

Mock Set B – Paper 2

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Factorise fully $10(x + 4)^3 - 15(x + 4)^2$

Both terms have a factor of 5 and a factor of $(x + 4)^2$.

$$10(x + 4)^3 - 15(x + 4)^2 = 5(x + 4)^2 [2(x + 4) - 3]$$

Simplify the square bracket: $2(x + 4) - 3 = 2x + 8 - 3 = 2x + 5$

So the answer is $5(x + 4)^2(2x + 5)$

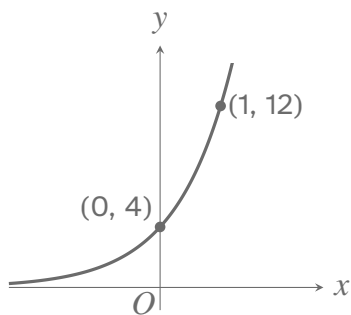
Answer: $5(x + 4)^2(2x + 5)$

Tip: Do not expand the brackets: take out the highest common factor, $5(x + 4)^2$, and then simplify what is left inside the bracket.

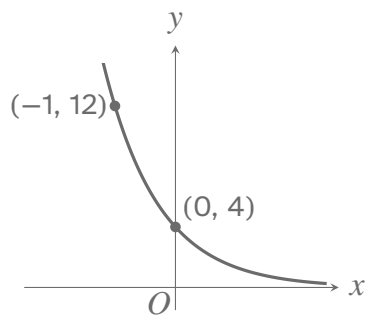
Marks: 1 mark for taking out a correct common factor, e.g. $5(x + 4)^2[2(x + 4) - 3]$. 1 mark for $5(x + 4)^2(2x + 5)$.

2 (1 mark)

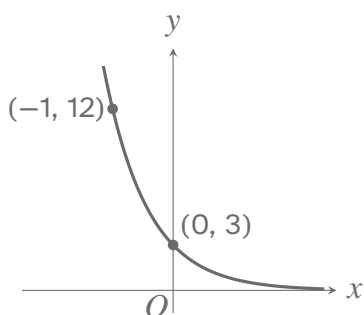
Here are four sketch graphs.



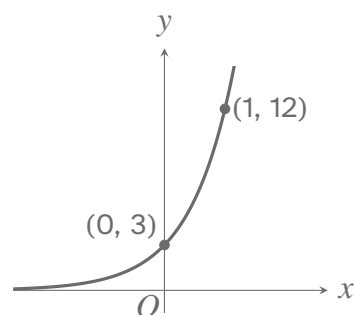
A



B



C



D

Which sketch shows the graph of $y = 4 \times 3^{-x}$?

Options: A, B, C, D

y-intercept: put $x = 0$. $3^0 = 1$, so $y = 4 \times 1 = 4$. That leaves A or B.

$3^{-x} = \left(\frac{1}{3}\right)^x$ gets smaller as x increases, so the curve goes **down** from left to right: that is B.

Check: when $x = -1$, $y = 4 \times 3^1 = 12$, so B passes through $(-1, 12)$ ✓

Answer: B

Tip: For $y = a \times b^x$ or $y = a \times b^{-x}$ the curve always crosses the y-axis at a . The minus sign in the power makes the curve fall from left to right.

3 (3 marks)

$P(-7, 2)$ and $Q(3, -4)$ are the ends of a diameter of a circle.

Work out the equation of the circle.

Give your answer in the form $(x - a)^2 + (y - b)^2 = k$

The centre is the midpoint of PQ : $\left(\frac{-7+3}{2}, \frac{2+(-4)}{2}\right) = (-2, -1)$

$$\begin{aligned}\text{Radius}^2 &= (\text{distance from the centre to } Q)^2 \\ &= (3 - (-2))^2 + (-4 - (-1))^2 = 5^2 + (-3)^2 = 34\end{aligned}$$

(Or: $PQ^2 = 10^2 + 6^2 = 136$, so the diameter is $\sqrt{136}$ and $\text{radius}^2 = \frac{136}{4} = 34$.)

$$\text{Equation: } (x + 2)^2 + (y + 1)^2 = 34$$

Answer: $(x + 2)^2 + (y + 1)^2 = 34$

Tip: PQ is the **diameter**, so halve it to get the radius. The number on the right is the radius **squared**. A centre of $(-2, -1)$ gives $(x + 2)^2$ and $(y + 1)^2$.

Marks: 1 mark for the centre $(-2, -1)$. 1 mark for $\text{radius}^2 = 34$ (or $\text{radius} = \sqrt{34}$). 1 mark for the full equation.

4 (5 marks)

Triangle ABC has vertices $A(-5, -1)$, $B(1, 2)$ and $C(3, -2)$

(a) Use gradients to show that angle ABC is a right angle.

$$\text{Gradient of } AB: \frac{2 - (-1)}{1 - (-5)} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Gradient of } BC: \frac{-2 - 2}{3 - 1} = \frac{-4}{2} = -2$$

$$\text{Product of the gradients: } \frac{1}{2} \times (-2) = -1$$

The product is -1 , so AB is perpendicular to BC and angle ABC is a right angle.

Answer: $m_{AB} = \frac{1}{2}$, $m_{BC} = -2$, $\frac{1}{2} \times (-2) = -1$, so AB and BC are perpendicular and angle $ABC = 90^\circ$

Tip: Gradient is change in y divided by change in x ; keep the points in the same order on the top and the bottom. Finish with a statement: “the product is -1 , so the lines are perpendicular”.

Marks: 1 mark for the gradient of AB . 1 mark for the gradient of BC . 1 mark for showing the product is -1 (or that one is the negative reciprocal of the other) and saying the angle is 90° .

- (b) Work out the area of triangle ABC .

AB and BC are perpendicular, so use them as the base and the height.

$$AB = \sqrt{6^2 + 3^2} = \sqrt{45} \quad BC = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\text{Area} = \frac{1}{2} \times \sqrt{45} \times \sqrt{20} = \frac{1}{2} \times \sqrt{900} = \frac{1}{2} \times 30 = 15$$

Answer: 15 units²

Tip: The two sides that meet at the right angle are the base and the height. Keep the surds (or full calculator values) so that you get exactly 15.

Marks: 1 mark for both lengths $\sqrt{45}$ and $\sqrt{20}$ (or 6.70 ... and 4.47 ...). 1 mark for 15.

5 (5 marks)

Here are the first four terms of a quadratic sequence.

−3 4 17 36

- (a) Work out an expression for the n th term.

First differences: 7, 13, 19

Second differences: 6, 6, so the n^2 term is $\frac{6}{2}n^2 = 3n^2$

Subtract $3n^2$ (3, 12, 27, 48) from the terms: −6, −8, −10, −12

This linear sequence has n th term $-2n - 4$

$$n\text{th term} = 3n^2 - 2n - 4$$

Check: $n = 3$ gives $27 - 6 - 4 = 17$ ✓

Answer: $3n^2 - 2n - 4$

Tip: The coefficient of n^2 is **half** the second difference. Then subtract $3n^2$ from each term and find the n th term of what is left.

Marks: 1 mark for $3n^2$. 1 mark for the remaining linear sequence ($-6, -8, -10, -12$ or $-2n - 4$). 1 mark for $3n^2 - 2n - 4$.

(b) The m th term of the sequence is 1041

Work out the value of m .

You must show your working.

$$3n^2 - 2n - 4 = 1041$$

$$3n^2 - 2n - 1045 = 0$$

$$(n - 19)(3n + 55) = 0 \quad (\text{or use the quadratic formula})$$

$$n = 19 \quad \text{or} \quad n = -\frac{55}{3}$$

n must be a positive whole number, so $m = 19$.

Answer: $m = 19$

Tip: n is a position, so it must be a positive whole number: reject $-\frac{55}{3}$. Give the position (19), not the value.

Marks: 1 mark for $3n^2 - 2n - 4 = 1041$ with a correct method to solve it. 1 mark for 19 only.

6 (4 marks)

A curve has equation $y = \frac{6x^4 - 5x + 8}{2x^2}$. Work out the gradient of the curve at the point where $x = -2$

Divide each term on the top by $2x^2$:

$$y = \frac{6x^4}{2x^2} - \frac{5x}{2x^2} + \frac{8}{2x^2} = 3x^2 - \frac{5}{2}x^{-1} + 4x^{-2}$$

Differentiate each term: $\frac{dy}{dx} = 6x + \frac{5}{2}x^{-2} - 8x^{-3} = 6x + \frac{5}{2x^2} - \frac{8}{x^3}$

$$\text{At } x = -2: 6(-2) + \frac{5}{2(-2)^2} - \frac{8}{(-2)^3} = -12 + \frac{5}{8} + 1$$

$$= -10.375 \left(= -\frac{83}{8} \right)$$

Answer: $-10.375 \left(-\frac{83}{8} \right)$

Tip: You cannot differentiate the top and the bottom separately. Split the fraction into separate terms first: $\frac{8}{2x^2} = 4x^{-2}$, not $16x^{-2}$. Use brackets when you substitute $x = -2$.

Marks: 1 mark for writing one term as a power of x , e.g. $3x^2$. 1 mark for all three terms $(3x^2 - \frac{5}{2}x^{-1} + 4x^{-2})$. 1 mark for differentiating at least two terms correctly. 1 mark for -10.375 .

7 (3 marks)

(a) $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Describe fully the single transformation represented by $\mathbf{A}$.

Find the images of the unit vectors: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

A point (x, y) is mapped to $(-y, -x)$, which is a reflection in the line $y = -x$

Answer: Reflection in the line $y = -x$

Tip: The columns of the matrix are the images of $(1, 0)$ and $(0, 1)$. Sketch them on the unit square to see the transformation.

(b) Matrix $\mathbf{B}$ represents a rotation of 90° clockwise about the origin.

Work out $\mathbf{B}^3$ and describe fully the single transformation that $\mathbf{B}^3$ represents.

Rotating 90° clockwise: $(1, 0) \rightarrow (0, -1)$ and $(0, 1) \rightarrow (1, 0)$, so $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$\mathbf{B}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{B}^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Three turns of 90° clockwise = 270° clockwise = 90° anticlockwise.

$\mathbf{B}^3$ is a rotation of 90° anticlockwise about the origin.

Answer: $\mathbf{B}^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$: rotation 90° anticlockwise about the origin

Tip: “Describe fully” a rotation needs the angle, the direction and the centre: write “about the origin”.

Marks: 1 mark for $\mathbf{B}^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. 1 mark for rotation, 90° anticlockwise (or 270° clockwise), about the origin.

8 (2 marks)

$$7n^2 + 12 < 160$$

List all the integer values of n that satisfy this inequality.

$$7n^2 < 148$$

$$n^2 < \frac{148}{7} = 21.14 \dots$$

$$\text{So } -4.59 \dots < n < 4.59 \dots$$

The integers are $-4, -3, -2, -1, 0, 1, 2, 3, 4$

Check: $n = \pm 5$ gives $7 \times 25 + 12 = 187$, which is not less than 160.

Answer: $-4, -3, -2, -1, 0, 1, 2, 3, 4$

Tip: A squared inequality has two boundaries, so include the negative integers and 0. Give a list, not an inequality.

Marks: 1 mark for $n^2 < 21.14 \dots$ (or for most of the list). 1 mark for all nine integers and no others.

9 (3 marks)

Solve $2x + 3y = 4k$ and $y = kx + 1$, where k is a positive constant, giving x and y in terms of k , each as a single fraction.

Substitute $y = kx + 1$ into the first equation: $2x + 3(kx + 1) = 4k$

$$2x + 3kx + 3 = 4k, \text{ so } 2x + 3kx = 4k - 3$$

$$\text{Factorise: } x(2 + 3k) = 4k - 3, \text{ so } x = \frac{4k - 3}{3k + 2}$$

$$y = kx + 1 = \frac{k(4k - 3)}{3k + 2} + \frac{3k + 2}{3k + 2} = \frac{4k^2 - 3k + 3k + 2}{3k + 2} = \frac{4k^2 + 2}{3k + 2}$$

Answer: $x = \frac{4k - 3}{3k + 2}, y = \frac{4k^2 + 2}{3k + 2}$

Tip: You are solving for x and y , not k . Collect the x terms and factorise out x before dividing. For y , write the 1 as $\frac{3k+2}{3k+2}$ so you can add the fractions.

Marks: 1 mark for substituting to get $2x + 3(kx + 1) = 4k$. 1 mark for factorising to get $x = \frac{4k-3}{3k+2}$. 1 mark for both answers, with y written as the single fraction $\frac{4k^2+2}{3k+2}$.

10 (3 marks)

The function f is given by

$$\begin{aligned} f(x) &= x^2 + px - 6 & x < 2 \\ &= 5 - px & x \geq 2 \end{aligned}$$

where p is a constant.

$$f(-5) = f(3)$$

Work out the value of p .

$$-5 < 2, \text{ so use the first part: } f(-5) = (-5)^2 + p(-5) - 6 = 25 - 5p - 6 = 19 - 5p$$

$$3 \geq 2, \text{ so use the second part: } f(3) = 5 - 3p$$

$$19 - 5p = 5 - 3p$$

$$19 - 5 = 5p - 3p, \text{ so } 2p = 14$$

$$p = 7$$

Answer: $p = 7$

Tip: Check the domain before you substitute: -5 is less than 2 , so it goes into $x^2 + px - 6$; 3 is at least 2 , so it goes into $5 - px$. Using the same part for both values gives the wrong answer.

Marks: 1 mark for $f(-5) = 19 - 5p$ from the first part. 1 mark for $f(3) = 5 - 3p$ from the second part and an equation. 1 mark for $p = 7$.

11 (3 marks)

Simplify $\frac{\sqrt{x^5}(\sqrt[3]{x^2} + \sqrt{x})}{x^2}$, giving your answer in the form $x^{\frac{a}{b}} + x^c$.

Write each root as a power: $\sqrt{x^5} = x^{\frac{5}{2}}$, $\sqrt[3]{x^2} = x^{\frac{2}{3}}$, $\sqrt{x} = x^{\frac{1}{2}}$

Divide first: $\frac{x^{\frac{5}{2}}}{x^2} = x^{\frac{5}{2}-2} = x^{\frac{1}{2}}$

So the expression is $x^{\frac{1}{2}}(x^{\frac{2}{3}} + x^{\frac{1}{2}})$

Multiply out (add the powers): $x^{\frac{1}{2}+\frac{2}{3}} + x^{\frac{1}{2}+\frac{1}{2}} = x^{\frac{7}{6}} + x^1$

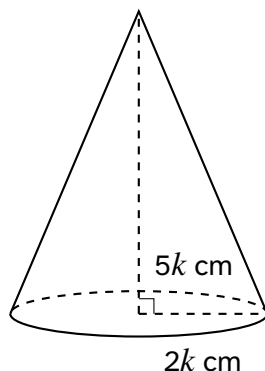
Answer: $x^{\frac{7}{6}} + x$ (that is, $a = 7$, $b = 6$, $c = 1$)

Tip: When you multiply powers of x , add the indices ($\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$); do not multiply them. Divide **both** terms by x^2 .

Marks: 1 mark for a correct step with powers, e.g. $x^{\frac{1}{2}}(x^{\frac{2}{3}} + x^{\frac{1}{2}})$. 1 mark for one correct term. 1 mark for $x^{\frac{7}{6}} + x$.

12 (3 marks)

A solid cone has base radius $2k$ cm and perpendicular height $5k$ cm.



Not drawn accurately

A cone has radius $2k$ cm, perpendicular height $5k$ cm and volume 250π cm³. Work out k to 3 significant figures.

$$\frac{1}{3}\pi(2k)^2(5k) = 250\pi$$

$$(2k)^2 = 4k^2, \text{ so } \frac{1}{3}\pi \times 4k^2 \times 5k = \frac{20}{3}\pi k^3$$

$$\frac{20}{3}\pi k^3 = 250\pi \Rightarrow 20k^3 = 750 \Rightarrow k^3 = 37.5$$

$$k = \sqrt[3]{37.5} = 3.347 \dots = 3.35 \text{ (3 s.f.)}$$

Answer: $k = 3.35$

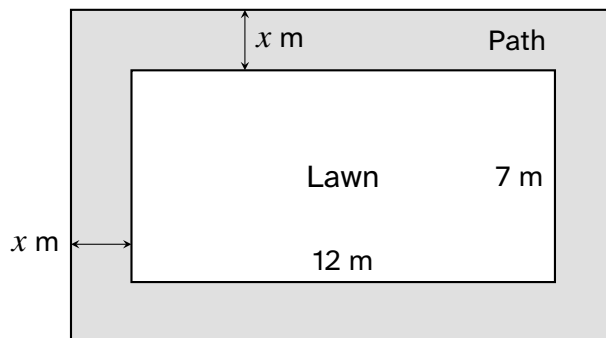
Tip: Square the whole radius: $(2k)^2 = 4k^2$, not $2k^2$. The π cancels on both sides. Use the **cube** root at the end.

Marks: 1 mark for $\frac{1}{3}\pi(2k)^2(5k) = 250\pi$. 1 mark for $k^3 = 37.5$. 1 mark for 3.35.

13 (3 marks)

A rectangular lawn is 12 m long and 7 m wide.

A gravel path of width x metres goes all the way round the lawn.



Not drawn accurately

A 12 m by 7 m lawn has a path of width x m all the way round it. The area of the path is 50 m^2 . Work out x to 3 significant figures.

The lawn and the path together make a rectangle $(12 + 2x)$ m by $(7 + 2x)$ m.

Area of path = outer rectangle – lawn: $(12 + 2x)(7 + 2x) - 12 \times 7 = 50$

$$84 + 24x + 14x + 4x^2 - 84 = 50$$

$$4x^2 + 38x - 50 = 0, \text{ so } 2x^2 + 19x - 25 = 0$$

$$x = \frac{-19 \pm \sqrt{19^2 - 4 \times 2 \times (-25)}}{2 \times 2} = \frac{-19 \pm \sqrt{561}}{4}$$

$$x = 1.171 \dots \text{ or } x = -10.67 \dots$$

x is a width, so it must be positive: $x = 1.17$ (3 s.f.)

Answer: $x = 1.17$

Tip: The path goes round **both** ends, so each side of the big rectangle is $2x$ longer, not x . A length cannot be negative, so reject the negative solution.

Marks: 1 mark for a correct equation, e.g. $(12 + 2x)(7 + 2x) - 84 = 50$. 1 mark for using the quadratic formula correctly on $2x^2 + 19x - 25 = 0$ (or equivalent). 1 mark for 1.17.

14 (4 marks)

Simplify fully $\frac{6y^2}{2y^2 - y - 15} \div \frac{4y}{9 - 3y}$

Dividing by a fraction is the same as multiplying by its reciprocal:

$$\frac{6y^2}{2y^2 - y - 15} \times \frac{9 - 3y}{4y}$$

Factorise the quadratic: $2y^2 - y - 15 = (2y + 5)(y - 3)$

Factorise $9 - 3y$ so that it has a factor $(y - 3)$: $9 - 3y = -3(y - 3)$

$$\frac{6y^2}{(2y + 5)(y - 3)} \times \frac{-3(y - 3)}{4y}$$

Cancel $(y - 3)$ and y : $\frac{6y \times (-3)}{4(2y + 5)} = \frac{-18y}{4(2y + 5)}$

$$= -\frac{9y}{2(2y + 5)} \quad \text{or} \quad -\frac{9y}{4y + 10}$$

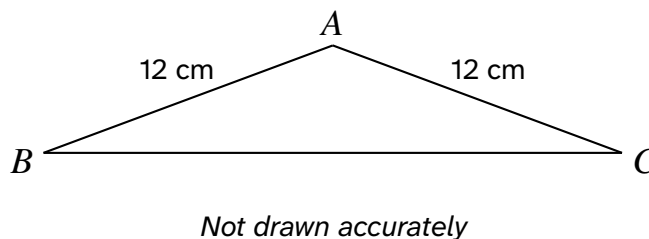
Answer: $-\frac{9y}{2(2y + 5)}$

Tip: Turn the second fraction upside down and multiply. $(3 - y)$ and $(y - 3)$ are not the same: $(3 - y) = -(y - 3)$. Take out -3 from $9 - 3y$ so it cancels, and keep the minus sign in your answer.

Marks: 1 mark for $(2y + 5)(y - 3)$. 1 mark for $9 - 3y = -3(y - 3)$ (or $3(3 - y)$). 1 mark for multiplying by the reciprocal and cancelling to a correct single fraction. 1 mark for $-\frac{9y}{2(2y+5)}$.

15 (4 marks)

Triangle ABC is isosceles with $AB = AC = 12$ cm.



Triangle ABC has $AB = AC = 12$ cm and area 47 cm^2 . Angle BAC is obtuse. Work out angle ABC to 1 decimal place.

$$\text{Area} = \frac{1}{2}bc \sin A: \frac{1}{2} \times 12 \times 12 \times \sin BAC = 47$$

$$72 \sin BAC = 47, \text{ so } \sin BAC = \frac{47}{72}$$

$$\sin^{-1}\left(\frac{47}{72}\right) = 40.75 \dots^\circ, \text{ but this is acute and angle } BAC \text{ is obtuse.}$$

$$\text{Angle } BAC = 180^\circ - 40.75 \dots^\circ = 139.24 \dots^\circ$$

$$AB = AC, \text{ so the base angles are equal: } \text{angle } ABC = \frac{180 - 139.24 \dots}{2} = 20.37 \dots^\circ$$

$$\text{Angle } ABC = 20.4^\circ \text{ (1 d.p.)}$$

Answer: 20.4°

Tip: $\sin^{-1}$ on a calculator only gives the acute angle. Here angle BAC is obtuse, so it is 180° minus the calculator value. Then use the equal base angles of the isosceles triangle.

Marks: 1 mark for $\frac{1}{2} \times 12 \times 12 \times \sin BAC = 47$. 1 mark for $40.75 \dots^\circ$ (or $\sin BAC = \frac{47}{72}$). 1 mark for using the obtuse angle $139.24 \dots^\circ$ to find a base angle. 1 mark for 20.4 .

16 (4 marks)

$$\text{Rearrange } y = \frac{3 + 4k^3}{2k^3 - 5} \text{ to make } k \text{ the subject.}$$

$$\text{Multiply both sides by } (2k^3 - 5): y(2k^3 - 5) = 3 + 4k^3$$

Expand: $2yk^3 - 5y = 3 + 4k^3$

Collect the k^3 terms on one side: $2yk^3 - 4k^3 = 5y + 3$

Factorise: $k^3(2y - 4) = 5y + 3$

Divide: $k^3 = \frac{5y + 3}{2y - 4}$

Cube root: $k = \sqrt[3]{\frac{5y + 3}{2y - 4}}$

Answer: $k = \sqrt[3]{\frac{5y + 3}{2y - 4}}$

Tip: k appears twice, so collect both k^3 terms on one side and factorise. Make sure the cube root covers the whole fraction.

Marks: 1 mark for $y(2k^3 - 5) = 3 + 4k^3$. 1 mark for collecting the k^3 terms. 1 mark for factorising, $k^3(2y - 4) = 5y + 3$. 1 mark for the final answer.

17 (5 marks)

$(x + 5)$ is a factor of $3x^3 + 14x^2 + ax - 10$ where a is a constant.

(a) Use the factor theorem to work out the value of a .

$(x + 5)$ is a factor, so $f(-5) = 0$.

$$3(-5)^3 + 14(-5)^2 + a(-5) - 10 = 0$$

$$-375 + 350 - 5a - 10 = 0$$

$$-35 - 5a = 0, \text{ so } a = -7$$

Answer: $a = -7$

Tip: For the factor $(x + 5)$ substitute $x = -5$, not $x = 5$. Use brackets for $(-5)^3$ and $(-5)^2$.

Marks: 1 mark for substituting $x = -5$ and setting it equal to 0. 1 mark for $a = -7$.

(b) Factorise $3x^3 + 14x^2 + ax - 10$ fully.

With $a = -7$: $3x^3 + 14x^2 - 7x - 10 = (x + 5)(3x^2 + bx - 2)$

(The $3x^2$ makes the $3x^3$, and $5 \times (-2) = -10$.)

Compare the x^2 terms: $15x^2 + bx^2 = 14x^2$, so $b = -1$

Quadratic factor: $3x^2 - x - 2 = (3x + 2)(x - 1)$

$3x^3 + 14x^2 - 7x - 10 = (x + 5)(3x + 2)(x - 1)$

Answer: $(x + 5)(3x + 2)(x - 1)$

Tip: Check the quadratic factor by expanding $(x + 5)(3x^2 - x - 2)$. “Factorise fully” means three linear brackets; do not solve the equation.

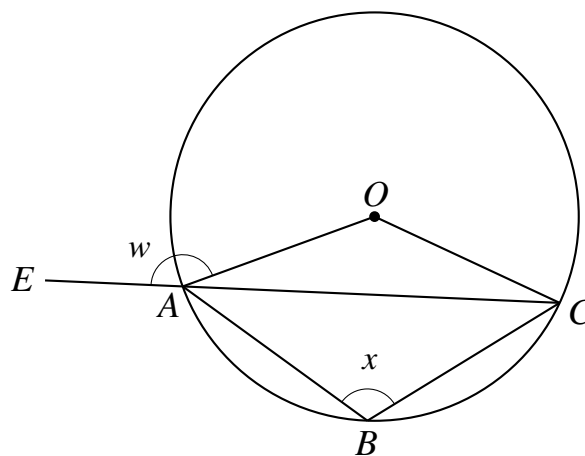
Marks: 1 mark for a quadratic factor of the form $3x^2 + bx - 2$. 1 mark for $3x^2 - x - 2$ (or another correct linear factor). 1 mark for the full factorisation.

18 (5 marks)

A , B and C are points on a circle, centre O .

EAC is a straight line.

Angle $ABC = x$ Angle $OAE = w$



Not drawn accurately

Prove that $w = 270^\circ - x$

Give a reason for each stage of your working.

Reflex angle $AOC = 2x$ (the angle at the centre is twice the angle at the circumference)

Angle $AOC = 360^\circ - 2x$ (angles around a point add up to 360°)

$OA = OC$ (radii), so triangle OAC is isosceles and angle $OAC =$ angle OCA (base angles of an isosceles triangle are equal)

Angle $OAC = \frac{180^\circ - (360^\circ - 2x)}{2} = \frac{2x - 180^\circ}{2} = x - 90^\circ$ (angles in a triangle add up to 180°)

EAC is a straight line, so $w + (x - 90^\circ) = 180^\circ$ (angles on a straight line add up to 180°)

$$w = 180^\circ - x + 90^\circ = 270^\circ - x$$

Answer: Reflex $AOC = 2x$, so $AOC = 360^\circ - 2x$; $OAC = x - 90^\circ$ (isosceles);
 $w = 180^\circ - (x - 90^\circ) = 270^\circ - x$, with reasons

Tip: B is on the minor arc, so $2x$ is the **reflex** angle at the centre. Use brackets when you subtract: $180 - (360 - 2x)$. Every step needs a reason, in words such as “base angles of an isosceles triangle are equal”.

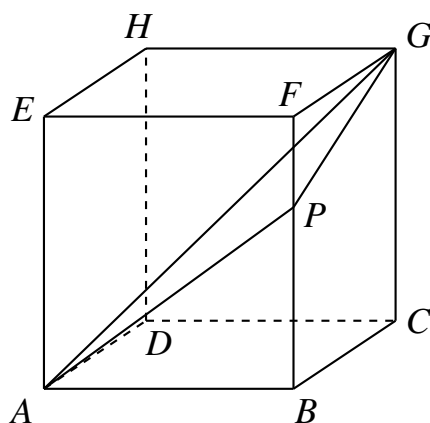
Marks: 1 mark for reflex angle $AOC = 2x$. 1 mark for angle $AOC = 360^\circ - 2x$. 1 mark for angle $OAC = x - 90^\circ$. 1 mark for $w = 180^\circ - (x - 90^\circ)$. 1 mark for reaching $w = 270^\circ - x$ with a correct reason at every step.

19 (5 marks)

$ABCDEFGH$ is a cuboid.

$$AB = 11 \text{ cm} \quad BC = 6 \text{ cm} \quad BF = 12 \text{ cm}$$

P is the point on BF such that $BP:PF = 2:1$



Not drawn accurately

Cuboid $ABCDEFGH$ with $AB = 11 \text{ cm}$, $BC = 6 \text{ cm}$, $BF = 12 \text{ cm}$; P on BF with $BP:PF = 2:1$. Work out angle APG .

$$BP = \frac{2}{3} \times 12 = 8 \text{ cm} \quad \text{and} \quad PF = 4 \text{ cm}$$

$$AP \text{ (face } ABFE\text{): } AP^2 = 11^2 + 8^2 = 185$$

$$PG \text{ (face } BCGF\text{): } PG^2 = FG^2 + PF^2 = 6^2 + 4^2 = 52$$

$$AG \text{ (space diagonal): } AG^2 = 11^2 + 6^2 + 12^2 = 301$$

$$\text{Cosine rule in triangle } APG: \cos APG = \frac{AP^2 + PG^2 - AG^2}{2 \times AP \times PG} = \frac{185 + 52 - 301}{2\sqrt{185}\sqrt{52}}$$

$$\cos APG = \frac{-64}{196.16 \dots} = -0.3262 \dots$$

$$\text{Angle } APG = \cos^{-1}(-0.3262 \dots) = 109.04 \dots^\circ = 109.0^\circ \text{ (1 d.p.)}$$

Answer: 109.0°

Tip: In the cosine rule the side opposite the angle you want (AG) is the one that is subtracted. A negative cosine means the angle is obtuse; that is correct here.

Marks: 1 mark for AP . 1 mark for PG . 1 mark for AG . 1 mark for a correct cosine rule for angle APG . 1 mark for 109.0.

20 (4 marks)

The curve $y = 2x^3 + kx^2 - 24x + 5$ has a stationary point where $x = 4$. Work out the coordinates of the other stationary point.

$$\frac{dy}{dx} = 6x^2 + 2kx - 24$$

$$\text{At } x = 4, \frac{dy}{dx} = 0: 6(4)^2 + 2k(4) - 24 = 0, \text{ so } 96 + 8k - 24 = 0, 8k = -72 \text{ and } k = -9$$

$$\text{So } \frac{dy}{dx} = 6x^2 - 18x - 24 = 6(x^2 - 3x - 4) = 6(x - 4)(x + 1)$$

$$\frac{dy}{dx} = 0 \text{ when } x = 4 \text{ (the point we were given) or } x = -1$$

$$x = -1: y = 2(-1)^3 - 9(-1)^2 - 24(-1) + 5 = -2 - 9 + 24 + 5 = 18$$

The other stationary point is $(-1, 18)$.

Answer: $(-1, 18)$

Tip: Use the stationary point you are given to find k first: $\frac{dy}{dx} = 0$ at $x = 4$. Then $x = 4$ must be one root of $\frac{dy}{dx} = 0$, which helps you factorise. Find the y -coordinate from the equation of the curve, not from $\frac{dy}{dx}$.

Marks: 1 mark for $\frac{dy}{dx} = 6x^2 + 2kx - 24$. 1 mark for $k = -9$. 1 mark for $x = -1$ from solving $\frac{dy}{dx} = 0$. 1 mark for $(-1, 18)$.

21 (6 marks)

$f(x) = 3x - 4$, $g(x) = 2x^2 + 5x - 1$. Solve $fg(x) = f^{-1}(x)$, giving your answers to 3 significant figures.

Inverse: $y = 3x - 4 \Rightarrow x = \frac{y+4}{3}$, so $f^{-1}(x) = \frac{x+4}{3}$

Composite: $fg(x)$ means do g first, then f : $fg(x) = 3(2x^2 + 5x - 1) - 4 = 6x^2 + 15x - 7$

Equation: $6x^2 + 15x - 7 = \frac{x+4}{3}$

Multiply both sides by 3: $18x^2 + 45x - 21 = x + 4$, so $18x^2 + 44x - 25 = 0$

Quadratic formula: $x = \frac{-44 \pm \sqrt{44^2 - 4 \times 18 \times (-25)}}{2 \times 18} = \frac{-44 \pm \sqrt{3736}}{36}$

$x = 0.47563 \dots$ or $x = -2.92007 \dots$

$$x = 0.476 \text{ or } x = -2.92 \text{ (3 s.f.)}$$

Answer: $x = 0.476$ or $x = -2.92$

Tip: $fg(x)$ means g first, then f : put the whole of $g(x)$ into f , so $fg(x) = 3g(x) - 4$. Multiply every term by 3 to clear the fraction before you rearrange.

Marks: 1 mark for $f^{-1}(x) = \frac{x+4}{3}$. 1 mark for $3(2x^2 + 5x - 1) - 4$. 1 mark for forming the equation and clearing the fraction. 1 mark for $18x^2 + 44x - 25 = 0$. 1 mark for substituting correctly into the quadratic formula. 1 mark for both answers.

22 (3 marks)

Show that $\frac{7 - 7\cos^2 x + 2\sin^2 x}{3\sin x \cos x}$ can be written as $k \tan x$, where k is an integer.

$$7 - 7\cos^2 x = 7(1 - \cos^2 x) = 7\sin^2 x \quad (\text{using } \sin^2 x + \cos^2 x = 1)$$

$$\text{So the numerator is } 7\sin^2 x + 2\sin^2 x = 9\sin^2 x$$

$$\frac{9\sin^2 x}{3\sin x \cos x} = \frac{3\sin x}{\cos x} \quad (\text{cancel } 3\sin x)$$

$$= 3 \tan x \quad (\text{using } \tan x = \frac{\sin x}{\cos x}), \text{ so } k = 3$$

Answer: $\frac{9\sin^2 x}{3\sin x \cos x} = \frac{3\sin x}{\cos x} = 3 \tan x$, so $k = 3$

Tip: Look for $1 - \cos^2 x$ and replace it with $\sin^2 x$. Show every step; do not substitute a value for x , which does not prove the result for all x .

Marks: 1 mark for replacing $7 - 7\cos^2 x$ by $7\sin^2 x$. 1 mark for $\frac{9\sin^2 x}{3\sin x \cos x}$ simplified to $\frac{3\sin x}{\cos x}$. 1 mark for $3 \tan x$ with $k = 3$.