

Mock Set C – Paper 2

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Expand and simplify fully $(x^2 - 4x + 7)(3x + 2) - 2x(x - 5)^2$

Expand the first product: multiply every term in the first bracket by $3x$ and then by 2 .

$$\begin{aligned}(x^2 - 4x + 7)(3x + 2) &= 3x^3 + 2x^2 - 12x^2 - 8x + 21x + 14 \\ &= 3x^3 - 10x^2 + 13x + 14\end{aligned}$$

Square the bracket first: $(x - 5)^2 = x^2 - 10x + 25$

$$2x(x^2 - 10x + 25) = 2x^3 - 20x^2 + 50x$$

Subtract, keeping a bracket round the second expression:

$$\begin{aligned}3x^3 - 10x^2 + 13x + 14 - (2x^3 - 20x^2 + 50x) \\ = 3x^3 - 10x^2 + 13x + 14 - 2x^3 + 20x^2 - 50x \\ = x^3 + 10x^2 - 37x + 14\end{aligned}$$

Answer: $x^3 + 10x^2 - 37x + 14$

Tip: $(x - 5)^2 = x^2 - 10x + 25$, not $x^2 + 25$. Put a bracket round $2x^3 - 20x^2 + 50x$ before you subtract it, so that every sign changes.

Marks: 1 mark for expanding $(x^2 - 4x + 7)(3x + 2)$ correctly. 1 mark for expanding $2x(x - 5)^2$ correctly. 1 mark for the final answer.

2 (2 marks)

The equation of a circle is $(x + 3)^2 + (y - 8)^2 = 144$. Write down the coordinates of the centre and work out the radius.

Compare with $(x - a)^2 + (y - b)^2 = r^2$, which has centre (a, b) and radius r .

$(x + 3)^2 = (x - (-3))^2$, so $a = -3$. $(y - 8)^2$ gives $b = 8$.

Centre $(-3, 8)$

$r^2 = 144$, so $r = \sqrt{144} = 12$

Answer: Centre $(-3, 8)$, radius 12

Tip: The signs in the brackets flip: $(x + 3)^2$ gives $x = -3$. The number on the right is r^2 , so the radius is $\sqrt{144} = 12$, not 144.

Marks: 1 mark for the centre. 1 mark for the radius.

3 (2 marks)

The function g is given by $g(x) = 5 - 3x^3$ with domain $-2 < x \leq 1$. Work out the range of the function.

x^3 increases as x increases, so $5 - 3x^3$ **decreases**: the greatest value of $g(x)$ is at the left end of the domain and the least value is at the right end.

$g(-2) = 5 - 3 \times (-2)^3 = 5 - 3 \times (-8) = 5 + 24 = 29$, but $x = -2$ is **not** in the domain (the sign is $<$), so $g(x)$ never equals 29: $g(x) < 29$

$g(1) = 5 - 3 \times 1^3 = 5 - 3 = 2$, and $x = 1$ **is** in the domain, so $g(x)$ can equal 2: $g(x) \geq 2$

Range: $2 \leq g(x) < 29$

Answer: $2 \leq g(x) < 29$

Tip: Work out g at both ends of the domain. The function is decreasing, so the end values swap round: the smaller value of $g(x)$ comes from $x = 1$, which is included, so that end has $\leq$. Bracket the negative number: $(-2)^3 = -8$, so $5 - 3 \times (-8) = 29$.

Marks: 1 mark for one end of the range correct (value and inequality sign). 1 mark for the full range.

4 (5 marks)

$A(h, -3)$ and $B(9, k)$ are points, where h and k are constants.
The midpoint of AB is $M(4, 1)$

- (a) Work out the values of h and k .

The midpoint is the mean of the coordinates.

$$x: \frac{h+9}{2} = 4, \text{ so } h+9 = 8 \text{ and } h = -1$$

$$y: \frac{-3+k}{2} = 1, \text{ so } -3+k = 2 \text{ and } k = 5$$

Answer: $h = -1, k = 5$

Tip: Set up one equation for each coordinate: the mean of the two x -coordinates is 4 and the mean of the two y -coordinates is 1.

Marks: 1 mark for each value.

- (b) Work out the equation of the perpendicular bisector of AB .
Give your answer in the form $y = mx + c$

$A(-1, -3)$ and $B(9, 5)$

$$\text{Gradient of } AB = \frac{5 - (-3)}{9 - (-1)} = \frac{8}{10} = 0.8$$

The bisector is perpendicular to AB , so its gradient is $-\frac{1}{0.8} = -1.25$

It passes through the midpoint $M(4, 1)$.

$$y - 1 = -1.25(x - 4)$$

$$y = -1.25x + 5 + 1$$

$$y = -1.25x + 6$$

Answer: $y = -1.25x + 6$ (or $y = -\frac{5}{4}x + 6$)

Tip: A perpendicular bisector goes through the **midpoint**, so use $M(4, 1)$, not A or B . For the perpendicular gradient, flip the fraction and change the sign.

Marks: 1 mark for the gradient of AB . 1 mark for using the perpendicular gradient -1.25 with the point $M(4, 1)$. 1 mark for $y = -1.25x + 6$.

5 (3 marks)

$$\begin{pmatrix} a & -2 \\ 3 & b \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 2 & a \end{pmatrix} = \begin{pmatrix} -14 & 2 \\ 8 & 10 \end{pmatrix}. \text{ Work out the values of } a \text{ and } b.$$

Multiply each row of the first matrix by each column of the second.

$$\text{Top left: } 5a + (-2) \times 2 = 5a - 4 \quad \text{Top right: } a \times 1 + (-2) \times a = -a$$

$$\text{Bottom left: } 3 \times 5 + 2b = 15 + 2b \quad \text{Bottom right: } 3 \times 1 + b \times a = 3 + ab$$

$$\text{Top left: } 5a - 4 = -14, \text{ so } 5a = -10 \text{ and } a = -2 \quad (\text{top right: } -a = 2 \checkmark)$$

$$\text{Bottom left: } 15 + 2b = 8, \text{ so } 2b = -7 \text{ and } b = -3.5$$

$$\text{Check bottom right: } 3 + (-3.5) \times (-2) = 3 + 7 = 10 \checkmark$$

Answer: $a = -2, b = -3.5$

Tip: Write each entry of the product as an expression and set it equal to the matching entry on the right. Check your values in the entries you did not use.

Marks: 1 mark for a correct equation from one entry of the product. 1 mark for $a = -2$. 1 mark for $b = -3.5$.

6 (4 marks)

The n th term of a sequence is $\frac{6n+5}{4n-5}$

- (a) Write down the limiting value of the sequence as $n \rightarrow \infty$

For very large n , the $+5$ and -5 make almost no difference, so the terms get close to $\frac{6n}{4n} = \frac{6}{4}$

Limiting value = 1.5

Answer: 1.5

Tip: Compare the coefficients of n on the top and the bottom.

- (b) The k th term of the sequence is $\frac{1}{30}$ more than the limiting value.

Work out the value of k .

You must show your working.

$$\text{The } k\text{th term is } \frac{3}{2} + \frac{1}{30} = \frac{45}{30} + \frac{1}{30} = \frac{46}{30} = \frac{23}{15}$$

$$\text{So } \frac{6k+5}{4k-5} = \frac{23}{15}$$

$$\text{Cross-multiply: } 15(6k+5) = 23(4k-5)$$

$$90k + 75 = 92k - 115$$

$$75 + 115 = 92k - 90k, \text{ so } 190 = 2k$$

$$k = 95$$

$$\text{Check: } \frac{6 \times 95 + 5}{4 \times 95 - 5} = \frac{575}{375} = \frac{23}{15} \quad \checkmark$$

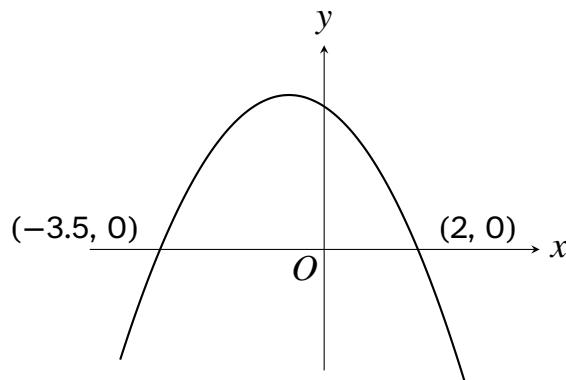
Answer: $k = 95$

Tip: Work with exact fractions, not rounded decimals: $1.5 + \frac{1}{30} = \frac{23}{15}$. A rounded value such as 1.53 gives the wrong k . Then cross-multiply and collect the k terms.

Marks: 1 mark for setting the k th term equal to $1.5 + \frac{1}{30}$ (or $\frac{23}{15}$). 1 mark for a correct equation with no fractions, such as $90k + 75 = 92k - 115$. 1 mark for 95.

7 (2 marks)

Here is a sketch of $y = f(x)$, where $f(x)$ is a quadratic function.
The graph crosses the x -axis at $(-3.5, 0)$ and $(2, 0)$



Not drawn accurately

Solve $f(x) < 0$

$f(x) < 0$ where the graph is **below** the x -axis.

The curve is $\cap$ -shaped, so it is below the x -axis to the left of $x = -3.5$ and to the right of $x = 2$.

$$x < -3.5 \text{ or } x > 2$$

Answer: $x < -3.5$ or $x > 2$

Tip: Write two separate inequalities. $-3.5 < x < 2$ is where the curve is **above** the axis, and $-3.5 > x > 2$ is impossible.

Marks: 1 mark for one correct inequality. 1 mark for both.

8 (3 marks)

4-digit serial numbers use only the digits 1 to 7, with repeats allowed. Each serial number is less than 5000 and a multiple of 5. How many serial numbers are possible?

Count the choices for each digit, starting with the restricted ones.

1st digit: the number is less than 5000, so the first digit is 1, 2, 3 or 4: 4 choices

4th digit: a multiple of 5 ends in 0 or 5. There is no 0, so the last digit must be 5: 1 choice

2nd and 3rd digits: any of the 7 digits (repeats are allowed): 7 choices each

Number of serial numbers = $4 \times 7 \times 7 \times 1 = 196$

Answer: 196

Tip: There is no 0 among the digits, so a multiple of 5 must end in 5. Deal with the restricted positions first, then multiply.

Marks: 1 mark for 4 choices for the first digit and 1 for the last. 1 mark for $4 \times 7 \times 7 \times 1$. 1 mark for 196.

9 (4 marks)

$y = 3x^4 - \frac{2}{x^3}$. Work out the value of $\frac{d^2y}{dx^2}$ when $x = 0.9$, to 3 significant figures.

Write the fraction as a power of x : $\frac{2}{x^3} = 2x^{-3}$, so $y = 3x^4 - 2x^{-3}$

$$\frac{dy}{dx} = 12x^3 - 2 \times (-3)x^{-4} = 12x^3 + 6x^{-4}$$

$$\frac{d^2y}{dx^2} = 36x^2 + 6 \times (-4)x^{-5} = 36x^2 - 24x^{-5}$$

$$\text{When } x = 0.9: 36 \times 0.9^2 - \frac{24}{0.9^5} = 29.16 - 40.644 \dots$$

$$= -11.484 \dots$$

$$= -11.5 \text{ (3 s.f.)}$$

Answer: -11.5

Tip: Change $\frac{2}{x^3}$ to $2x^{-3}$ before differentiating, and take care with the signs: $-2x^{-3}$ differentiates to $+6x^{-4}$, and $+6x^{-4}$ to $-24x^{-5}$.

Marks: 1 mark for one correct term of $\frac{dy}{dx}$. 1 mark for $\frac{dy}{dx} = 12x^3 + 6x^{-4}$. 1 mark for differentiating again correctly. 1 mark for -11.5.

10 (4 marks)

(a) Simplify fully $\frac{72a^5b^9}{(6a^3b^2)^2}$

Square the bracket: square the number and double each index:

$$(6a^3b^2)^2 = 36a^6b^4$$

$$\frac{72a^5b^9}{36a^6b^4} = 2a^{5-6}b^{9-4} = 2a^{-1}b^5$$

$$= \frac{2b^5}{a}$$

Answer: $\frac{2b^5}{a}$ (or $2a^{-1}b^5$)

Tip: Square the 6 as well as the letters: $(6a^3b^2)^2 = 36a^6b^4$, not $6a^6b^4$ or $12a^6b^4$.

Marks: 1 mark for $36a^6b^4$, or for an answer with two of the three parts correct. 1 mark for the fully simplified answer.

(b) $p^4y\sqrt{y} = pq^6$. Write y in the form $p^m q^n$.

$$y\sqrt{y} = y^1 \times y^{\frac{1}{2}} = y^{\frac{3}{2}}$$

Divide both sides by p^4 : $y^{\frac{3}{2}} = \frac{p^1q^6}{p^4} = p^{1-4}q^6 = p^{-3}q^6$

Raise both sides to the power $\frac{2}{3}$ (multiply each index by $\frac{2}{3}$):

$$y = p^{-3 \times \frac{2}{3}} q^{6 \times \frac{2}{3}} = p^{-2}q^4$$

Answer: $y = p^{-2}q^4$

Tip: $\sqrt{y} = y^{\frac{1}{2}}$, so $y\sqrt{y} = y^{\frac{3}{2}}$. To undo a power of $\frac{3}{2}$, raise both sides to the power $\frac{2}{3}$: multiply each index by $\frac{2}{3}$.

Marks: 1 mark for $y^{\frac{3}{2}} = p^{-3}q^6$. 1 mark for $y = p^{-2}q^4$.

11 (3 marks)

Rearrange $v = 2\sqrt[3]{\frac{w+k}{w}}$ to make w the subject.

Divide by 2: $\frac{v}{2} = \sqrt[3]{\frac{w+k}{w}}$

Cube both sides: $\frac{v^3}{8} = \frac{w+k}{w}$

Multiply both sides by $8w$: $v^3w = 8(w+k) = 8w + 8k$

w appears twice, so collect the w terms on one side: $v^3w - 8w = 8k$

Take out w as a factor: $w(v^3 - 8) = 8k$

Divide by $(v^3 - 8)$: $w = \frac{8k}{v^3 - 8}$

Answer: $w = \frac{8k}{v^3 - 8}$

Tip: Divide by 2 before you cube, and remember $2^3 = 8$. When the new subject appears twice, get both w terms on the same side and take w out as a factor.

Marks: 1 mark for cubing correctly. 1 mark for collecting the w terms and factorising, $w(v^3 - 8) = 8k$. 1 mark for $w = \frac{8k}{v^3 - 8}$.

12 (4 marks)

(a) Write $4 + 20x - 5x^2$ in the form $a - b(x - c)^2$ where a , b and c are integers.

Take out -5 from the two x terms: $4 + 20x - 5x^2 = 4 - 5(x^2 - 4x)$

Complete the square: $x^2 - 4x = (x - 2)^2 - 4$

$4 - 5[(x - 2)^2 - 4] = 4 - 5(x - 2)^2 + 20$

$= 24 - 5(x - 2)^2$

Check: $24 - 5(x^2 - 4x + 4) = 24 - 5x^2 + 20x - 20 = 4 + 20x - 5x^2 \checkmark$

Answer: $24 - 5(x - 2)^2$

Tip: Take out -5 (not 5) from both x terms, and multiply the -4 inside the square bracket by -5 as well. Expand your answer to check it.

Marks: 1 mark for $b = 5$ (or $4 - 5(x^2 - 4x)$). 1 mark for $c = 2$. 1 mark for $a = 24$.

- (b) Hence, write down the coordinates of the maximum point of the curve
 $y = 4 + 20x - 5x^2$

$-5(x - 2)^2$ is never positive, so y is greatest when $(x - 2)^2 = 0$, that is when $x = 2$.

Then $y = 24$.

Maximum point $(2, 24)$

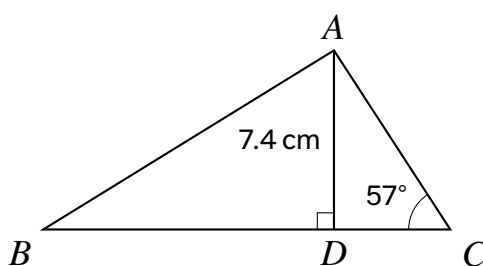
Answer: $(2, 24)$

Tip: The sign in the bracket flips: $(x - 2)^2$ gives $x = 2$.

13 (3 marks)

ABC is a triangle.

D is the point on BC such that AD is perpendicular to BC .



Not drawn accurately

$AD = 7.4$ cm, angle $ACD = 57^\circ$, $BD:DC = 5:2$. Work out the length AB to 3 significant figures.

In the right-angled triangle ADC : $\tan 57^\circ = \frac{\text{opp}}{\text{adj}} = \frac{AD}{DC} = \frac{7.4}{DC}$

$$DC = \frac{7.4}{\tan 57^\circ} = 4.8056 \dots \text{ cm}$$

$BD:DC = 5:2$, so DC is 2 parts and BD is 5 parts.

$$BD = \frac{5}{2} \times 4.8056 \dots = 12.014 \dots \text{ cm}$$

Triangle ABD is right-angled at D , so use Pythagoras:

$$AB = \sqrt{7.4^2 + 12.014 \dots^2} = \sqrt{199.09 \dots} = 14.110 \dots$$

$$= 14.1 \text{ cm (3 s.f.)}$$

Answer: 14.1 cm

Tip: Label the ratio on the diagram: $BD = 5$ parts and $DC = 2$ parts. AB is the hypotenuse of the right-angled triangle ABD , so you need BD first.

Marks: 1 mark for $DC = \frac{7.4}{\tan 57^\circ}$ (4.81...). 1 mark for $BD = \frac{5}{2} \times DC$ used in Pythagoras (or trigonometry) in triangle ABD . 1 mark for 14.1.

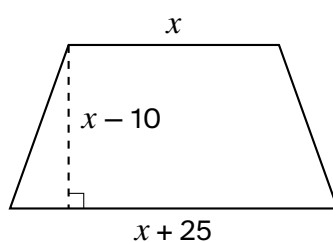
14 (5 marks)

The diagram shows the cross-section of a raised garden bed.

The cross-section is a trapezium.

All lengths are in centimetres.

The area of the cross-section is 2000 cm^2



Not drawn accurately

(a) Show that $2x^2 + 5x - 4250 = 0$

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

$$\frac{1}{2}(x + (x + 25))(x - 10) = 2000$$

$$\frac{1}{2}(2x + 25)(x - 10) = 2000$$

Multiply by 2: $(2x + 25)(x - 10) = 4000$

$$2x^2 - 20x + 25x - 250 = 4000$$

$$2x^2 + 5x - 250 - 4000 = 0$$

$$2x^2 + 5x - 4250 = 0, \text{ as required.}$$

Answer: $\frac{1}{2}(2x + 25)(x - 10) = 2000$, so $2x^2 + 5x - 250 = 4000$, which gives $2x^2 + 5x - 4250 = 0$

Tip: In a “show that” write every step, including the expansion of the brackets, and finish with the exact equation you were given.

Marks: 1 mark for a correct equation for the area. 1 mark for expanding and rearranging to the given equation with no errors.

- (b) Work out the height of the cross-section.
Give your answer to 3 significant figures.

Use the quadratic formula with $a = 2$, $b = 5$, $c = -4250$:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times (-4250)}}{2 \times 2} = \frac{-5 \pm \sqrt{34\,025}}{4}$$

$$x = 44.864 \dots \text{ or } x = -47.364 \dots$$

x is a length, so $x = 44.864 \dots$

$$\text{Height} = x - 10 = 34.864 \dots$$

$$= 34.9 \text{ cm (3 s.f.)}$$

Answer: 34.9 cm

Tip: Reject the negative solution because x is a length, then answer the question asked: the height is $x - 10$, not x .

Marks: 1 mark for substituting correctly into the quadratic formula. 1 mark for $x = 44.86 \dots$ 1 mark for the height, 34.9 cm.

15 (3 marks)

Simplify fully $\frac{x^2 - 9}{2x^2 + 7x + 3} \div \frac{x^2 - x - 6}{4x + 2}$

Factorise each expression:

$$x^2 - 9 = (x + 3)(x - 3) \text{ (difference of two squares)}$$

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

$$x^2 - x - 6 = (x - 3)(x + 2) \quad 4x + 2 = 2(2x + 1)$$

Dividing by a fraction is the same as multiplying by its reciprocal:

$$\frac{(x + 3)(x - 3)}{(2x + 1)(x + 3)} \times \frac{2(2x + 1)}{(x - 3)(x + 2)}$$

$$\text{Cancel } (x + 3), (x - 3) \text{ and } (2x + 1): = \frac{2}{x + 2}$$

Answer: $\frac{2}{x + 2}$

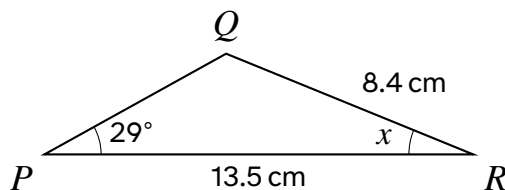
Tip: Factorise first, then cancel whole brackets. Don't forget the common factor 2 in $4x + 2 = 2(2x + 1)$.

Marks: 1 mark for multiplying by the reciprocal of the second fraction. 1 mark for factorising at least two of the expressions correctly. 1 mark for $\frac{2}{x+2}$.

16 (3 marks)

PQR is a triangle.

Angle PQR is obtuse.



Not drawn accurately

Triangle PQR : $PR = 13.5$ cm, $QR = 8.4$ cm, angle $QPR = 29^\circ$, and angle PQR is obtuse. Work out angle $x =$ angle PRQ to 1 decimal place.

Angle x is not opposite a known side, so first find angle PQR .

Angle P (29°) is opposite QR (8.4 cm). Angle PQR is opposite PR (13.5 cm).

$$\text{Sine rule: } \frac{\sin PQR}{13.5} = \frac{\sin 29^\circ}{8.4}$$

$$\sin PQR = \frac{13.5 \times \sin 29^\circ}{8.4} = 0.7791 \dots$$

$\sin^{-1}(0.7791 \dots) = 51.183 \dots^\circ$, but this is acute and angle PQR is obtuse.

$$\sin(180^\circ - \theta) = \sin \theta, \text{ so angle } PQR = 180^\circ - 51.183 \dots^\circ = 128.816 \dots^\circ$$

$$\text{Angles in a triangle add up to } 180^\circ: x = 180^\circ - 29^\circ - 128.816 \dots^\circ = 22.183 \dots^\circ$$

$$x = 22.2^\circ \text{ (1 d.p.)}$$

Answer: 22.2°

Tip: $\sin^{-1}$ on a calculator only gives an acute angle. Angle PQR is obtuse, so subtract the calculator value from 180° before you use the angle sum. Using 51.2° gives 99.8° , which is wrong.

Marks: 1 mark for a correct sine rule equation. 1 mark for angle $PQR = 128.8^\circ$ (the obtuse angle). 1 mark for 22.2° .

17 (3 marks)

$$f(x) = (x - 4)^2$$

$$g(x) = ax + b, \text{ where } a \text{ and } b \text{ are constants and } a > 0$$

$$fg(x) = (3x + 1)^2$$

(a) Work out $g(x)$.

$$fg(x) \text{ means put } g(x) \text{ into } f: fg(x) = (g(x) - 4)^2 = (ax + b - 4)^2$$

$$\text{Compare with } (3x + 1)^2: ax + b - 4 = 3x + 1, \text{ since } a > 0$$

$$\text{So } a = 3 \text{ and } b - 4 = 1, \text{ giving } b = 5$$

$$g(x) = 3x + 5$$

(Check: $f(3x + 5) = (3x + 5 - 4)^2 = (3x + 1)^2$ ✓. $ax + b - 4 = -(3x + 1)$ would give $a = -3$, which is not allowed.)

Answer: $g(x) = 3x + 5$

Tip: $fg(x)$ means do g first, then f . Here f takes 4 away and then squares, so $g(x) - 4$ must be $3x + 1$.

(b) Solve $gf(x) = 32$

$$gf(x) \text{ means put } f(x) \text{ into } g: gf(x) = 3(x - 4)^2 + 5$$

$$3(x - 4)^2 + 5 = 32$$

$$3(x - 4)^2 = 27, \text{ so } (x - 4)^2 = 9$$

$$\text{Square root, remembering } \pm: x - 4 = 3 \text{ or } x - 4 = -3$$

$$x = 7 \text{ or } x = 1$$

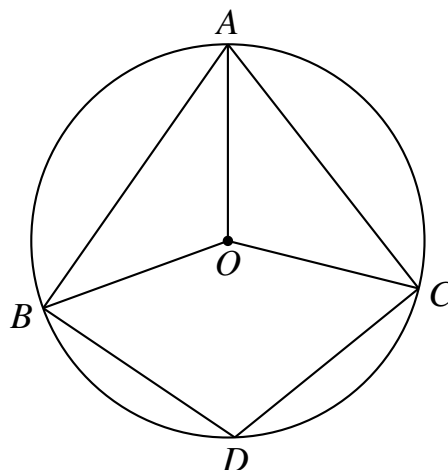
Answer: $x = 1$ or $x = 7$

Tip: $gf(x)$ is not the same as $fg(x)$: here f is done first. When you square root, use $\pm$ so that you find both solutions.

Marks: 1 mark for $3(x - 4)^2 + 5 = 32$. 1 mark for both solutions.

18 (4 marks)

A, B, C and D are points on a circle, centre O .



Not drawn accurately

Angle $OAB = 2x + 7^\circ$, angle $OCA = 3x - 4^\circ$ and angle $BDC = 8x - 5^\circ$. Work out the value of x .

$OA = OB$ (radii), so triangle OAB is isosceles: angle $OBA = 2x + 7$

Angle $AOB = 180 - 2(2x + 7) = 166 - 4x$

$OA = OC$ (radii), so triangle OAC is isosceles: angle $OAC = 3x - 4$

Angle $AOC = 180 - 2(3x - 4) = 188 - 6x$

D is on the minor arc BC , so angle BDC stands on the major arc BAC . The angle at the centre on that arc is the **reflex** angle BOC .

Angle at the centre = $2 \times$ angle at the circumference: reflex angle

$BOC = 2(8x - 5) = 16x - 10$

Reflex angle BOC is made up of angles AOB and AOC :

$(166 - 4x) + (188 - 6x) = 16x - 10$

$354 - 10x = 16x - 10$, so $364 = 26x$ and $x = 14$

Another way: angle $BAC = (2x + 7) + (3x - 4) = 5x + 3$. $ABDC$ is a cyclic quadrilateral, so $5x + 3 + 8x - 5 = 180$, giving $13x = 182$ and $x = 14$.

Answer: $x = 14$

Tip: D is on the minor arc, so angle BDC is half of the **reflex** angle BOC . Or spot that $ABDC$ is a cyclic quadrilateral: its opposite angles add up to 180° .

Marks: 1 mark for using an isosceles triangle (e.g. angle $AOB = 166 - 4x$ or angle $OAC = 3x - 4$). 1 mark for reflex angle $BOC = 2(8x - 5)$, or angle $BAC = 5x + 3$. 1 mark for a correct equation. 1 mark for $x = 14$.

19 (6 marks)

$$f(x) = 3x^3 + 16x^2 - 21x + 6$$

(a) Use the factor theorem to show that $(3x - 2)$ is a factor of $f(x)$.

$$3x - 2 = 0 \text{ when } x = \frac{2}{3}$$

$$\begin{aligned} f\left(\frac{2}{3}\right) &= 3\left(\frac{2}{3}\right)^3 + 16\left(\frac{2}{3}\right)^2 - 21\left(\frac{2}{3}\right) + 6 \\ &= \frac{8}{9} + \frac{64}{9} - 14 + 6 = 8 - 14 + 6 = 0 \end{aligned}$$

$f\left(\frac{2}{3}\right) = 0$, so by the factor theorem $(3x - 2)$ is a factor of $f(x)$.

Answer: $f\left(\frac{2}{3}\right) = \frac{8}{9} + \frac{64}{9} - 14 + 6 = 0$, so $(3x - 2)$ is a factor of $f(x)$

Tip: Substitute the value of x that makes $3x - 2 = 0$, which is $x = \frac{2}{3}$, not $x = 2$ or $x = -\frac{2}{3}$. Finish with a sentence.

Marks: 1 mark for substituting $x = \frac{2}{3}$. 1 mark for showing that it equals 0 and stating that $(3x - 2)$ is a factor.

(b) Hence, solve $f(x) = 0$

Give your answers in exact form, with any surds fully simplified.

$$\text{Write } f(x) = (3x - 2)(x^2 + bx + c)$$

$$\text{Constant terms: } -2c = 6, \text{ so } c = -3$$

$$x^2 \text{ terms: } 3b - 2 = 16, \text{ so } b = 6 \quad (\text{check } x \text{ terms: } 3c - 2b = -9 - 12 = -21 \checkmark)$$

$$f(x) = (3x - 2)(x^2 + 6x - 3)$$

$x^2 + 6x - 3 = 0$ does not factorise. Complete the square: $(x + 3)^2 - 9 - 3 = 0$, so $(x + 3)^2 = 12$

$$x + 3 = \pm\sqrt{12} = \pm 2\sqrt{3}, \text{ so } x = -3 \pm 2\sqrt{3}$$

$$\text{Solutions: } x = \frac{2}{3}, x = -3 + 2\sqrt{3}, x = -3 - 2\sqrt{3}$$

$$\textbf{Answer: } x = \frac{2}{3}, x = -3 + 2\sqrt{3}, x = -3 - 2\sqrt{3}$$

Tip: Exact means leave the surds in and simplify them: $\sqrt{12} = 2\sqrt{3}$. Don't forget the solution $x = \frac{2}{3}$ from the factor you were given.

Marks: 1 mark for a quadratic factor with at least two terms correct. 1 mark for $x^2 + 6x - 3$.
1 mark for a correct method to solve it. 1 mark for all three solutions, with $\sqrt{12}$ simplified.

20 (5 marks)

$y = (x^2 - 7)(2x - 5)$. Use calculus to work out the range of values of x for which y is a decreasing function, with values to 3 significant figures.

$$\text{Expand first: } y = 2x^3 - 5x^2 - 14x + 35$$

$$\frac{dy}{dx} = 6x^2 - 10x - 14$$

y is decreasing where $\frac{dy}{dx} < 0$: $6x^2 - 10x - 14 < 0$, or $3x^2 - 5x - 7 < 0$

$$\text{Solve } 3x^2 - 5x - 7 = 0: x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times (-7)}}{2 \times 3} = \frac{5 \pm \sqrt{109}}{6}$$

$$x = -0.9067 \dots \text{ or } x = 2.5733 \dots$$

$3x^2 - 5x - 7$ is a U-shaped quadratic, so it is negative **between** its roots.

$$-0.907 < x < 2.57$$

Answer: $-0.907 < x < 2.57$

Tip: Expand first, then differentiate: don't differentiate each bracket separately. Decreasing means $\frac{dy}{dx} < 0$, and a U-shaped quadratic is negative between its roots.

Marks: 1 mark for expanding the brackets. 1 mark for $\frac{dy}{dx} = 6x^2 - 10x - 14$. 1 mark for solving $\frac{dy}{dx} = 0$ (or < 0) correctly. 1 mark for -0.907 and 2.57 . 1 mark for the inequality.

21 (5 marks)

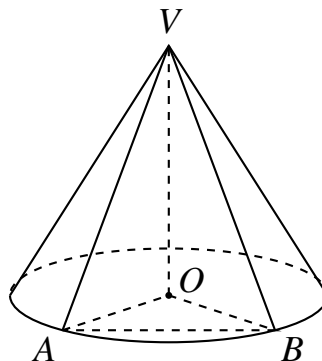
The diagram shows a cone.

V is the vertex of the cone and O is the centre of its circular base.

V is vertically above O .

A and B are points on the circumference of the base.

- The radius of the base is 6.4 cm
- $AB = 10.8$ cm
- The angle between VA and the base is 66°



Not drawn accurately

- (a) Show that the vertical height VO is 14.4 cm, correct to 3 significant figures.

Triangle VOA is right-angled at O , with $OA = 6.4$ cm (a radius) and angle $VAO = 66^\circ$.

$$\tan 66^\circ = \frac{\text{opp}}{\text{adj}} = \frac{VO}{6.4}$$

$$VO = 6.4 \tan 66^\circ = 14.374 \dots$$

= 14.4 cm (3 s.f.), as required.

Answer: $VO = 6.4 \tan 66^\circ = 14.374 \dots = 14.4 \text{ cm (3 s.f.)}$

Tip: In a “show that” with a rounded answer, write down a more accurate value (such as 14.374 ...) before you round it.

Marks: 1 mark for $VO = 6.4 \tan 66^\circ$. 1 mark for showing the more accurate value 14.37 ... before rounding to 14.4.

- (b) Work out the size of angle AVB .
Give your answer to 1 decimal place.

VA : in triangle VOA , $\cos 66^\circ = \frac{OA}{VA} = \frac{6.4}{VA}$, so $VA = \frac{6.4}{\cos 66^\circ} = 15.734 \dots \text{ cm}$

(or $VA = \sqrt{6.4^2 + 14.374 \dots^2}$)

$VB = VA$ (every point on the circumference is the same distance from V), so triangle VAB is isosceles.

Split it into two right-angled triangles at M , the midpoint of AB :

$AM = 10.8 \div 2 = 5.4 \text{ cm}$

$\sin AVM = \frac{AM}{VA} = \frac{5.4}{15.734 \dots} = 0.3431 \dots$, so angle $AVM = 20.070 \dots^\circ$

Angle $AVB = 2 \times 20.070 \dots^\circ = 40.141 \dots^\circ$

= 40.1° (1 d.p.)

(Or use the cosine rule: $\cos AVB = \frac{VA^2 + VB^2 - AB^2}{2 \times VA \times VB}$.)

Answer: 40.1°

Tip: Find the triangle that contains the angle you want: AVB is in triangle VAB , whose sides are VA , VB and AB . The height VO is not one of its sides.

Marks: 1 mark for the length VA (15.73 ...). 1 mark for a correct method for angle AVB in triangle VAB (right-angled triangles or the cosine rule). 1 mark for 40.1° .

22 (4 marks)

Solve $7 \sin x \cos x = -3 \cos x$ for $0^\circ \leq x \leq 360^\circ$, giving answers to 1 decimal place where appropriate.

Bring everything to one side: $7 \sin x \cos x + 3 \cos x = 0$

Factorise: $\cos x (7 \sin x + 3) = 0$

So $\cos x = 0$ or $\sin x = -\frac{3}{7}$

$\cos x = 0$: $x = 90^\circ, 270^\circ$

$\sin x = -\frac{3}{7}$: $\sin^{-1}\left(\frac{3}{7}\right) = 25.376 \dots^\circ$

$\sin x$ is negative in the 3rd and 4th quadrants: $x = 180^\circ + 25.376 \dots^\circ = 205.4^\circ$ and $x = 360^\circ - 25.376 \dots^\circ = 334.6^\circ$

Solutions: $90^\circ, 205.4^\circ, 270^\circ, 334.6^\circ$

Answer: $90^\circ, 205.4^\circ, 270^\circ, 334.6^\circ$

Tip: Don't divide both sides by $\cos x$: that loses the solutions where $\cos x = 0$. Take out the common factor $\cos x$ instead.

Marks: 1 mark for factorising (or for $\cos x = 0$ and $\sin x = -\frac{3}{7}$). 1 mark for 90° and 270° . 1 mark for using $\sin^{-1} \frac{3}{7}$ (or 25.4°). 1 mark for 205.4° and 334.6° .
