

Mock Set C – Paper 1

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

$p = 2.5 \times 10^5$ and $q = 3.6 \times 10^{-3}$. Work out the value of $\sqrt{pq}$

Multiply the number parts and the powers of 10 separately:

$$pq = (2.5 \times 3.6) \times (10^5 \times 10^{-3})$$

$$2.5 \times 3.6 = 9 \text{ and } 10^5 \times 10^{-3} = 10^{5+(-3)} = 10^2$$

$$pq = 9 \times 10^2 = 900$$

$$\sqrt{900} = 30$$

Answer: 30

Tip: When you multiply powers of 10, add the indices: $10^5 \times 10^{-3} = 10^2$, not 10^8 or 10^{-15} . Then $\sqrt{900} = 30$.

Marks: 1 mark for 900 (or 9×10^2). 1 mark for 30.

2 (4 marks)

(a) $f(x) = 7 - 3x$ and $g(x) = x^2 - 2$. Work out $fg(x)$ in its simplest form.

$fg(x)$ means put $g(x)$ into f : do g first, then f .

$$fg(x) = f(x^2 - 2) = 7 - 3(x^2 - 2)$$

$$= 7 - 3x^2 + 6$$

$$= 13 - 3x^2$$

Answer: $fg(x) = 13 - 3x^2$

Tip: Put the whole of $g(x)$ in a bracket before you multiply by -3 : $-3 \times (-2) = +6$.
Writing $7 - 3x^2 - 2$ is the most common error.

Marks: 1 mark for $7 - 3(x^2 - 2)$. 1 mark for $13 - 3x^2$.

(b) $h(x) = \frac{5}{x-2}$, $x > 2$. Work out $h^{-1}(x)$

Write $y = \frac{5}{x-2}$ and make x the subject.

Multiply both sides by $(x - 2)$: $y(x - 2) = 5$

Divide by y : $x - 2 = \frac{5}{y}$

Add 2: $x = \frac{5}{y} + 2$

Write the inverse in terms of x : $h^{-1}(x) = \frac{5}{x} + 2$ (or $\frac{5+2x}{x}$)

Check: $h(7) = \frac{5}{5} = 1$ and $h^{-1}(1) = 5 + 2 = 7$; $h(12) = \frac{5}{10} = \frac{1}{2}$ and

$h^{-1}\left(\frac{1}{2}\right) = 10 + 2 = 12$ ✓

Answer: $h^{-1}(x) = \frac{5}{x} + 2$

Tip: $h^{-1}(x)$ is the inverse function, not $\frac{1}{h(x)}$: the answer is not $\frac{x-2}{5}$. Undo the operations in reverse order and give the final answer in terms of x .

Marks: 1 mark for a correct first step in making x the subject (e.g. $x - 2 = \frac{5}{y}$). 1 mark for $\frac{5}{x} + 2$.

3 (2 marks)

Line L has equation $y + 2x = 5$. Line M passes through $(-1, 3)$ and $(2, -3)$. Show that L and M are parallel.

Line L : $y + 2x = 5$ so $y = -2x + 5$, which has gradient -2

Line M : gradient = $\frac{\text{change in } y}{\text{change in } x} = \frac{-3 - 3}{2 - (-1)} = \frac{-6}{3} = -2$

Both lines have gradient -2 , so the lines are parallel.

(They are different lines: when $x = -1$, line L has $y = 7$, not 3.)

Answer: Gradient of $L = -2$ and gradient of $M = \frac{-6}{3} = -2$: the gradients are equal, so the lines are parallel.

Tip: Rearrange L to $y = mx + c$ to read off its gradient, and use change in $y \div$ change in x for M . A “show that” needs a conclusion: say that the gradients are equal, so the lines are parallel.

Marks: 1 mark for one correct gradient (-2 from $y = -2x + 5$, or $\frac{-6}{3}$ for M). 1 mark for both gradients -2 and the statement that the lines are parallel.

4 (2 marks)

Factorise fully $12x^2y - 14xy - 6y$

Every term has a common factor of $2y$: $12x^2y - 14xy - 6y = 2y(6x^2 - 7x - 3)$

Factorise the quadratic. Find two numbers that multiply to $6 \times (-3) = -18$ and add to -7 : -9 and 2

$6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (3x + 1)(2x - 3)$

$$\text{So } 12x^2y - 14xy - 6y = 2y(3x + 1)(2x - 3)$$

Answer: $2y(3x + 1)(2x - 3)$

Tip: “Fully” means take out the whole common factor, $2y$, not just y or 2 . Check by expanding your answer.

Marks: 1 mark for taking out a common factor (e.g. $2y(6x^2 - 7x - 3)$). 1 mark for $2y(3x + 1)(2x - 3)$.

5 (4 marks)

- (a) Which matrix represents a rotation of 270° anticlockwise about the origin?
Circle your answer.

Options: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

A rotation of 270° anticlockwise is the same as 90° clockwise.

$(1, 0)$ goes to $(0, -1)$ and $(0, 1)$ goes to $(1, 0)$.

These images are the columns of the matrix: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Answer: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Tip: Find where $(1, 0)$ and $(0, 1)$ go. Their images are the first and second columns of the matrix.

- (b) P is rotated 270° anticlockwise about the origin, then transformed by $\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$.
The final image is $(-4, 5)$. Work out the coordinates of P .

Work backwards. Let $Q(x, y)$ be the point after the rotation. Then

$$\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Multiply out: $2x + 3y = -4$ (1) and $x - 2y = 5$ (2)

$2 \times (2)$: $2x - 4y = 10$ (3)

(1) – (3): $7y = -14$, so $y = -2$. Substitute into (2): $x + 4 = 5$, so $x = 1$. Q is $(1, -2)$

Q is P rotated 270° anticlockwise, so P is Q rotated 90° anticlockwise:

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(Or: the combined matrix is $\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & 1 \end{pmatrix}$, and solving $-3x + 2y = -4$, $2x + y = 5$ gives $x = 2$, $y = 1$.)

Check: rotating $(2, 1)$ by 270° anticlockwise gives $(1, -2)$, and

$$\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 - 6 \\ 1 + 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \checkmark$$

Answer: $P = (2, 1)$

Tip: Undo the transformations in reverse order: undo the matrix first, then undo the rotation (rotate 90° anticlockwise). If you combine the matrices, the rotation matrix from part (a) goes on the **right**, because it is done first.

Marks: 1 mark for the point $(1, -2)$ before the matrix was used (or the combined matrix $\begin{pmatrix} -3 & 2 \\ 2 & 1 \end{pmatrix}$). 1 mark for a correct method to finish (rotating $(1, -2)$ by 90° anticlockwise, or solving the two equations). 1 mark for $(2, 1)$.

6 (3 marks)

Here are the first five terms of a quadratic sequence: 18, 29, 34, 33, 26. Work out an expression for the n th term.

First differences: 11, 5, -1 , -7

Second differences: -6 , -6 , -6

Half of the second difference is -3 , so the n th term starts $-3n^2$.

Work out sequence $-(-3n^2)$, i.e. add $3n^2$: $18 + 3 = 21$, $29 + 12 = 41$, $34 + 27 = 61$, $33 + 48 = 81$

21, 41, 61, 81 is linear with n th term $20n + 1$

So the n th term is $-3n^2 + 20n + 1$

Check $n = 5$: $-75 + 100 + 1 = 26$ ✓

Answer: $-3n^2 + 20n + 1$

Tip: The second difference is negative, so the n^2 coefficient is negative. Take care with signs when you add back $3n^2$, and check your formula with a term you did not use.

Marks: 1 mark for $-3n^2$. 1 mark for $20n + 1$ (or for two correct equations in b and c). 1 mark for $-3n^2 + 20n + 1$.

7 (3 marks)

Expand and simplify $(3 - 2\sqrt{7})^2 - (5 + \sqrt{7})(5 - \sqrt{7})$ in the form $a + b\sqrt{7}$.

$$(3 - 2\sqrt{7})^2 = (3 - 2\sqrt{7})(3 - 2\sqrt{7}) = 9 - 6\sqrt{7} - 6\sqrt{7} + 4 \times 7$$

$$= 9 - 12\sqrt{7} + 28 = 37 - 12\sqrt{7}$$

$$(5 + \sqrt{7})(5 - \sqrt{7}) = 25 - 5\sqrt{7} + 5\sqrt{7} - 7 = 18$$

$$37 - 12\sqrt{7} - 18 = 19 - 12\sqrt{7}$$

Answer: $19 - 12\sqrt{7}$

Tip: $(2\sqrt{7})^2 = 4 \times 7 = 28$, not 14. The second product is a difference of two squares, so its $\sqrt{7}$ terms cancel.

Marks: 1 mark for $37 - 12\sqrt{7}$ (or four correct terms). 1 mark for 18. 1 mark for $19 - 12\sqrt{7}$.

8 (3 marks)

$y = \frac{3}{4}x^4 + 5x$. The rate of change of y with respect to x is -19 . Work out the value of x .

The rate of change of y with respect to x is $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{3}{4} \times 4x^3 + 5 = 3x^3 + 5$$

$$3x^3 + 5 = -19$$

$$3x^3 = -24$$

$$x^3 = -8$$

$$x = \sqrt[3]{-8} = -2$$

Answer: $x = -2$

Tip: “Rate of change” means differentiate. A negative number has a negative cube root:
 $(-2)^3 = -8$.

Marks: 1 mark for $3x^3 + 5$. 1 mark for $3x^3 + 5 = -19$ (or $x^3 = -8$). 1 mark for -2 .

9 (2 marks)

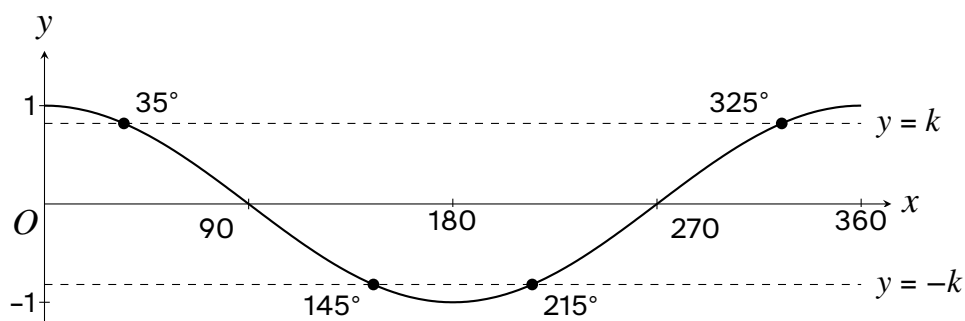
The graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ is sketched below.

$\cos 35^\circ = k$. Find the two angles x , in the range $0^\circ \leq x \leq 360^\circ$, where $\cos x = -k$

The graph of $y = \cos x$ is symmetrical about $x = 180^\circ$.

$\cos x = k$ at $x = 35^\circ$ and at $x = 360^\circ - 35^\circ = 325^\circ$.

$\cos x = -k$ where the graph is 35° either side of 180° : $180^\circ - 35^\circ = 145^\circ$ and $180^\circ + 35^\circ = 215^\circ$



Answer: 145° and 215°

Tip: Cos is negative in the second and third quadrants (90° to 270°). Draw the line $y = -k$ on the sketch to see where it meets the curve.

Marks: 1 mark for one correct angle. 1 mark for both.

10 (4 marks)

A circle has its centre at the origin, O .

The circle passes through the point $P(6, -1)$.

- (a) Write down the equation of the circle.

The circle has centre $(0, 0)$, so its equation is $x^2 + y^2 = r^2$.

P is on the circle: $r^2 = 6^2 + (-1)^2 = 36 + 1 = 37$

$$x^2 + y^2 = 37$$

Answer: $x^2 + y^2 = 37$

Tip: The number on the right is r^2 , so do not square root it: $x^2 + y^2 = \sqrt{37}$ is wrong.

- (b) Work out the equation of the tangent to the circle at P , in the form $ax + by = c$.

The tangent is perpendicular to the radius OP .

$$\text{Gradient of } OP = \frac{-1 - 0}{6 - 0} = -\frac{1}{6}$$

$$\text{Gradient of the tangent} = -1 \div \left(-\frac{1}{6}\right) = 6$$

$$\text{Tangent: } y - (-1) = 6(x - 6)$$

$$y + 1 = 6x - 36$$

$$y = 6x - 37, \text{ so } 6x - y = 37$$

Answer: $6x - y = 37$

Tip: Use the negative reciprocal of the radius gradient. Then rearrange into exactly the form asked for, with whole numbers.

Marks: 1 mark for the gradient of OP ($-\frac{1}{6}$). 1 mark for the tangent gradient 6. 1 mark for $6x - y = 37$.

11 (3 marks)

Write $\frac{2}{x-5} - \frac{3x+19}{3x^2-13x-10}$ as a single fraction in its simplest form.

Factorise the quadratic denominator: $3x^2 - 13x - 10 = (3x + 2)(x - 5)$

So the common denominator is $(3x + 2)(x - 5)$.

$$\frac{2}{x-5} = \frac{2(3x+2)}{(3x+2)(x-5)}$$

$$\frac{2(3x+2) - (3x+19)}{(3x+2)(x-5)} = \frac{6x+4-3x-19}{(3x+2)(x-5)} = \frac{3x-15}{(3x+2)(x-5)}$$

Factorise the numerator: $3x - 15 = 3(x - 5)$

$$\frac{3(x-5)}{(3x+2)(x-5)} = \frac{3}{3x+2} \quad (\text{cancel the common factor } x-5)$$

Answer: $\frac{3}{3x+2}$

Tip: Put the second numerator in a bracket before subtracting: $-(3x+19) = -3x-19$.

“Simplest form” means you must factorise the new numerator and cancel the common factor.

Marks: 1 mark for factorising $3x^2 - 13x - 10$. 1 mark for the single fraction $\frac{3x-15}{(3x+2)(x-5)}$. 1 mark for $\frac{3}{3x+2}$.

12 (3 marks)

When $(2+ax)^3$ and $(3-x)^4$ are expanded, the two terms in x^2 are the same. Work out the possible values of a .

Pascal's triangle rows: 1 3 3 1 and 1 4 6 4 1

$$x^2 \text{ term of } (2+ax)^3: 3 \times 2^1 \times (ax)^2 = 6a^2x^2$$

$$x^2 \text{ term of } (3-x)^4: 6 \times 3^2 \times (-x)^2 = 6 \times 9 \times x^2 = 54x^2$$

$$6a^2 = 54$$

$$a^2 = 9$$

$$a = 3 \text{ or } a = -3$$

Answer: $a = 3$ or $a = -3$

Tip: Bracket the whole term before squaring: $(ax)^2 = a^2x^2$ and $(-x)^2 = +x^2$. When you square root $a^2 = 9$, remember there are two answers, ± 3 .

Marks: 1 mark for $6a^2$. 1 mark for 54. 1 mark for both 3 and -3 .

13 (4 marks)

Here is the graph of $y = x^2 - 4x + 2$ for values of x from -1 to 5

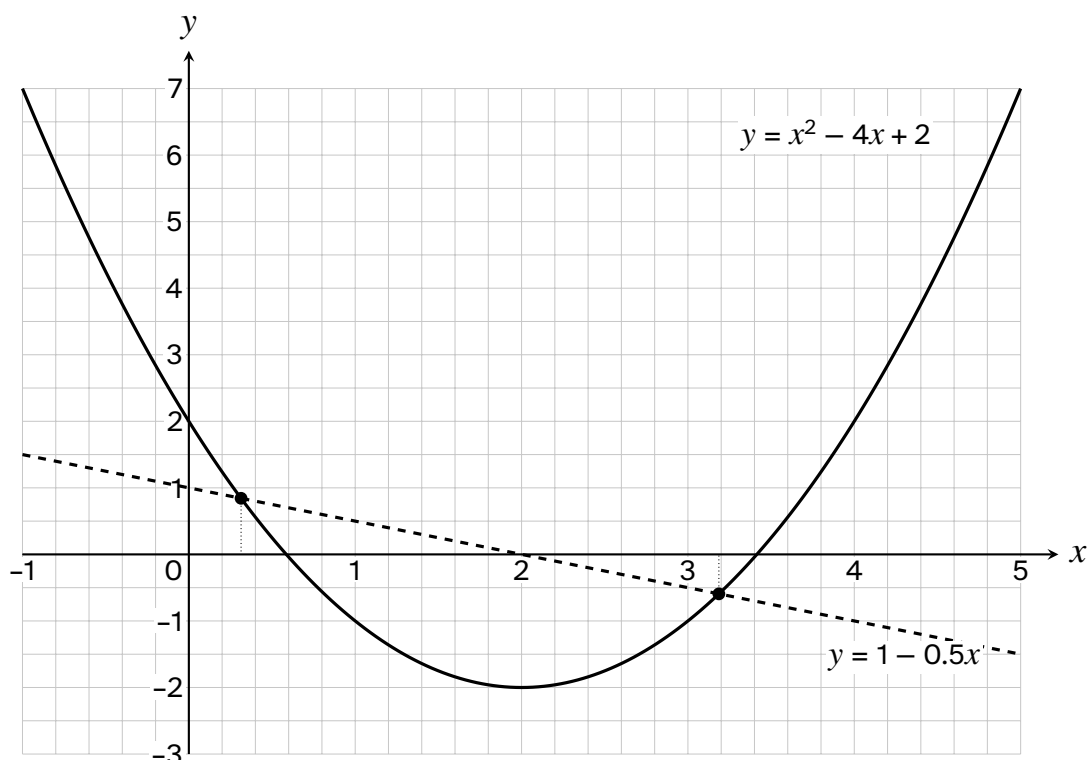
By drawing a suitable straight line on the grid, work out approximate solutions to $2x^2 - 7x + 2 = 0$, to 1 decimal place.

Divide the equation by 2 so that it starts with x^2 : $x^2 - \frac{7}{2}x + 1 = 0$

The graph is $y = x^2 - 4x + 2$, so rearrange to get $x^2 - 4x + 2$ on the left.

Subtract $\frac{1}{2}x$ from both sides and add 1 to both sides: $x^2 - 4x + 2 = 1 - \frac{1}{2}x$

So draw the line $y = 1 - 0.5x$. It passes through $(-1, 1.5)$, $(0, 1)$, $(2, 0)$ and $(4, -1)$.



Read the x -values where the line crosses the curve: $x \approx 0.3$ and $x \approx 3.2$

Answers from 0.2 to 0.4 and from 3.1 to 3.3 are fine if read from an accurate line.

Answer: $x \approx 0.3$ and $x \approx 3.2$

Tip: Rearrange so that the curve's expression, $x^2 - 4x + 2$, is on one side; the other side is the line to draw. Draw it with a ruler across the whole grid and read the x -values carefully.

Marks: 1 mark for rearranging to $x^2 - 4x + 2 = 1 - 0.5x$ (or stating the line $y = 1 - 0.5x$). 1 mark for drawing the line. 1 mark for each solution.

14 (4 marks)

x is negative. Solve $2x + \frac{12}{x} \geq -11$

x is negative, so when you multiply both sides by x you must **reverse** the inequality sign.

$$2x^2 + 12 \leq -11x$$

Rearrange: $2x^2 + 11x + 12 \leq 0$

Factorise: $(2x + 3)(x + 4) \leq 0$

Critical values: $x = -\frac{3}{2}$ and $x = -4$

$y = (2x + 3)(x + 4)$ is a U-shaped curve, so it is ≤ 0 between the critical values.

$$-4 \leq x \leq -\frac{3}{2} \text{ (these values are all negative)}$$

Check $x = -2$: $2(-2) + \frac{12}{-2} = -4 - 6 = -10$, and $-10 \geq -11$ ✓

Answer: $-4 \leq x \leq -\frac{3}{2}$

Tip: Multiplying an inequality by a negative number reverses the sign. If you keep $\geq$, you get the region outside the critical values, which is wrong. Test a value inside your range in the original inequality.

Marks: 1 mark for multiplying every term by x (e.g. $2x^2 + 11x + 12 \leq 0$). 1 mark for $(2x + 3)(x + 4)$. 1 mark for the critical values -4 and $-\frac{3}{2}$. 1 mark for $-4 \leq x \leq -\frac{3}{2}$.

15 (3 marks)

Solve $16^{x-1} = \frac{8^{2x+1}}{64}$

Write every number as a power of 2: $16 = 2^4$, $8 = 2^3$, $64 = 2^6$

Left side: $(2^4)^{x-1} = 2^{4x-4}$

Right side: $\frac{(2^3)^{2x+1}}{2^6} = \frac{2^{6x+3}}{2^6} = 2^{6x-3}$

Equate the powers: $4x - 4 = 6x - 3$

$-1 = 2x$

$x = -\frac{1}{2}$

Answer: $x = -\frac{1}{2}$

Tip: Multiply every term in the bracket: $3(2x + 1) = 6x + 3$, not $6x + 1$. Dividing powers of the same base means subtracting the indices.

Marks: 1 mark for writing the numbers as powers of 2. 1 mark for a correct linear equation, e.g. $4(x - 1) = 3(2x + 1) - 6$. 1 mark for $-\frac{1}{2}$.

16 (4 marks)

$y = 2x^5 - \frac{6}{x}$. The tangent to the curve at the point where $x = -1$ has equation $y = mx + c$. Work out m and c .

When $x = -1$: $y = 2(-1)^5 - \frac{6}{-1} = -2 + 6 = 4$, so the point is $(-1, 4)$.

Write $\frac{6}{x}$ as $6x^{-1}$: $y = 2x^5 - 6x^{-1}$

$$\frac{dy}{dx} = 10x^4 - 6 \times (-1)x^{-2} = 10x^4 + 6x^{-2} = 10x^4 + \frac{6}{x^2}$$

$$\text{At } x = -1: \frac{dy}{dx} = 10(-1)^4 + \frac{6}{(-1)^2} = 10 + 6 = 16$$

Tangent through $(-1, 4)$ with gradient 16: $y - 4 = 16(x + 1)$

$$y = 16x + 16 + 4$$

$$y = 16x + 20, \text{ so } m = 16 \text{ and } c = 20$$

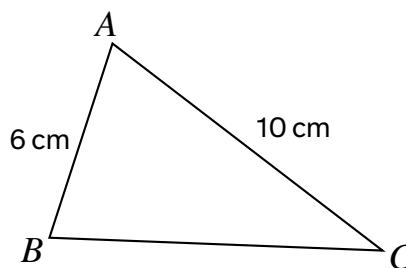
Answer: $m = 16$, $c = 20$ (tangent $y = 16x + 20$)

Tip: Differentiating $-6x^{-1}$ gives $+6x^{-2}$: two minus signs make a plus. The tangent uses the gradient itself, not the negative reciprocal (that is the normal).

Marks: 1 mark for one term of $\frac{dy}{dx}$ correct. 1 mark for $10x^4 + 6x^{-2}$. 1 mark for the gradient 16 used in a line through $(-1, 4)$. 1 mark for $m = 16$ and $c = 20$.

17 (3 marks)

Here is triangle ABC .



Not drawn accurately

$AB = 6$ cm, $AC = 10$ cm and $\cos A = \frac{1}{3}$. Work out BC as a surd in its simplest form.

Two sides and the angle between them are known, so use the cosine rule.

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$$

$$BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \frac{1}{3}$$

$$= 36 + 100 - 40 = 96$$

$$BC = \sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6} \text{ cm}$$

Answer: $4\sqrt{6}$ cm

Tip: Use the value of $\cos A$ directly; you do not need angle A . Work out $2 \times 6 \times 10 \times \frac{1}{3} = 40$ before subtracting.

Marks: 1 mark for a correct cosine rule substitution. 1 mark for 96. 1 mark for $4\sqrt{6}$.

18 (5 marks)

Solve $2x + 4y - z = 2$, $3x - 2y + 2z = 14$, $4x - 2y = 13$

Label the equations (1) $2x + 4y - z = 2$, (2) $3x - 2y + 2z = 14$, (3) $4x - 2y = 13$

Equation (3) has no z , so eliminate z from (1) and (2).

$$2 \times (1): 4x + 8y - 2z = 4$$

$$\text{Add (2): } 7x + 6y = 18 \quad (4)$$

$$3 \times (3): 12x - 6y = 39 \quad (5)$$

$$(4) + (5): 19x = 57, \text{ so } x = 3$$

$$\text{Substitute into (3): } 12 - 2y = 13, \text{ so } y = -\frac{1}{2}$$

$$\text{Substitute into (1): } 6 - 2 - z = 2, \text{ so } z = 2$$

$$\text{Check in (2): } 9 + 1 + 4 = 14 \quad \checkmark$$

$$\textbf{Answer: } x = 3, y = -\frac{1}{2}, z = 2$$

Tip: Eliminate the letter that is missing from one equation (z here) so that you get two equations in x and y . Substitute your answers into all three equations to check.

Marks: 1 mark for eliminating z correctly. 1 mark for a correct method to eliminate x or y next. 1 mark for one correct value. 1 mark for substituting to find the others. 1 mark for all three correct.

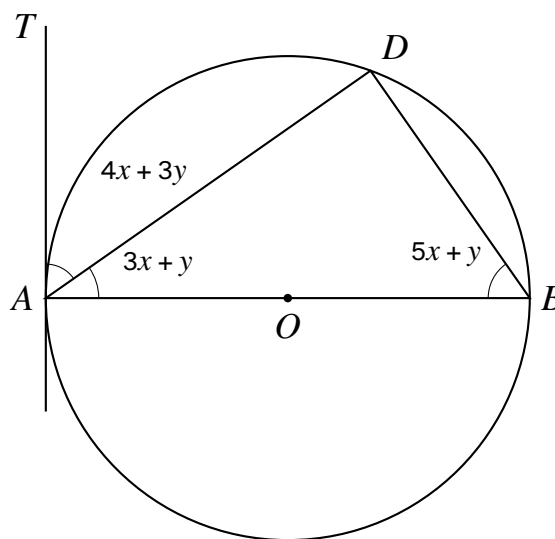
19 (5 marks)

A , B and D are points on a circle, centre O .

AB is a diameter of the circle.

TA is the tangent to the circle at A .

All angles are in degrees.



Not drawn accurately

- (a) Give a reason why angle $ADB = 90^\circ$ and a reason why angle $TAB = 90^\circ$

Angle ADB : AB is a diameter, and the angle in a semicircle is 90° .

Angle TAB : OA is a radius, and a tangent meets a radius at 90° (the tangent is perpendicular to the radius at the point of contact).

Answer: ADB : angle in a semicircle is 90° . TAB : the tangent is perpendicular to the radius OA .

Tip: Use the standard wording of the circle theorems. " AB is a diameter" on its own is not a reason.

Marks: 1 mark for each correct reason.

- (b) Work out the value of x and the value of y .
You must show your working.

Angle $TAB = 90^\circ$: $(4x + 3y) + (3x + y) = 90$, so $7x + 4y = 90$ (1)

Triangle ADB has a right angle at D , so the other two angles add to 90° :

$(3x + y) + (5x + y) = 90$, so $8x + 2y = 90$, i.e. $4x + y = 45$ (2)

$$4 \times (2): 16x + 4y = 180 \quad (3)$$

$$(3) - (1): 9x = 90, \text{ so } x = 10$$

$$\text{Substitute into (2): } 40 + y = 45, \text{ so } y = 5$$

Check: angles $4x + 3y = 55^\circ$, $3x + y = 35^\circ$, $5x + y = 55^\circ$; $55 + 35 = 90$ and $35 + 55 = 90$ ✓

Answer: $x = 10$, $y = 5$

Tip: Use the two 90° angles from part (a) to write two equations, then solve them simultaneously. (Angle $TAD =$ angle ABD by the alternate segment theorem also gives a correct equation.)

Marks: 1 mark for one correct equation. 1 mark for two correct equations and a correct method to eliminate a variable. 1 mark for $x = 10$ and $y = 5$.

20 (3 marks)

n is a positive integer. Prove that $(4n + 2)^2 - (2n - 5)(2n + 4)$ is a multiple of 6

$$(4n + 2)^2 = (4n + 2)(4n + 2) = 16n^2 + 8n + 8n + 4 = 16n^2 + 16n + 4$$

$$(2n - 5)(2n + 4) = 4n^2 + 8n - 10n - 20 = 4n^2 - 2n - 20$$

$$(16n^2 + 16n + 4) - (4n^2 - 2n - 20) = 16n^2 + 16n + 4 - 4n^2 + 2n + 20$$

$$= 12n^2 + 18n + 24$$

$$= 6(2n^2 + 3n + 4)$$

$2n^2 + 3n + 4$ is an integer, so $6(2n^2 + 3n + 4)$ is a multiple of 6.

Answer: $12n^2 + 18n + 24 = 6(2n^2 + 3n + 4)$, which is a multiple of 6 because $2n^2 + 3n + 4$ is an integer.

Tip: Put the whole second expansion in a bracket and change every sign: $-(-2n) = +2n$ and $-(-20) = +20$. Finish with a sentence explaining why your expression is a multiple of 6.

Marks: 1 mark for expanding one product correctly. 1 mark for $12n^2 + 18n + 24$. 1 mark for $6(2n^2 + 3n + 4)$ with a conclusion.

21 (5 marks)

The cost of running a delivery van is C pence per mile when the van travels at a steady speed of v miles per hour, where

$$C = \frac{v}{5} + \frac{720}{v} \quad 20 \leq v \leq 70$$

Use calculus to work out the minimum cost per mile.

You must show that your answer is a minimum.

Write $C = \frac{1}{5}v + 720v^{-1}$

$$\frac{dC}{dv} = \frac{1}{5} - 720v^{-2} = \frac{1}{5} - \frac{720}{v^2}$$

At a minimum, $\frac{dC}{dv} = 0$: $\frac{1}{5} = \frac{720}{v^2}$

$$v^2 = 3600$$

$v = 60$ (v is a speed, so take the positive root; 60 is in the range $20 \leq v \leq 70$)

$$\frac{d^2C}{dv^2} = -720 \times (-2)v^{-3} = \frac{1440}{v^3}$$

When $v = 60$, $\frac{1440}{60^3}$ is positive, so C has a minimum value when $v = 60$.

$$C = \frac{60}{5} + \frac{720}{60} = 12 + 12 = 24$$

The minimum cost is 24 pence per mile.

Answer: $v = 60$ and $\frac{d^2C}{dv^2} = \frac{1440}{v^3} > 0$, so a minimum; minimum cost 24 pence per mile.

Tip: Write $\frac{720}{v}$ as $720v^{-1}$ before differentiating. Finding $v = 60$ is not the end: substitute it back into C to get the cost, and use the second derivative (or the sign of $\frac{dC}{dv}$ either side of $v = 60$) to show it is a minimum. A sketch is not a test.

Marks: 1 mark for differentiating (one term correct). 1 mark for setting $\frac{dC}{dv} = 0$ and getting $v^2 = 3600$. 1 mark for $v = 60$. 1 mark for showing it is a minimum (e.g. $\frac{d^2C}{dv^2} = \frac{1440}{v^3}$, positive at $v = 60$). 1 mark for 24 pence.

22 (6 marks)

Work out the coordinates of the points where the curve $y = 2x^2 - 7x - 5$ and the line $4x + y = 4$ intersect.

Rearrange the line: $y = 4 - 4x$

Substitute into the curve: $2x^2 - 7x - 5 = 4 - 4x$

$$2x^2 - 7x + 4x - 5 - 4 = 0$$

$$2x^2 - 3x - 9 = 0$$

Two numbers that multiply to $2 \times (-9) = -18$ and add to -3 : -6 and 3

$$2x^2 - 6x + 3x - 9 = 2x(x - 3) + 3(x - 3) = (2x + 3)(x - 3) = 0$$

$$x = -\frac{3}{2} \text{ or } x = 3$$

$$\text{When } x = 3: y = 4 - 12 = -8$$

$$\text{When } x = -\frac{3}{2}: y = 4 + 6 = 10$$

Check in the curve: $18 - 21 - 5 = -8$ and $\frac{9}{2} + \frac{21}{2} - 5 = 10$ ✓

Answer: $(3, -8)$ and $\left(-\frac{3}{2}, 10\right)$

Tip: Substitute for y , collect everything on one side and factorise. With $2x^2$ the brackets are $(2x \quad)(x \quad)$: expand them to check. Then find each y -value from the line and pair the coordinates correctly.

Marks: 1 mark for $y = 4 - 4x$. 1 mark for substituting into the curve. 1 mark for $2x^2 - 3x - 9 = 0$. 1 mark for factorising. 1 mark for $x = 3$ and $x = -\frac{3}{2}$. 1 mark for both points.

23 (3 marks)

Show that $\frac{6 \sin^4 x + 6 \sin^2 x \cos^2 x}{2 \cos^2 x + 5 \sin^2 x - 5}$ can be written as $k \tan^2 x$ where k is an integer.

Use $\sin^2 x + \cos^2 x = 1$.

Numerator: take out the common factor $6 \sin^2 x$:

$$6 \sin^4 x + 6 \sin^2 x \cos^2 x = 6 \sin^2 x (\sin^2 x + \cos^2 x) = 6 \sin^2 x \times 1 = 6 \sin^2 x$$

Denominator:

$$2 \cos^2 x + 5 \sin^2 x - 5 = 2 \cos^2 x + 5(\sin^2 x - 1) = 2 \cos^2 x - 5 \cos^2 x = -3 \cos^2 x$$

$$\frac{6 \sin^2 x}{-3 \cos^2 x} = -2 \times \frac{\sin^2 x}{\cos^2 x}$$

$$= -2 \tan^2 x \quad \left(\text{using } \tan x = \frac{\sin x}{\cos x}\right), \text{ so } k = -2$$

Answer: $\frac{6 \sin^2 x}{-3 \cos^2 x} = -2 \tan^2 x$, so $k = -2$

Tip: Factorise the numerator first: $6 \sin^2 x$ is a common factor, and the bracket left is $\sin^2 x + \cos^2 x = 1$. In the denominator write $\sin^2 x - 1$ as $-\cos^2 x$. Work on the expression step by step; do not treat it as an equation.

Marks: 1 mark for the numerator $6 \sin^2 x$. 1 mark for the denominator $-3 \cos^2 x$. 1 mark for $-2 \tan^2 x$.