15x-10}{(x+6)(3x-2)}$$

Add the numerators: $15x - 10 + 34 - 11x = 4x + 24$

$$\frac{4x+24}{(x+6)(3x-2)} = \frac{4(x+6)}{(x+6)(3x-2)}$$

Cancel the common factor $(x + 6)$: $\frac{4}{3x - 2}$

Answer: $\frac{4}{3x - 2}$

Tip: Factorise the quadratic first: it already contains $(x + 6)$, so that is the lowest common denominator. "Simplest form" means you must factorise the top and cancel.

Marks: 1 mark for $(x + 6)(3x - 2)$. 1 mark for the single fraction $\frac{4x+24}{(x+6)(3x-2)}$. 1 mark for $\frac{4}{3x-2}$.

10 (3 marks)

$A(-2.4, 8.8)$, $B(3.6, 0.8)$. AB is extended to C so that $AB:BC = 5:3$. Work out the equation of the line OC .

C is beyond B , on the same straight line, with $BC = \frac{3}{5}$ of AB .

From A to B : x changes by $3.6 - (-2.4) = 6$ and y changes by $0.8 - 8.8 = -8$

From B to C : x changes by $\frac{3}{5} \times 6 = 3.6$ and y changes by $\frac{3}{5} \times (-8) = -4.8$

C is $(3.6 + 3.6, 0.8 - 4.8) = (7.2, -4)$

The line through $O(0, 0)$ and $C(7.2, -4)$ has gradient $\frac{-4}{7.2} = -\frac{5}{9}$ and y-intercept 0

$$y = -\frac{5}{9}x$$

Answer: $y = -\frac{5}{9}x$

Tip: Draw a quick sketch: C is **beyond** B , so add $\frac{3}{5}$ of the step from A to B on to B . It is not the point that divides AB in the ratio 5:3.

Marks: 1 mark for a correct method for one coordinate of C . 1 mark for $C(7.2, -4)$. 1 mark for $y = -\frac{5}{9}x$.

11 (4 marks)

$y = \frac{2x^7 - 9x^2}{6x^4}$. Work out the value of x for which $\frac{d^2y}{dx^2} = 0$. Give your answer to 3 significant figures.

Divide each term on the top by $6x^4$ first:

$$y = \frac{2x^7}{6x^4} - \frac{9x^2}{6x^4} = \frac{1}{3}x^3 - \frac{3}{2}x^{-2}$$

$$\frac{dy}{dx} = x^2 - \frac{3}{2} \times (-2)x^{-3} = x^2 + 3x^{-3}$$

$$\frac{d^2y}{dx^2} = 2x + 3 \times (-3)x^{-4} = 2x - 9x^{-4}$$

Set it equal to 0: $2x - \frac{9}{x^4} = 0$

Multiply by x^4 : $2x^5 - 9 = 0$, so $x^5 = 4.5$

$$x = \sqrt[5]{4.5} = 1.3509 \dots = 1.35 \text{ (3 s.f.)}$$

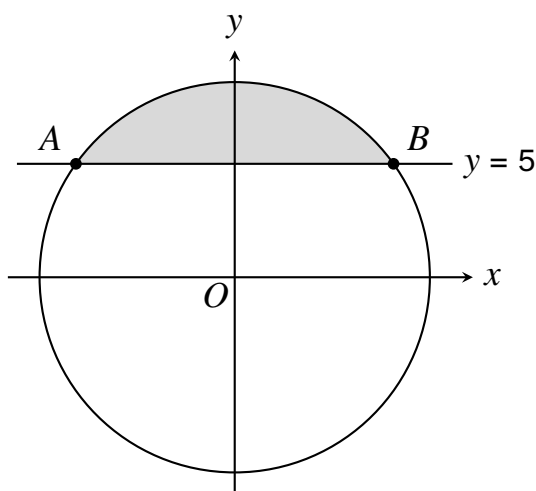
Answer: $x = 1.35$

Tip: You cannot differentiate the top and the bottom separately. Split the fraction into powers of x , then differentiate twice. With negative powers, subtract 1: $x^{-2} \rightarrow -2x^{-3}$. To solve, multiply through by x^4 .

Marks: 1 mark for writing y as $\frac{1}{3}x^3 - \frac{3}{2}x^{-2}$. 1 mark for $\frac{dy}{dx} = x^2 + 3x^{-3}$. 1 mark for $2x - 9x^{-4} = 0$ (or $x^5 = 4.5$). 1 mark for 1.35.

12 (4 marks)

The diagram shows the circle $x^2 + y^2 = 74$ and the straight line $y = 5$.
The line meets the circle at the points A and B .



Not drawn accurately

The line $y = 5$ meets the circle $x^2 + y^2 = 74$ at A and B . Work out the area of the shaded segment. Give your answer to 3 significant figures.

At A and B , $y = 5$: $x^2 + 25 = 74$, so $x^2 = 49$ and $x = \pm 7$

A is $(-7, 5)$ and B is $(7, 5)$, so $AB = 14$

The radius is $\sqrt{74}$

Let M be $(0, 5)$, the midpoint of AB . In the right-angled triangle OMB :

$$\tan(\text{angle } MOB) = \frac{7}{5}$$

$$\text{Angle } MOB = \tan^{-1} \frac{7}{5} = 54.462 \dots^\circ, \text{ so angle } AOB = 108.924 \dots^\circ$$

$$\text{Area of sector } OAB = \frac{108.924 \dots}{360} \times \pi \times 74 = 70.340 \dots$$

$$\text{Area of triangle } OAB = \frac{1}{2} \times 14 \times 5 = 35$$

Shaded area = $70.340 \dots - 35 = 35.340 \dots = 35.3$ (3 s.f.)

Answer: 35.3 units²

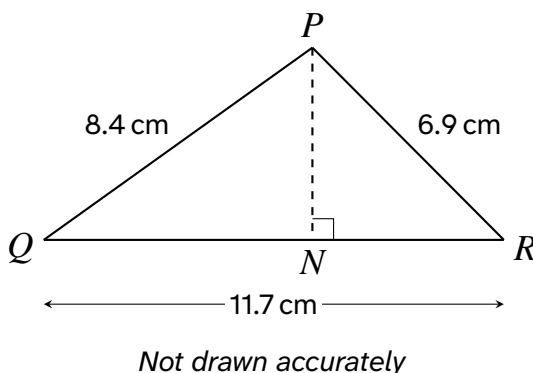
Tip: The circle's equation gives $r^2 = 74$, so the radius is $\sqrt{74}$, not 74. Keep the full angle in your calculator so that the final answer is accurate.

Marks: 1 mark for $x = \pm 7$ (or $AB = 14$). 1 mark for angle $AOB = 108.9^\circ$ (or half of it). 1 mark for sector minus triangle. 1 mark for 35.3.

13 (4 marks)

PQR is a triangle.

N is the point on QR such that PN is perpendicular to QR .



In triangle PQR , $PQ = 8.4$ cm, $PR = 6.9$ cm and $QR = 11.7$ cm. N is on QR and PN is perpendicular to QR . Work out PN .

Use the cosine rule to find an angle, for example angle Q :

$$\cos Q = \frac{8.4^2 + 11.7^2 - 6.9^2}{2 \times 8.4 \times 11.7} = \frac{70.56 + 136.89 - 47.61}{196.56} = \frac{159.84}{196.56} = 0.8131 \dots$$

$$Q = \cos^{-1} 0.8131 \dots = 35.59 \dots^\circ$$

Triangle PQN is right-angled at N , with hypotenuse $PQ = 8.4$ cm.

$$PN = 8.4 \times \sin 35.59 \dots^\circ = 4.888 \dots = 4.89 \text{ cm (3 s.f.)}$$

(Or: the largest angle is $P = 99.29 \dots^\circ$, area $= \frac{1}{2} \times 8.4 \times 6.9 \times \sin P = 28.59 \dots$, and $PN = \frac{2 \times 28.59 \dots}{11.7} = 4.89 \text{ cm.}$)

Answer: 4.89 cm

Tip: Keep the angle in your calculator memory. Rounding it to 36° gives 4.94, which is outside the accepted range.

Marks: 1 mark for a correct cosine rule for one of the angles. 1 mark for the angle. 1 mark for a correct method for PN . 1 mark for 4.89.

14 (6 marks)

The straight line $2x + y = 7$ and the circle $(x - 3)^2 + (y + 1)^2 = 18$ intersect at the points P and Q .

- (a) Show that the x -coordinates of P and Q satisfy the equation $5x^2 - 38x + 55 = 0$

Rearrange the line: $y = 7 - 2x$

Substitute into the equation of the circle. $y + 1 = 7 - 2x + 1 = 8 - 2x$, so

$$(x - 3)^2 + (8 - 2x)^2 = 18$$

$$(x - 3)^2 = x^2 - 6x + 9$$

$$(8 - 2x)^2 = 64 - 32x + 4x^2$$

$$x^2 - 6x + 9 + 64 - 32x + 4x^2 = 18$$

$$5x^2 - 38x + 73 = 18$$

$$5x^2 - 38x + 55 = 0 \text{ as required.}$$

Answer: $(x - 3)^2 + (8 - 2x)^2 = 18$ gives $5x^2 - 38x + 55 = 0$

Tip: Make y the subject of the line first. Simplify $y + 1$ to $8 - 2x$ before squaring, and expand $(8 - 2x)^2$ as $(8 - 2x)(8 - 2x)$: it has a middle term.

Marks: 1 mark for a correct expansion of $(8 - 2x)^2$ or $(x - 3)^2$. 1 mark for a full equation with both expansions. 1 mark for $5x^2 - 38x + 55 = 0$ with no errors.

- (b) Work out the distance between P and Q to 3 significant figures.

$$x = \frac{38 \pm \sqrt{(-38)^2 - 4 \times 5 \times 55}}{2 \times 5} = \frac{38 \pm \sqrt{344}}{10}$$

$$x = 5.6547... \text{ or } x = 1.9452...$$

Use the line for the y -coordinates: $y = 7 - 2 \times 5.6547... = -4.3094...$ and
 $y = 7 - 2 \times 1.9452... = 3.1094...$

Differences: $5.6547... - 1.9452... = 3.7094...$ and
 $3.1094... - (-4.3094...) = 7.4188...$

$$PQ = \sqrt{3.7094...^2 + 7.4188...^2} = \sqrt{68.8} = 8.2945... = 8.29 \text{ (3 s.f.)}$$

Answer: 8.29 units

Tip: Find **both** points in full: work out each y -coordinate from the line $y = 7 - 2x$, then use Pythagoras on the differences. Keep the unrounded values in your calculator.

Marks: 1 mark for solving the quadratic (both x values). 1 mark for the y -coordinates and a correct Pythagoras method for PQ . 1 mark for 8.29.

15 (4 marks)

In the expansion of $(a + 3x)^6$, where $a > 0$, coefficient of x^2 : coefficient of $x^3 = 5:8$.
Work out a .

Row 6 of Pascal's triangle: 1 6 15 20 15 6 1

x^2 term: $15 \times a^4 \times (3x)^2 = 15 \times a^4 \times 9x^2 = 135a^4x^2$, so the coefficient is $135a^4$

x^3 term: $20 \times a^3 \times (3x)^3 = 20 \times a^3 \times 27x^3 = 540a^3x^3$, so the coefficient is $540a^3$

$$\frac{135a^4}{540a^3} = \frac{5}{8}$$

$$\frac{a}{4} = \frac{5}{8} \quad (a \text{ is positive, so } a^3 \neq 0 \text{ and it cancels})$$

$$a = \frac{4 \times 5}{8} = 2.5$$

Answer: $a = 2.5$

Tip: Bracket $(3x)$ before raising it to a power: $(3x)^3 = 27x^3$, not $3x^3$. A ratio $p:q = 5:8$ means $\frac{p}{q} = \frac{5}{8}$, so $8p = 5q$.

Marks: 1 mark for $135a^4$. 1 mark for $540a^3$. 1 mark for an equation from the ratio, such as $8 \times 135a^4 = 5 \times 540a^3$. 1 mark for 2.5.

16 (4 marks)

The functions f and g are given by

$$f(x) = x^2 - 4 \quad x \geq 0$$

$$g(x) = \sqrt{2x+10} \quad x \geq -5$$

(a) Work out $f^{-1}(x)$

Write $y = x^2 - 4$ and make x the subject.

Add 4: $x^2 = y + 4$

Square root: $x = \sqrt{y+4}$ (the positive root, because the domain of f is $x \geq 0$)

Write the inverse in terms of x : $f^{-1}(x) = \sqrt{x+4}$

Check: $f(3) = 5$ and $f^{-1}(5) = \sqrt{9} = 3$ ✓

Answer: $f^{-1}(x) = \sqrt{x+4}$

Tip: Undo the operations in reverse order: f squares and then subtracts 4, so add 4 and then square root. The domain of f is $x \geq 0$, so the inverse uses only the positive square root.

Marks: 1 mark for $x^2 = y + 4$ (or $y^2 = x + 4$). 1 mark for $\sqrt{x+4}$.

(b) Solve $gf(x) = 5$. Give your answer to 3 significant figures.

In $gf(x)$, f acts first: $gf(x) = g(x^2 - 4) = \sqrt{2(x^2 - 4) + 10} = \sqrt{2x^2 + 2}$

$$\sqrt{2x^2 + 2} = 5$$

Square both sides: $2x^2 + 2 = 25$

$$2x^2 = 23, \text{ so } x^2 = 11.5$$

$$x = \pm\sqrt{11.5} = \pm 3.3911 \dots$$

The domain of f is $x \geq 0$, so reject the negative value.

$$x = 3.39 \text{ (3 s.f.)}$$

Answer: $x = 3.39$

Tip: Square-rooting $x^2 = 11.5$ gives ± 3.39 , but f is only defined for $x \geq 0$, so -3.39 is not allowed. Always check answers against the domain.

Marks: 1 mark for $\sqrt{2(x^2 - 4) + 10} = 5$ (or $2x^2 + 2 = 25$). 1 mark for 3.39 only.

17 (4 marks)

Solve $4x - 9x^{\frac{1}{2}} = 2$. Give your answer to 3 significant figures.

$x = \left(x^{\frac{1}{2}}\right)^2$, so this is a quadratic in $x^{\frac{1}{2}}$.

Let $u = x^{\frac{1}{2}}$. Then $x = u^2$ and the equation becomes $4u^2 - 9u = 2$, i.e. $4u^2 - 9u - 2 = 0$

$$u = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 4 \times (-2)}}{2 \times 4} = \frac{9 \pm \sqrt{113}}{8}$$

$$u = 2.4537 \dots \text{ or } u = -0.2037 \dots$$

$$u = x^{\frac{1}{2}} = \sqrt{x} \text{ cannot be negative, so } x^{\frac{1}{2}} = 2.4537 \dots$$

$$x = 2.4537 \dots^2 = 6.0209 \dots$$

$$x = 6.02 \text{ (3 s.f.)}$$

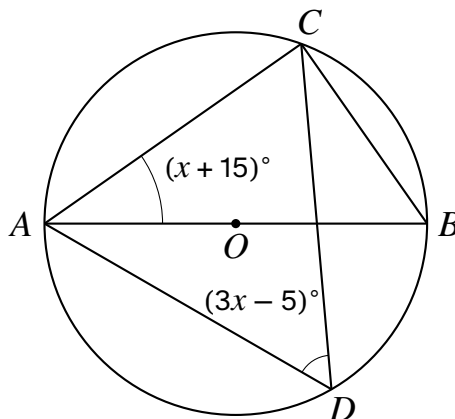
Answer: $x = 6.02$

Tip: x is the square of $x^{\frac{1}{2}}$, so replace $x^{\frac{1}{2}}$ by u and x by u^2 . Solve for u , reject the negative value (a square root cannot be negative), then square to get x . Giving $u = 2.45$ as the answer is a common mistake.

Marks: 1 mark for the quadratic $4u^2 - 9u - 2 = 0$ where $u = x^{\frac{1}{2}}$. 1 mark for using the formula. 1 mark for $x^{\frac{1}{2}} = 2.4537 \dots$ (negative value rejected). 1 mark for 6.02.

18 (3 marks)

The points A , B , C and D lie on a circle with centre O .
The line AB is a diameter.



Not drawn accurately

AB is a diameter. Angle $CAB = (x + 15)^\circ$ and angle $ADC = (3x - 5)^\circ$. Work out angle ADC .

Angle $ACB = 90^\circ$ (the angle in a semicircle is 90°)

Angle $ABC = \text{angle } ADC = (3x - 5)^\circ$ (angles in the same segment are equal)

Angles in triangle ABC : $x + 15 + 90 + 3x - 5 = 180$

$4x + 100 = 180$, so $4x = 80$ and $x = 20$

Angle $ADC = 3 \times 20 - 5 = 55^\circ$

Answer: 55°

Tip: Look for the right angle: AB is a diameter, so the angle at C in triangle ABC is 90° .

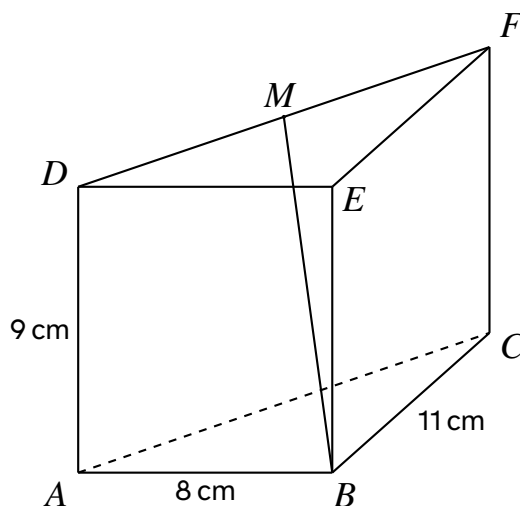
Angles ADC and ABC stand on the same chord AC , so they are equal.

Marks: 1 mark for angle $ACB = 90^\circ$ or angle $ABC = 3x - 5$. 1 mark for a correct equation using both. 1 mark for 55° .

19 (5 marks)

$ABCDEF$ is a triangular prism.

- The base ABC is horizontal and angle $ABC = 90^\circ$
- AD , BE and CF are vertical edges.
- $AB = 8$ cm $BC = 11$ cm $AD = 9$ cm
- M is the midpoint of DF .



Not drawn accurately

Triangular prism: base ABC horizontal, angle $ABC = 90^\circ$, $AB = 8$ cm, $BC = 11$ cm, vertical edges 9 cm. M is the midpoint of DF . Work out the angle between MB and the plane ABC .

Let N be the point on the base directly below M . Then $MN = 9$ cm and N is the midpoint of AC .

Measured from B , N is half of AB along one edge and half of BC along the other: 4 cm and 5.5 cm.

$$BN = \sqrt{4^2 + 5.5^2} = \sqrt{46.25} = 6.800 \dots \text{ cm} \quad (\text{or } AC = \sqrt{8^2 + 11^2} = \sqrt{185} \text{ and } BN = \frac{1}{2}AC)$$

Triangle MNB is right-angled at N . The angle between MB and the base is angle MBN .

$$\tan(\text{angle } MBN) = \frac{MN}{BN} = \frac{9}{6.800 \dots}$$

$$\text{Angle } MBN = \tan^{-1} 1.323 \dots = 52.92 \dots = 52.9^\circ \text{ (1 d.p.)}$$

Answer: 52.9°

Tip: Draw the right-angled triangle MNB separately. The point under M is the midpoint of AC , so BN is half of AC , not half of AB or BC .

Marks: 1 mark for AC (or for finding where N is). 1 mark for $BN = 6.80$. 1 mark for setting up the right-angled triangle MNB with $MN = 9$. 1 mark for a correct trig ratio for the angle. 1 mark for 52.9 .

20 (4 marks)

$y = kx^3 + 3x^2$ passes through $P(-2, 8)$. Work out the equation of the normal to the curve at P , in the form $ax + by = c$ where a , b and c are integers.

$$P \text{ is on the curve: } 8 = k \times (-2)^3 + 3 \times (-2)^2 = -8k + 12$$

$$8k = 4, \text{ so } k = \frac{1}{2}$$

$$\frac{dy}{dx} = 3kx^2 + 6x = \frac{3}{2}x^2 + 6x$$

$$\text{At } P: \frac{dy}{dx} = \frac{3}{2} \times 4 + 6 \times (-2) = 6 - 12 = -6$$

This is the gradient of the tangent. The normal is perpendicular, so its gradient is $\frac{1}{6}$ (the negative reciprocal).

$$y - 8 = \frac{1}{6}(x + 2)$$

$$\text{Multiply by 6: } 6y - 48 = x + 2$$

$$x - 6y = -50$$

Answer: $x - 6y = -50$

Tip: Use the point P to find k first. $\frac{dy}{dx}$ gives the gradient of the **tangent**; the normal is perpendicular, so use the negative reciprocal. Bracket -2 when you substitute: $(-2)^2 = 4$ and $(-2)^3 = -8$.

Marks: 1 mark for $k = \frac{1}{2}$ from the point P . 1 mark for $\frac{dy}{dx} = 3kx^2 + 6x$. 1 mark for the normal gradient $\frac{1}{6}$ used in an equation through $(-2, 8)$. 1 mark for $x - 6y = -50$.

21 (4 marks)

Solve $7 \cos^2 x - 9 \sin x = 8$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place.

Use $\cos^2 x = 1 - \sin^2 x$: $7(1 - \sin^2 x) - 9 \sin x = 8$

$7 - 7 \sin^2 x - 9 \sin x = 8$, so $7 \sin^2 x + 9 \sin x + 1 = 0$

This is a quadratic in $\sin x$: $\sin x = \frac{-9 \pm \sqrt{9^2 - 4 \times 7 \times 1}}{2 \times 7} = \frac{-9 \pm \sqrt{53}}{14}$

$\sin x = -0.12284 \dots$ or $\sin x = -1.162 \dots$

$\sin x$ cannot be less than -1 , so $\sin x = -1.162 \dots$ has no solutions.

$\sin^{-1}(0.12284 \dots) = 7.056 \dots^\circ$. $\sin x$ is negative, so x is in the third or fourth quadrant.

$x = 180 + 7.056 \dots = 187.1^\circ$ or $x = 360 - 7.056 \dots = 352.9^\circ$

Answer: $x = 187.1^\circ$ or $x = 352.9^\circ$

Tip: $\cos^2 x = 1 - \sin^2 x$, never $1 - \sin x$. Reject the value of $\sin x$ outside -1 to 1 . For a negative sine, the angles are $180^\circ + \alpha$ and $360^\circ - \alpha$, where $\alpha = \sin^{-1} 0.123$.

Marks: 1 mark for replacing $\cos^2 x$ with $1 - \sin^2 x$. 1 mark for solving the quadratic in $\sin x$ (to get $\sin x = -0.123$). 1 mark for one correct angle. 1 mark for both angles and no others.