

Mock Set D – Paper 1

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

A straight line has gradient $\frac{7}{24}$

The line passes through the point $A(-24, -11)$

The line crosses the y -axis at the point B .

(a) Work out the coordinates of B .

Use $y - y_1 = m(x - x_1)$ with $m = \frac{7}{24}$ and the point $(-24, -11)$:

$$y - (-11) = \frac{7}{24}(x - (-24)), \text{ so } y + 11 = \frac{7}{24}(x + 24)$$

B is on the y -axis, so $x = 0$ there.

$$y + 11 = \frac{7}{24} \times 24 = 7$$

$$y = 7 - 11 = -4$$

Answer: $(0, -4)$

Tip: The y -axis is where $x = 0$, so the answer is the point $(0, c)$. Once you have it, stop: extra working after the answer can cost the mark.

Marks: 1 mark for a correct equation of the line (or $c = -4$). 1 mark for $(0, -4)$.

(b) Work out the length AB .

From $A(-24, -11)$ to $B(0, -4)$: 24 across and 7 up.

$$AB^2 = 24^2 + 7^2 = 576 + 49 = 625$$

$$AB = \sqrt{625} = 25$$

Answer: 25 units

Tip: On a non-calculator paper, look for Pythagorean triples: 7, 24, 25 is one of them. A gradient of $\frac{7}{24}$ means 7 up for every 24 across.

Marks: 1 mark for $24^2 + 7^2$ (or $\sqrt{625}$). 1 mark for 25.

2 (2 marks)

Factorise fully $24a^2b^3 - 40a^4b$

Highest common factor of 24 and 40: 8

Lowest power of a in both terms: a^2 . Lowest power of b : b

So the HCF is $8a^2b$.

$$24a^2b^3 \div 8a^2b = 3b^2 \quad \text{and} \quad 40a^4b \div 8a^2b = 5a^2$$

$$24a^2b^3 - 40a^4b = 8a^2b(3b^2 - 5a^2)$$

Answer: $8a^2b(3b^2 - 5a^2)$

Tip: "Fully" means take out the **highest** common factor: the number, every letter that is in both terms, each to its lowest power. Expand your answer to check it.

Marks: 2 marks for $8a^2b(3b^2 - 5a^2)$. 1 mark for a correct factorisation that is not complete, such as $4a^2b(6b^2 - 10a^2)$ or $8a^2(3b^3 - 5a^2b)$.

3 (3 marks)

$$\mathbf{A} = \begin{pmatrix} a & -3 \\ 0 & a \end{pmatrix}, \quad \mathbf{A}^2 = \begin{pmatrix} 49 & b \\ 0 & 49 \end{pmatrix}, \quad b > 0. \quad \text{Work out } a \text{ and } b.$$

$$\mathbf{A}^2 = \begin{pmatrix} a & -3 \\ 0 & a \end{pmatrix} \begin{pmatrix} a & -3 \\ 0 & a \end{pmatrix}$$

$$\text{Top left: } a \times a + (-3) \times 0 = a^2$$

$$\text{Top right: } a \times (-3) + (-3) \times a = -6a$$

Bottom left: $0 \times a + a \times 0 = 0$ Bottom right: $0 \times (-3) + a \times a = a^2$

$$\text{So } \begin{pmatrix} a^2 & -6a \\ 0 & a^2 \end{pmatrix} = \begin{pmatrix} 49 & b \\ 0 & 49 \end{pmatrix}$$

$$a^2 = 49, \text{ so } a = 7 \text{ or } a = -7$$

$b = -6a$ and b is positive, so a must be negative.

$$a = -7 \text{ and } b = -6 \times (-7) = 42$$

Answer: $a = -7, b = 42$

Tip: $a^2 = 49$ has two solutions, 7 and -7 . Use the other piece of information (b is positive) to decide which one works.

Marks: 1 mark for $\mathbf{A}^2$ with at least two correct entries (such as a^2 and $-6a$). 1 mark for $a = \pm 7$ or $b = -6a$. 1 mark for $a = -7$ and $b = 42$.

4 (3 marks)

A curve has equation $y = (2x + 5)(x - 1)^2$. Work out the gradient of the curve at the point where $x = -2$

$$\text{Expand first: } (x - 1)^2 = x^2 - 2x + 1$$

$$(2x + 5)(x^2 - 2x + 1) = 2x^3 - 4x^2 + 2x + 5x^2 - 10x + 5$$

$$y = 2x^3 + x^2 - 8x + 5$$

$$\text{Differentiate each term: } \frac{dy}{dx} = 6x^2 + 2x - 8$$

$$\text{At } x = -2: \frac{dy}{dx} = 6 \times (-2)^2 + 2 \times (-2) - 8 = 24 - 4 - 8 = 12$$

Answer: 12

Tip: You cannot differentiate a product of brackets bracket by bracket. Expand first, then differentiate each term. Put -2 in brackets when you square it: $(-2)^2 = 4$.

Marks: 1 mark for expanding to $2x^3 + x^2 - 8x + 5$. 1 mark for differentiating your cubic. 1 mark for 12.

5 (3 marks)

$\sqrt{147} + \sqrt{12} - \frac{18}{\sqrt{3}}$ can be written as $\sqrt{n}$, where n is an integer. Work out n .

Write each term as a multiple of $\sqrt{3}$.

$$147 = 49 \times 3, \text{ so } \sqrt{147} = 7\sqrt{3}. \quad 12 = 4 \times 3, \text{ so } \sqrt{12} = 2\sqrt{3}$$

$$\text{Rationalise: } \frac{18}{\sqrt{3}} = \frac{18\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$$

$$7\sqrt{3} + 2\sqrt{3} - 6\sqrt{3} = 3\sqrt{3}$$

$$3\sqrt{3} = \sqrt{9} \times \sqrt{3} = \sqrt{27}, \text{ so } n = 27$$

Answer: $n = 27$

Tip: Write every term as a multiple of $\sqrt{3}$ before you add or subtract: $\sqrt{147} + \sqrt{12}$ is not $\sqrt{159}$.
To write $k\sqrt{3}$ as one square root, square the k : $3\sqrt{3} = \sqrt{9 \times 3}$.

Marks: 1 mark for simplifying one of the terms ($7\sqrt{3}$, $2\sqrt{3}$ or $6\sqrt{3}$). 1 mark for $3\sqrt{3}$. 1 mark for 27.

6 (2 marks)

The graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ is sketched on the question paper.

$$\text{Solve } \sin x = -\frac{\sqrt{2}}{2} \text{ for } 0^\circ \leq x \leq 360^\circ$$

$$\text{Exact value: } \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ (from the 1:1:}\sqrt{2}\text{ triangle)}$$

$\sin x$ is negative between 180° and 360° , where the graph is below the x -axis.

The graph is symmetrical, so the angles are 45° after 180° and 45° before 360° .

$$x = 180^\circ + 45^\circ = 225^\circ$$

$$x = 360^\circ - 45^\circ = 315^\circ$$

Answer: 225° and 315°

Tip: Learn the exact values: $\frac{\sqrt{2}}{2}$ is the same as $\frac{1}{\sqrt{2}}$, so it comes from 45° . Then use the sketch: the line $y = -\frac{\sqrt{2}}{2}$ crosses the curve twice, once between 180° and 270° and once between 270° and 360° . 135° gives $+\frac{\sqrt{2}}{2}$, not $-\frac{\sqrt{2}}{2}$.

Marks: 1 mark for each correct angle.

7 (3 marks)

Make p the subject of $\frac{3p+2}{q} = p-5$

Multiply both sides by q : $3p+2 = q(p-5)$

Expand the bracket: $3p+2 = pq-5q$

Get the p terms on one side and the other terms on the other side: $2+5q = pq-3p$

Factorise out p : $2+5q = p(q-3)$

Divide by $(q-3)$: $p = \frac{5q+2}{q-3}$

Answer: $p = \frac{5q+2}{q-3}$

Tip: Multiply **all** of the right-hand side by q , then expand. When the new subject appears twice, collect both terms on one side and take it out as a factor. $p = \frac{-5q-2}{3-q}$ is also correct.

Marks: 1 mark for $3p+2 = pq-5q$. 1 mark for collecting the p terms and factorising. 1 mark for the answer.

8 (4 marks)

The n th term of a quadratic sequence is $an^2 + bn + 5$, where a and b are integers. The 3rd term is 26 and the 5th term is 25 more than the 4th term. Work out the 1st term and the 2nd term.

3rd term ($n = 3$): $9a + 3b + 5 = 26$, so $9a + 3b = 21$, so $3a + b = 7$

5th term – 4th term: $(25a + 5b + 5) - (16a + 4b + 5) = 25$, so $9a + b = 25$

Subtract: $(9a + b) - (3a + b) = 25 - 7$, so $6a = 18$ and $a = 3$

$3 \times 3 + b = 7$, so $b = -2$

n th term: $3n^2 - 2n + 5$

1st term: $3 - 2 + 5 = 6$ 2nd term: $12 - 4 + 5 = 13$

Check: 3rd term $27 - 6 + 5 = 26$ ✓ 4th term 45, 5th term 70, and $70 - 45 = 25$ ✓

Answer: 1st term 6, 2nd term 13

Tip: Substitute the **position** n into the n th term and set it equal to the **value** of the term. For “25 more than the 4th term”, subtract the two expressions in brackets: the +5 cancels.

Marks: 1 mark for one correct equation in a and b (such as $9a + b = 25$). 1 mark for both equations and a correct method to eliminate a or b . 1 mark for $a = 3$ and $b = -2$. 1 mark for 6 and 13.

9 (4 marks)

(a) Which expression is equal to $8x^{-3}$?

Circle your answer.

Options: $(8x)^{-3}$, $\frac{1}{8x^3}$, $\frac{8}{x^3}$, $8x^3$

The power -3 belongs to x only, not to the 8.

$$x^{-3} = \frac{1}{x^3}, \text{ so } 8x^{-3} = 8 \times \frac{1}{x^3} = \frac{8}{x^3}$$

Answer: $\frac{8}{x^3}$

Tip: $(8x)^{-3}$ would need brackets round the $8x$. It equals $\frac{1}{512x^3}$.

(b) Simplify fully $\frac{10x^{\frac{3}{2}}y^{-1} \times 3x^{\frac{1}{2}}y^4}{6x^{-2}y}$

Numerator: multiply the numbers and add the powers.

$$10 \times 3 = 30, \quad x^{\frac{3}{2}} \times x^{\frac{1}{2}} = x^{\frac{3}{2} + \frac{1}{2}} = x^2, \quad y^{-1} \times y^4 = y^3$$

$$\text{Numerator} = 30x^2y^3$$

Divide by $6x^{-2}y$: divide the numbers and subtract the powers.

$$30 \div 6 = 5, \quad x^{2 - (-2)} = x^4, \quad y^{3-1} = y^2$$

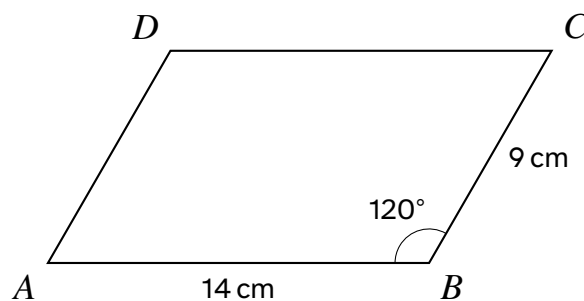
Answer: $5x^4y^2$

Tip: Dividing by a negative power adds to the power: $x^2 \div x^{-2} = x^{2 - (-2)} = x^4$.

Marks: 1 mark for each of 5, x^4 and y^2 in a single term.

10 (3 marks)

$ABCD$ is a parallelogram.



Not drawn accurately

Work out the area of the parallelogram.

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

The diagonal AC splits the parallelogram into two identical triangles.

$$\text{Area of triangle } ABC = \frac{1}{2} \times 14 \times 9 \times \sin 120^\circ$$

$$\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Area of parallelogram} = 2 \times \frac{1}{2} \times 14 \times 9 \times \frac{\sqrt{3}}{2} = 126 \times \frac{\sqrt{3}}{2} = 63\sqrt{3}$$

Answer: $63\sqrt{3} \text{ cm}^2$

Tip: The sine graph is symmetrical about 90° , so $\sin 120^\circ = \sin 60^\circ$. Remember the parallelogram is **two** triangles.

Marks: 1 mark for $\frac{1}{2} \times 14 \times 9 \times \sin 120^\circ$ (or $14 \times 9 \times \sin 120^\circ$). 1 mark for using $\sin 120^\circ = \frac{\sqrt{3}}{2}$. 1 mark for $63\sqrt{3}$.

11 (3 marks)

$(x - 4)(x^2 + qx + r) \equiv x^3 + px^2 - 11x + 12$. Work out the values of p , q and r .

Expand the left-hand side: $x^3 + qx^2 + rx - 4x^2 - 4qx - 4r$
 $= x^3 + (q - 4)x^2 + (r - 4q)x - 4r$

Constant terms: $-4r = 12$, so $r = -3$

x terms: $r - 4q = -11$, so $-3 - 4q = -11$, $-4q = -8$, $q = 2$

x^2 terms: $p = q - 4 = 2 - 4 = -2$

Answer: $p = -2$, $q = 2$, $r = -3$

Tip: Start with the coefficient that has only one unknown in it. Here that is the constant term, $-4r = 12$.

Marks: 1 mark for $r = -3$. 1 mark for a correct equation for q (or p) from the expansion. 1 mark for $p = -2$ and $q = 2$.

12 (3 marks)

Solve $(2x + 5)(2x - 7) \leq 4x - 3$

Expand the left-hand side: $(2x + 5)(2x - 7) = 4x^2 - 14x + 10x - 35 = 4x^2 - 4x - 35$
 $4x^2 - 4x - 35 \leq 4x - 3$

Subtract $4x$ and add 3 on both sides: $4x^2 - 8x - 32 \leq 0$

Divide by 4: $x^2 - 2x - 8 \leq 0$

Factorise: $(x - 4)(x + 2) \leq 0$

Critical values: $x = 4$ and $x = -2$

$y = (x - 4)(x + 2)$ is a U-shaped curve, so it is ≤ 0 between the critical values.

$$-2 \leq x \leq 4$$

Check $x = 0$: $5 \times (-7) = -35$ and $-35 \leq -3$ ✓

Answer: $-2 \leq x \leq 4$

Tip: Get everything on one side so that you are comparing a quadratic with 0, then find the critical values. Sketch the U-shaped curve to decide whether you want the values between them or outside them.

Marks: 1 mark for expanding and rearranging to $4x^2 - 8x - 32 \leq 0$ (or $x^2 - 2x - 8$). 1 mark for the critical values -2 and 4 . 1 mark for $-2 \leq x \leq 4$.

13 (5 marks)

A function f is given by

$$\begin{aligned} f(x) &= ax^2 + b & -3 \leq x \leq 1 \\ &= 4x + 1 & 1 < x \leq 4 \end{aligned}$$

where a and b are constants. The sketch shows the graph starts at $(-3, 21)$.

(a) Work out the values of a and b .

The two pieces join where $x = 1$. Use the linear piece: $f(1) = 4 \times 1 + 1 = 5$

So $(1, 5)$ is on $y = ax^2 + b$: $a + b = 5$

$(-3, 21)$ is on $y = ax^2 + b$: $9a + b = 21$

Subtract: $8a = 16$, so $a = 2$

$$2 + b = 5, \text{ so } b = 3$$

Answer: $a = 2, b = 3$

Tip: The pieces meet at $x = 1$, so work out the y -value there from the piece you know completely, $4x + 1$.

Marks: 1 mark for $a + b = 5$ (using the join). 1 mark for $9a + b = 21$. 1 mark for $a = 2$ and $b = 3$.

(b) Solve $f(x) = 11$

Quadratic piece: $2x^2 + 3 = 11, x^2 = 4, x = 2 \text{ or } x = -2$

This piece is only for $-3 \leq x \leq 1$, so $x = -2$ works but $x = 2$ does not.

Linear piece: $4x + 1 = 11, x = \frac{10}{4} = 2.5$

$1 < 2.5 \leq 4$, so $x = 2.5$ works.

(Check with the sketch: the line $y = 11$ crosses the graph twice.)

Answer: $x = -2$ and $x = 2\frac{1}{2}$

Tip: Check every solution against the domain of the piece it came from. $x = 2$ solves $2x^2 + 3 = 11$ but is not in $-3 \leq x \leq 1$.

Marks: 1 mark for $2x^2 + 3 = 11$ or $4x + 1 = 11$. 1 mark for -2 and 2.5 and no other values.

14 (4 marks)

A circle has equation $(x + 2)^2 + (y - 5)^2 = 13$. The point $P(1, 3)$ lies on the circle. Work out the equation of the tangent to the circle at P , in the form $ax + by = c$ where a, b and c are integers.

The centre of the circle is $(-2, 5)$.

Gradient of the radius from $(-2, 5)$ to $P(1, 3)$: $\frac{3 - 5}{1 - (-2)} = \frac{-2}{3} = -\frac{2}{3}$

The tangent is perpendicular to the radius, so its gradient is the negative reciprocal: $\frac{3}{2}$

$$y - 3 = \frac{3}{2}(x - 1)$$

Multiply by 2: $2y - 6 = 3x - 3$

$$3x - 2y = -3$$

Answer: $3x - 2y = -3$

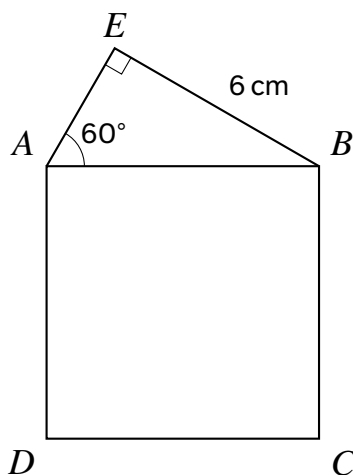
Tip: The centre is $(-2, 5)$, not $(2, -5)$: the signs in the brackets flip. Use the gradient of the radius from the centre to P , then flip it and change its sign for the tangent. Multiply by 2 to clear the fraction before you rearrange.

Marks: 1 mark for the centre $(-2, 5)$. 1 mark for the gradient of the radius, $-\frac{2}{3}$. 1 mark for gradient $\frac{3}{2}$ used in an equation through P . 1 mark for $3x - 2y = -3$ (or $2y - 3x = 3$).

15 (3 marks)

The shape is made from a square $ABCD$ and a right-angled triangle ABE .

Angle $AEB = 90^\circ$ Angle $EAB = 60^\circ$ $EB = 6 \text{ cm}$



Not drawn accurately

Work out the perimeter of the shape.

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

In triangle ABE , AB is the hypotenuse and EB is opposite the 60° angle.

$$\sin 60^\circ = \frac{EB}{AB}, \text{ so } AB = \frac{6}{\sin 60^\circ} = 6 \div \frac{\sqrt{3}}{2} = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\tan 60^\circ = \frac{EB}{AE}, \text{ so } AE = \frac{6}{\tan 60^\circ} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

The perimeter is $AE + EB + BC + CD + DA$ (AB is inside the shape).

$$BC = CD = DA = AB = 4\sqrt{3} \text{ (square)}$$

$$\text{Perimeter} = 2\sqrt{3} + 6 + 3 \times 4\sqrt{3} = 6 + 14\sqrt{3}$$

Answer: $(6 + 14\sqrt{3})$ cm

Tip: AB is not on the outside of the shape, so it is not part of the perimeter. Use exact values: $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\tan 60^\circ = \sqrt{3}$, and rationalise $\frac{12}{\sqrt{3}}$.

Marks: 1 mark for a correct method for AB or AE . 1 mark for $AB = 4\sqrt{3}$ and $AE = 2\sqrt{3}$ (any correct form). 1 mark for $6 + 14\sqrt{3}$.

16 (3 marks)

Simplify fully $\frac{6x^2}{5x+15} \div \frac{2x^2-10x}{x^2-2x-15}$

Factorise each expression: $5x+15 = 5(x+3)$, $2x^2-10x = 2x(x-5)$,
 $x^2-2x-15 = (x-5)(x+3)$

Dividing by a fraction means multiplying by its reciprocal:

$$\frac{6x^2}{5(x+3)} \times \frac{(x-5)(x+3)}{2x(x-5)}$$

Cancel $(x+3)$ and $(x-5)$: $\frac{6x^2}{5 \times 2x} = \frac{6x^2}{10x}$

$$= \frac{3x}{5}$$

Answer: $\frac{3x}{5}$

Tip: Factorise everything first, then cancel whole brackets. Cancelling single terms (such as the x^2 in $x^2 - 2x - 15$) is not allowed.

Marks: 1 mark for $(x - 5)(x + 3)$. 1 mark for the other two factorisations with the second fraction turned upside down. 1 mark for $\frac{3x}{5}$.

17 (3 marks)

The diagram shows a rectangle.

The width of the rectangle is $(1 + \sqrt{3})$ cm.

The area of the rectangle is $(8 + 6\sqrt{3})$ cm²



Not drawn accurately

Work out the length of the rectangle.

Give your answer in the form $p + q\sqrt{3}$ where p and q are integers.

You must show your working.

$$\text{Length} = \text{area} \div \text{width} = \frac{8 + 6\sqrt{3}}{1 + \sqrt{3}}$$

Multiply the top and bottom by $1 - \sqrt{3}$:

$$\text{Top: } (8 + 6\sqrt{3})(1 - \sqrt{3}) = 8 - 8\sqrt{3} + 6\sqrt{3} - 18 = -10 - 2\sqrt{3}$$

$$\text{Bottom: } (1 + \sqrt{3})(1 - \sqrt{3}) = 1 - 3 = -2$$

$$\text{Length} = \frac{-10 - 2\sqrt{3}}{-2} = 5 + \sqrt{3}$$

Check: $(5 + \sqrt{3})(1 + \sqrt{3}) = 5 + 5\sqrt{3} + \sqrt{3} + 3 = 8 + 6\sqrt{3}$ ✓

Answer: $(5 + \sqrt{3})$ cm

Tip: Multiply by the conjugate: change the sign between the terms.

$(1 + \sqrt{3})(1 - \sqrt{3}) = 1 - 3 = -2$, not $1 + 3$. Watch the negative signs when you divide by -2 .

Marks: 1 mark for $\frac{8+6\sqrt{3}}{1+\sqrt{3}}$ multiplied top and bottom by $1 - \sqrt{3}$. 1 mark for $\frac{-10 - 2\sqrt{3}}{-2}$. 1 mark for $5 + \sqrt{3}$.

18 (4 marks)

- (a) Factorise fully $2x^3 - 7x^2 - 12x + 45$

Use the factor theorem: try values of x that divide 45 (or halves of them).

Let $f(x) = 2x^3 - 7x^2 - 12x + 45$. $f(1) = 28$, $f(-1) = 48$

$f(3) = 2(27) - 7(9) - 12(3) + 45 = 54 - 63 - 36 + 45 = 0$

So $(x - 3)$ is a factor: $2x^3 - 7x^2 - 12x + 45 = (x - 3)(2x^2 + kx - 15)$

(The $2x^2$ gives $2x^3$, and $(-3) \times (-15) = 45$ gives the constant term.)

Compare the x^2 terms: $-6 + k = -7$, so $k = -1$

Check the x terms: $-3k - 15 = 3 - 15 = -12$ ✓

$2x^2 - x - 15 = (2x + 5)(x - 3)$

So $2x^3 - 7x^2 - 12x + 45 = (x - 3)(2x + 5)(x - 3) = (x - 3)^2(2x + 5)$

Answer: $(x - 3)^2(2x + 5)$

Tip: Try $x = \pm 1, \pm 3, \pm 5, \dots$ until $f(x) = 0$. If $f(3) = 0$, the factor is $(x - 3)$, not $(x + 3)$. Find the quadratic factor by inspection, then factorise it as well. "Fully" means three linear factors.

Marks: 1 mark for using the factor theorem to find a factor, such as $f(3) = 0$. 1 mark for $(x - 3)(2x^2 - x - 15)$. 1 mark for $(x - 3)^2(2x + 5)$.

- (b) Write down the coordinates of the point where the curve $y = 2x^3 - 7x^2 - 12x + 45$ touches the x -axis.

$y = (x - 3)^2(2x + 5)$ is zero when $x = 3$ or $x = -\frac{5}{2}$.

$(x - 3)$ is a repeated factor, so $x = 3$ is a repeated root: the curve touches the x -axis at $x = 3$ (it has a turning point there) instead of crossing it.

At $x = -\frac{5}{2}$ the factor is not repeated, so the curve crosses the x -axis there.

The curve touches the x -axis at $(3, 0)$.

Answer: $(3, 0)$

Tip: A squared factor gives a repeated root. At a repeated root the curve touches the x -axis and turns, rather than crossing it.

19 (3 marks)

- (a) Write $x^2 + 12x + 18$ in the form $(x + p)^2 + q$ where p and q are integers.

Halve the coefficient of x : $12 \div 2 = 6$, so start with $(x + 6)^2 = x^2 + 12x + 36$

$$x^2 + 12x + 18 = (x + 6)^2 - 36 + 18$$

$$= (x + 6)^2 - 18$$

Answer: $(x + 6)^2 - 18$

Tip: $(x + 6)^2$ gives an extra 36, so take 36 away: $18 - 36 = -18$. Expand your answer to check.

Marks: 1 mark for $(x + 6)^2$. 1 mark for $(x + 6)^2 - 18$.

- (b) Hence solve $x^2 + 12x + 18 = 0$
Give your answers in simplified surd form.

$$(x + 6)^2 - 18 = 0, \text{ so } (x + 6)^2 = 18$$

$$x + 6 = \pm\sqrt{18} = \pm 3\sqrt{2}$$

$$x = -6 + 3\sqrt{2} \text{ or } x = -6 - 3\sqrt{2}$$

Answer: $x = -6 \pm 3\sqrt{2}$

Tip: Square root both sides and remember the $\pm$: there are two answers.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}.$$

20 (5 marks)

Solve the simultaneous equations $2p + 3q - 4r = -2$, $p - q + 6r = 8$, $4p + q + 2r = 11$

Label the equations: (1) $2p + 3q - 4r = -2$, (2) $p - q + 6r = 8$, (3) $4p + q + 2r = 11$

Eliminate q from (1) and (2): (1) + 3×(2): $2p + 3q - 4r + 3p - 3q + 18r = -2 + 24$

$$5p + 14r = 22 \quad (4)$$

Eliminate q from (2) and (3): (2) + (3): $5p + 8r = 19 \quad (5)$

$$(4) - (5): 6r = 3, \text{ so } r = \frac{1}{2}$$

Substitute into (5): $5p + 4 = 19$, $5p = 15$, $p = 3$

Substitute into (2): $3 - q + 3 = 8$, so $q = -2$

Check in (3): $12 - 2 + 1 = 11 \quad \checkmark$

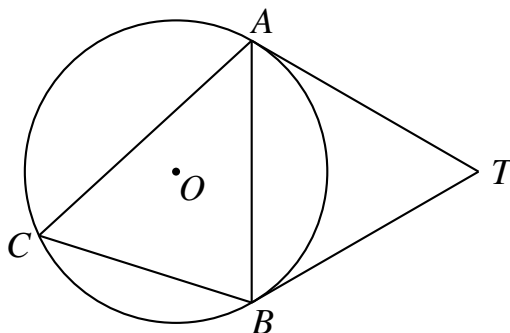
Answer: $p = 3$, $q = -2$, $r = \frac{1}{2}$

Tip: Eliminate the **same** letter from two different pairs of equations to get two equations in two unknowns. Check your answers in all three original equations.

Marks: 1 mark for eliminating one letter from a pair of equations. 1 mark for eliminating the same letter from a different pair. 1 mark for solving to find one value. 1 mark for one correct value. 1 mark for all three correct.

21 (4 marks)

A , B and C are points on a circle, centre O . TA and TB are tangents to the circle. Angle $ATB = \text{angle } ACB$.



Not drawn accurately

Prove that triangle ABT is equilateral.

Let angle $ATB = \text{angle } ACB = x$.

Angle $AOB = 2x$ (the angle at the centre is twice the angle at the circumference).

Angle $OAT = \text{angle } OBT = 90^\circ$ (a tangent is perpendicular to the radius).

In quadrilateral $OATB$: $2x + 90 + x + 90 = 360$ (angles in a quadrilateral add up to 360°)

$3x = 180$, so $x = 60$, so angle $ATB = 60^\circ$

$TA = TB$ (tangents from a point to a circle are equal), so triangle ABT is isosceles.

Angle $TAB = \text{angle } TBA = \frac{180 - 60}{2} = 60^\circ$ (base angles of an isosceles triangle are equal)

All three angles of triangle ABT are 60° , so triangle ABT is equilateral.

Answer: Angle $AOB = 2x$ and angles $OAT = OBT = 90^\circ$, so $3x + 180 = 360$ and $x = 60$. $TA = TB$, so angles $TAB = TBA = 60^\circ$: all angles are 60° , so ABT is equilateral.

Tip: Mark the angles you find on the diagram, and give a reason for every step using the proper theorem names. Another route: by the alternate segment theorem, angle TAB and angle TBA both equal angle ACB .

Marks: 1 mark for angle $AOB = 2x$ and the two 90° angles. 1 mark for $3x + 180 = 360$ (or $x = 60$). 1 mark for showing all three angles of ABT are 60° with a conclusion. 1 mark for the reasons.

22 (3 marks)

Prove that $\frac{(1 - \sin x)(1 + \sin x)}{\cos x} + \sin x \tan x \equiv \frac{1}{\cos x}$

Start with the left-hand side.

$$(1 - \sin x)(1 + \sin x) = 1 - \sin^2 x = \cos^2 x \quad (\text{using } \sin^2 x + \cos^2 x = 1)$$

So the first term is $\frac{\cos^2 x}{\cos x} = \cos x$

$$\tan x = \frac{\sin x}{\cos x}, \text{ so } \sin x \tan x = \frac{\sin^2 x}{\cos x}$$

$$\text{LHS} = \cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}$$

Answer: $\text{LHS} = \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}$

Tip: Work from one side only: start with the left-hand side and finish with the right-hand side. Do not cross-multiply or work on both sides at once. $\cos x \neq 1 - \sin x$.

Marks: 1 mark for $1 - \sin^2 x = \cos^2 x$. 1 mark for $\sin x \tan x = \frac{\sin^2 x}{\cos x}$. 1 mark for combining over $\cos x$ and using $\sin^2 x + \cos^2 x = 1$ to reach $\frac{1}{\cos x}$.

23 (6 marks)

A curve has equation

$$y = 4x^3 - 3x^2 - 6x + 2$$

- (a) Work out the coordinates of the two stationary points on the curve.

$$\frac{dy}{dx} = 12x^2 - 6x - 6$$

At a stationary point $\frac{dy}{dx} = 0$: $12x^2 - 6x - 6 = 0$

Divide by 6: $2x^2 - x - 1 = 0$

$$(2x + 1)(x - 1) = 0, \text{ so } x = -\frac{1}{2} \text{ or } x = 1$$

$$x = -\frac{1}{2}: y = 4\left(-\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) - 6\left(-\frac{1}{2}\right) + 2 = -\frac{1}{2} - \frac{3}{4} + 3 + 2 = \frac{15}{4}$$

$$x = 1: y = 4 - 3 - 6 + 2 = -3$$

Answer: $\left(-\frac{1}{2}, \frac{15}{4}\right)$ and $(1, -3)$

Tip: Stationary points are where $\frac{dy}{dx} = 0$. Substitute the x values into the equation of the **curve** (not into $\frac{dy}{dx}$) to get the y values. Use brackets: $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$.

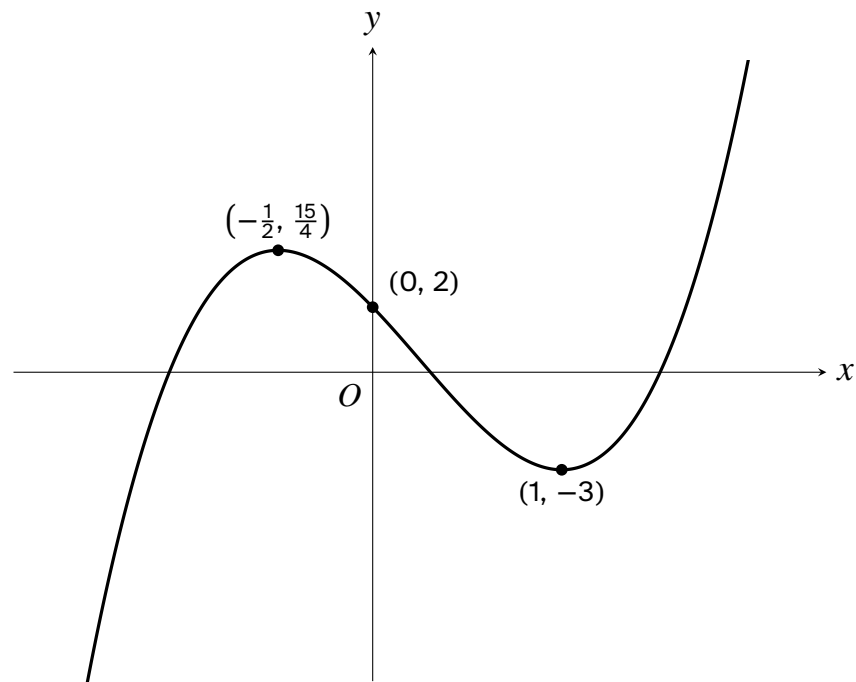
Marks: 1 mark for $12x^2 - 6x - 6$. 1 mark for setting it equal to 0 and factorising. 1 mark for both x values. 1 mark for both y values.

- (b) Sketch the curve. Label the coordinates of the stationary points and the point where the curve crosses the y -axis.

The x^3 term is positive, so the curve goes up, down, then up again (from bottom left to top right).

So $\left(-\frac{1}{2}, \frac{15}{4}\right)$ is the maximum and $(1, -3)$ is the minimum.

When $x = 0$, $y = 2$, so the curve crosses the y -axis at $(0, 2)$.



Answer: Positive cubic shape with maximum $\left(-\frac{1}{2}, \frac{15}{4}\right)$, minimum $(1, -3)$ and y-intercept $(0, 2)$, all labelled

Tip: The point on the left is the maximum because the curve comes up from the bottom left. Mark the y-intercept between the two stationary points, and draw smooth turning points, not sharp corners.

Marks: 1 mark for the correct shape with the maximum on the left above the x -axis and the minimum on the right below it. 1 mark for the three points labelled.