

Mock Set B – Paper 1

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

- (a) $f(x) = 11 - 3x$ with domain $-2 \leq x < 5$. Work out the range of f .

f is a straight line with a negative gradient, so the largest output comes from the smallest input.

$f(-2) = 11 + 6 = 17$, and $x = -2$ is included, so 17 is included.

$f(5) = 11 - 15 = -4$, but $x = 5$ is not included, so -4 is not included.

Answer: $-4 < f(x) \leq 17$

Tip: Because $-3x$ makes f decreasing, the inequality signs swap ends: the $\leq$ at $x = -2$ becomes the $\leq$ next to 17. Always write the range with $f(x)$ (or y), not x .

- (b) $g(x) = x^2 - 7$ with domain $-4 < x \leq 3$. Work out the range of g .

The domain includes $x = 0$, where x^2 is smallest.

Smallest value: $g(0) = 0 - 7 = -7$

Ends of the domain: $g(-4) = 16 - 7 = 9$ (not included) and $g(3) = 9 - 7 = 2$

The largest value is just below 9, and 9 itself is not reached because $x = -4$ is not in the domain.

Answer: $-7 \leq g(x) < 9$

Tip: Don't just substitute the two ends of the domain: that gives $2 < g(x) < 9$, which misses every value from -7 to 2 . Sketch $y = x^2 - 7$ and look for the lowest point.

Marks: 1 mark for finding -7 (the minimum at $x = 0$) or 9 . 1 mark for the complete range with the correct signs.

2 (3 marks)

$$\begin{pmatrix} 4 & -3 \\ 2 & a \end{pmatrix} \begin{pmatrix} b \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ -8 \end{pmatrix}$$

Work out the values of a and b .

Multiply each row of the matrix by the column:

Top row: $4 \times b + (-3) \times 5 = 9$, so $4b - 15 = 9$

$4b = 24$, so $b = 6$

Bottom row: $2 \times b + a \times 5 = -8$, so $2b + 5a = -8$

Substitute $b = 6$: $12 + 5a = -8$

$5a = -20$, so $a = -4$

Answer: $a = -4$, $b = 6$

Tip: Row times column: multiply across the row and down the column, then add. The top row has no a in it, so solve that equation first.

Marks: 1 mark for a correct equation from either row. 1 mark for $b = 6$. 1 mark for $a = -4$.

3 (5 marks)

A is the point $(6, -1)$

B is the point $(-3, 5)$

- (a) Work out the equation of the line AB .

Give your answer in the form $ax + by = c$ where a , b and c are integers.

$$\text{Gradient of } AB: \frac{5 - (-1)}{-3 - 6} = \frac{6}{-9} = -\frac{2}{3}$$

$$\text{Use } y - y_1 = m(x - x_1) \text{ with } A(6, -1): y + 1 = -\frac{2}{3}(x - 6)$$

$$\text{Multiply by 3: } 3y + 3 = -2(x - 6) = -2x + 12$$

$$\text{Rearrange: } 2x + 3y = 9$$

$$\text{Check with } B: 2(-3) + 3(5) = -6 + 15 = 9 \quad \checkmark$$

Answer: $2x + 3y = 9$

Tip: Change in y over change in x , keeping the points in the same order on the top and bottom. Multiply through by 3 to clear the fraction before rearranging.

Marks: 1 mark for the gradient $-\frac{2}{3}$. 1 mark for a correct equation in any form. 1 mark for $2x + 3y = 9$ (or any multiple with integers).

- (b) The line AB crosses the y -axis at C .

Show that $AC = 2\sqrt{13}$

At C , $x = 0$: $3y = 9$, so C is $(0, 3)$

$$AC^2 = (6 - 0)^2 + (-1 - 3)^2 = 36 + 16 = 52$$

$$AC = \sqrt{52} = \sqrt{4 \times 13} = \sqrt{4} \times \sqrt{13} = 2\sqrt{13}$$

Answer: $C = (0, 3)$, $AC = \sqrt{6^2 + 4^2} = \sqrt{52} = \sqrt{4} \times \sqrt{13} = 2\sqrt{13}$

Tip: In a 'show that' every step must be written down: show $\sqrt{52}$ and how it becomes $2\sqrt{13}$ (using $52 = 4 \times 13$).

Marks: 1 mark for using $C(0, 3)$ in Pythagoras ($6^2 + 4^2$ or 52). 1 mark for showing $\sqrt{52} = 2\sqrt{13}$.

4 (3 marks)

A linear sequence starts 4.6, 3.8, 3.0, 2.2. Work out the position of the first term in the sequence that is less than -30

Common difference: $3.8 - 4.6 = -0.8$

n th term: $-0.8n + 5.4$ (because $4.6 - (-0.8) = 5.4$)

Solve $5.4 - 0.8n < -30$

$5.4 + 30 < 0.8n$, so $0.8n > 35.4$

$$n > \frac{35.4}{0.8} = \frac{354}{8} = 44.25$$

The first whole number bigger than 44.25 is 45.

Check: 44th term = $5.4 - 35.2 = -29.8$ and 45th term = $5.4 - 36 = -30.6$ ✓

Answer: 45th term (position 45)

Tip: The question asks for the **position**, so the answer is 45, not the value -30.6 . After solving, check the terms either side: 44.25 rounds to 44, but the 44th term is -29.8 , which is not less than -30 .

Marks: 1 mark for $-0.8n$ in the n th term. 1 mark for a correct inequality or equation such as $5.4 - 0.8n < -30$. 1 mark for 45.

5 (3 marks)

Expand and simplify $(2x + 7)(3x - 1)(4 - x)$

First two brackets: $(2x + 7)(3x - 1) = 6x^2 - 2x + 21x - 7 = 6x^2 + 19x - 7$

Multiply by $(4 - x)$: $(6x^2 + 19x - 7)(4 - x)$

$$= 24x^2 + 76x - 28 - 6x^3 - 19x^2 + 7x$$

Collect like terms: $-6x^3 + 5x^2 + 83x - 28$

Answer: $-6x^3 + 5x^2 + 83x - 28$

Tip: Multiply two brackets and simplify before bringing in the third. Take care with $-x$:
 $-x \times (-7) = +7x$.

Marks: 1 mark for expanding two brackets with at least three of the four terms correct. 1 mark for a full expansion with the third bracket (at least four of six terms correct). 1 mark for the final answer.

6 (3 marks)

A curve has equation $y = x^3 - 2x^2 - 5x + 4$. The gradient of the curve is 2 at two points. Work out the x -coordinates of these two points.

The gradient is $\frac{dy}{dx} = 3x^2 - 4x - 5$

Set it equal to 2: $3x^2 - 4x - 5 = 2$

$$3x^2 - 4x - 7 = 0$$

Factorise: $(3x - 7)(x + 1) = 0$

$$x = \frac{7}{3} \text{ or } x = -1$$

Answer: $x = \frac{7}{3}$ and $x = -1$

Tip: The gradient is $\frac{dy}{dx}$, not y . Many students set $x^3 - 2x^2 - 5x + 4 = 2$, which finds where the curve's height is 2.

Marks: 1 mark for $\frac{dy}{dx} = 3x^2 - 4x - 5$. 1 mark for $3x^2 - 4x - 7 = 0$ factorised (or solved by the formula). 1 mark for both values.

7 (3 marks)

- (a) Circle the value of $\cos 210^\circ$

Options: $-\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$

$210^\circ = 180^\circ + 30^\circ$, so the related acute angle is 30° and $\cos 30^\circ = \frac{\sqrt{3}}{2}$

210° is between 180° and 270° , where the graph of $y = \cos x$ is below the x -axis, so $\cos 210^\circ$ is negative.

Answer: $-\frac{\sqrt{3}}{2}$

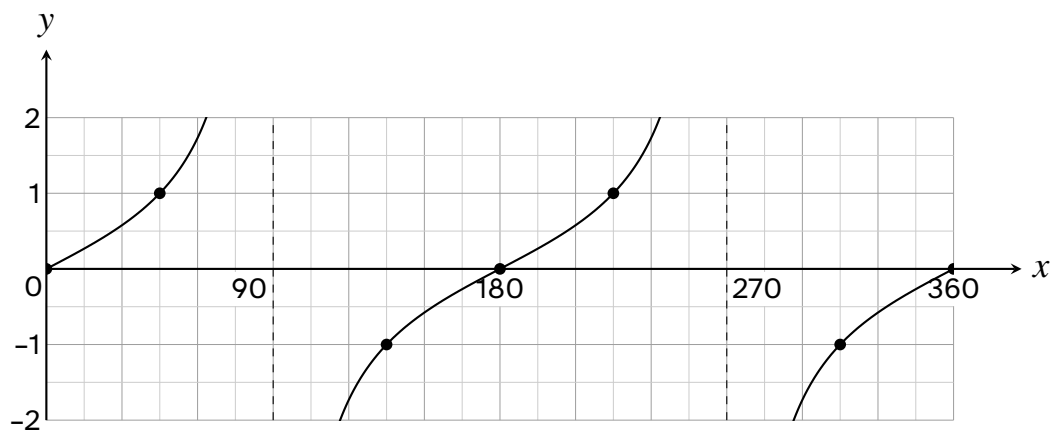
- (b) Sketch $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$

$\tan 0^\circ = 0$, $\tan 180^\circ = 0$ and $\tan 360^\circ = 0$, so the graph crosses the x -axis at 0, 180 and 360.

$\tan 45^\circ = 1$ and $\tan 135^\circ = -1$; the pattern repeats every 180° .

$\tan 90^\circ$ and $\tan 270^\circ$ do not exist: as x gets close to 90° or 270° the graph shoots off upwards (from the left) or downwards (from the right). These vertical lines are asymptotes.

The graph has three separate pieces, each going upwards from left to right.



Answer: Three increasing branches: through $(0, 0)$ up towards $x = 90$; from below near $x = 90$ through $(180, 0)$ up towards $x = 270$; from below near $x = 270$ to $(360, 0)$. Asymptotes at $x = 90$ and $x = 270$

Tip: $y = \tan x$ is not a wave like $\sin x$ and $\cos x$. It has no maximum or minimum value and it is not drawn across $x = 90$ or $x = 270$. Draw dashed vertical lines there first to guide your sketch.

Marks: 1 mark for a curve through $(0, 0)$, $(180, 0)$ and $(360, 0)$, going upwards from left to right. 1 mark for the correct shape near $x = 90$ and $x = 270$ (the curve gets closer and closer to these lines without touching them).

8 (4 marks)

Rationalise the denominator and simplify fully $\frac{\sqrt{5} + \sqrt{7}}{\sqrt{5} - \sqrt{7}}$. Give your answer in the form $m + n\sqrt{35}$.

Multiply the top and bottom by $\sqrt{5} + \sqrt{7}$: $\frac{(\sqrt{5} + \sqrt{7})(\sqrt{5} + \sqrt{7})}{(\sqrt{5} - \sqrt{7})(\sqrt{5} + \sqrt{7})}$

Top: $5 + \sqrt{35} + \sqrt{35} + 7 = 12 + 2\sqrt{35}$

Bottom: $5 - 7 = -2$ (the $\sqrt{35}$ terms cancel)

$$\frac{12 + 2\sqrt{35}}{-2} = -6 - \sqrt{35}$$

Answer: $-6 - \sqrt{35}$ ($m = -6, n = -1$)

Tip: The bottom is $(\sqrt{5})^2 - (\sqrt{7})^2 = 5 - 7$, which is negative. Dividing by -2 changes the sign of **both** terms on the top.

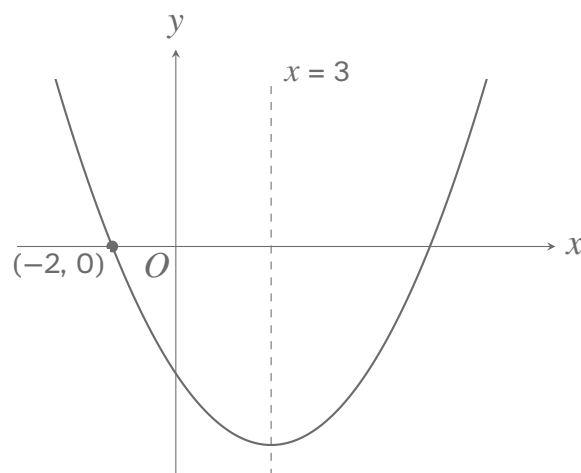
Marks: 1 mark for multiplying top and bottom by $\sqrt{5} + \sqrt{7}$. 1 mark for the denominator -2 . 1 mark for the numerator $12 + 2\sqrt{35}$. 1 mark for $-6 - \sqrt{35}$.

9 (4 marks)

Here is a sketch of $y = x^2 + bx + c$

The curve crosses the x -axis at $(-2, 0)$ and at one other point.

The line $x = 3$ is the line of symmetry of the curve.



(a) Work out the values of b and c .

The roots are the same distance either side of the line of symmetry $x = 3$.

-2 is 5 to the left of 3, so the other root is $3 + 5 = 8$.

So $y = (x + 2)(x - 8)$

$$= x^2 - 8x + 2x - 16 = x^2 - 6x - 16$$

Answer: $b = -6$, $c = -16$

Tip: Use the symmetry first: the line of symmetry is halfway between the two x -intercepts. Another route: the line of symmetry of $y = x^2 + bx + c$ is $x = -\frac{b}{2}$, so $-\frac{b}{2} = 3$.

Marks: 1 mark for the other root 8. 1 mark for $(x + 2)(x - 8)$ expanded (or one of b or c correct). 1 mark for both.

(b) Work out the coordinates of the turning point of the curve.

The turning point is on the line of symmetry, so $x = 3$.

$$y = 3^2 - 6(3) - 16 = 9 - 18 - 16 = -25$$

Answer: $(3, -25)$

10 (3 marks)

Solve $4x^3 + 4x^2 - 15x = 0$

You must show your working.

Every term has a factor x : $x(4x^2 + 4x - 15) = 0$

Factorise the quadratic: $4 \times (-15) = -60$, and 10 and -6 multiply to -60 and add to 4.

$$4x^2 + 10x - 6x - 15 = 2x(2x + 5) - 3(2x + 5) = (2x - 3)(2x + 5)$$

So $x(2x - 3)(2x + 5) = 0$

$$x = 0, x = \frac{3}{2} \text{ or } x = -\frac{5}{2}$$

Answer: $x = 0$, $x = \frac{3}{2}$, $x = -\frac{5}{2}$

Tip: Never divide both sides by x : that loses the solution $x = 0$. Take out x as a factor instead.

Marks: 1 mark for taking out the factor x (or stating $x = 0$ with the quadratic). 1 mark for $(2x - 3)(2x + 5)$. 1 mark for all three solutions.

11 (3 marks)

3-digit ticket numbers use three different digits from 0 to 9, and cannot start with 0.
How many are greater than 300 and a multiple of 5?

A multiple of 5 ends in 0 or 5. Greater than 300 means the first digit is 3, 4, 5, 6, 7, 8 or 9.

Last digit 0: first digit 7 choices (3 to 9), middle digit 8 choices (not the first or last digit).

$$7 \times 8 = 56$$

Last digit 5: first digit 6 choices (3, 4, 6, 7, 8, 9, because 5 is already used), middle digit 8 choices.

$$6 \times 8 = 48$$

$$\text{Total: } 56 + 48 = 104$$

Answer: 104

Tip: Split into cases by the last digit. When the number ends in 5, the first digit cannot also be 5, so there are only 6 choices for it.

Marks: 1 mark for one case correct (56 or 48). 1 mark for both cases added. 1 mark for 104.

12 (3 marks)

Expand and simplify fully $(2 - 3x)^4$

Row 4 of Pascal's triangle: 1 4 6 4 1

$$(2 - 3x)^4 = 2^4 + 4(2^3)(-3x) + 6(2^2)(-3x)^2 + 4(2)(-3x)^3 + (-3x)^4$$

$$= 16 + 4 \times 8 \times (-3x) + 6 \times 4 \times 9x^2 + 4 \times 2 \times (-27x^3) + 81x^4$$

$$= 16 - 96x + 216x^2 - 216x^3 + 81x^4$$

Answer: $16 - 96x + 216x^2 - 216x^3 + 81x^4$

Tip: Keep the minus sign inside the bracket: $(-3x)^2 = +9x^2$ and $(-3x)^3 = -27x^3$. Forgetting the brackets gives $3x^2$ instead of $9x^2$.

Marks: 1 mark for using 1, 4, 6, 4, 1. 1 mark for all five terms written correctly before simplifying. 1 mark for the final answer.

13 (4 marks)

(a) Write $\frac{x^{\frac{2}{3}} \times x^{\frac{1}{2}}}{x^{-\frac{1}{6}}}$ as a single power of x .

Options: x^{-2} , x , $x^{\frac{4}{3}}$, $x^{-\frac{1}{18}}$

Multiplying: add the powers. $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

Dividing: subtract the powers. $\frac{7}{6} - \left(-\frac{1}{6}\right) = \frac{7}{6} + \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$

Answer: $x^{\frac{4}{3}}$

(b) Solve $2x^{-\frac{3}{2}} = 250$

Divide both sides by 2: $x^{-\frac{3}{2}} = 125$

The negative power means a reciprocal: $x^{\frac{3}{2}} = \frac{1}{125}$

Cube root both sides: $x^{\frac{1}{2}} = \frac{1}{5}$

Square both sides: $x = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

Answer: $x = \frac{1}{25}$

Tip: Divide by 2 first: $2x^{-\frac{3}{2}}$ means $2 \times x^{-\frac{3}{2}}$, not $(2x)^{-\frac{3}{2}}$. Then undo the power one piece at a time: the minus sign (reciprocal), the 3 on top (cube root) and the 2 underneath (square).

Marks: 1 mark for $x^{-\frac{3}{2}} = 125$ (or $x^{\frac{3}{2}} = \frac{1}{125}$). 1 mark for $\sqrt{x} = \frac{1}{5}$ (or $x^{-\frac{1}{2}} = 5$). 1 mark for $\frac{1}{25}$.

14 (3 marks)

Write $\frac{3}{x-2} - \frac{5}{x+4} + 1$ as a single fraction in its simplest form.

Common denominator: $(x-2)(x+4)$

Write 1 as $\frac{(x-2)(x+4)}{(x-2)(x+4)}$

Numerator: $3(x+4) - 5(x-2) + (x-2)(x+4)$

$$= 3x + 12 - 5x + 10 + x^2 + 2x - 8$$

$$= x^2 + 14$$

So the fraction is $\frac{x^2 + 14}{(x-2)(x+4)}$

Answer: $\frac{x^2 + 14}{(x-2)(x+4)}$ (or $\frac{x^2 + 14}{x^2 + 2x - 8}$)

Tip: The +1 must go over the common denominator too. Use a bracket when subtracting:
 $-5(x-2) = -5x + 10$, not $-5x - 10$.

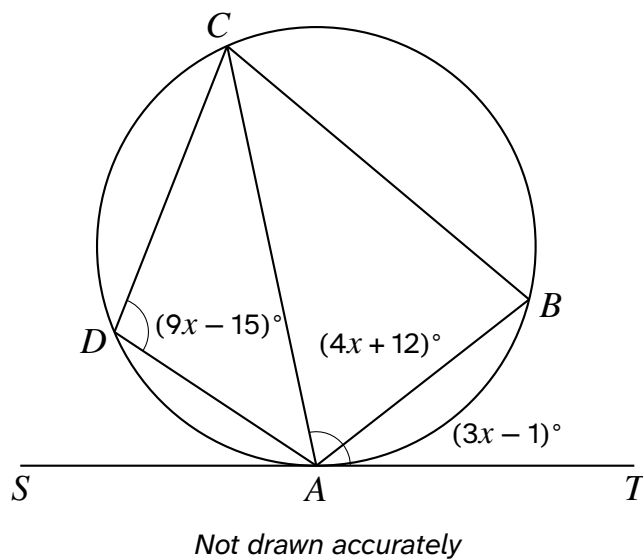
Marks: 1 mark for the common denominator with two of the three numerators correct. 1 mark for all three numerator terms correct. 1 mark for the simplified fraction.

15 (4 marks)

A, B, C and D are points on a circle.

SAT is a tangent to the circle at A .

Angle $TAB = (3x - 1)^\circ$ angle $BAC = (4x + 12)^\circ$ angle $ADC = (9x - 15)^\circ$



Work out the size of angle ABC .

You must show your working.

$$\text{Angle } TAC = \text{angle } TAB + \text{angle } BAC = (3x - 1) + (4x + 12) = 7x + 11$$

Alternate segment theorem: the angle between the tangent AT and the chord AC equals the angle in the alternate segment, angle ADC .

$$7x + 11 = 9x - 15$$

$$26 = 2x, \text{ so } x = 13$$

$$\text{Angle } ADC = 9 \times 13 - 15 = 102^\circ$$

Opposite angles of a cyclic quadrilateral add up to 180° : angle $ABC = 180 - 102 = 78^\circ$

(Check: angle $ACB = \text{angle } TAB = 38^\circ$ and angle $BAC = 64^\circ$, and $38 + 64 + 78 = 180$ ✓)

Answer: 78°

Tip: Look for the angle between the tangent and a chord. Here angle TAC (the two marked angles at A together) sits against chord AC , so it equals angle ADC . Another route: angle $ACB = \text{angle } TAB$, then use the angles in triangle ABC and opposite angles of the cyclic quadrilateral.

Marks: 1 mark for using the alternate segment theorem correctly. 1 mark for a correct equation in x . 1 mark for $x = 13$. 1 mark for 78° .

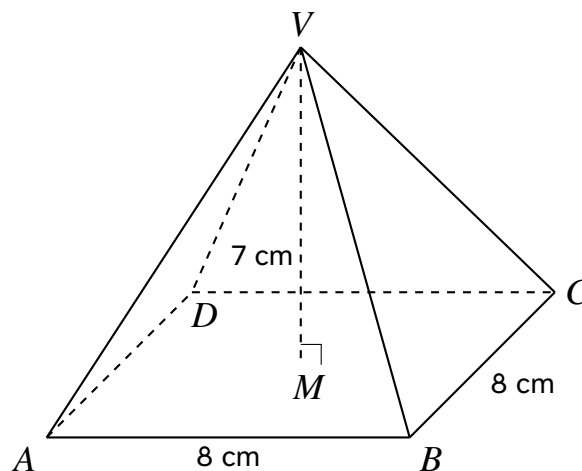
16 (3 marks)

$VABCD$ is a pyramid.

The base $ABCD$ is a square of side 8 cm.

M is the centre of the base.

V is vertically above M and $VM = 7$ cm



Not drawn accurately

Work out the length of the sloping edge VA .

M is the centre of the square, so it is the midpoint of the diagonal AC .

In the base: $AC^2 = AB^2 + BC^2 = 8^2 + 8^2 = 128$, so $AC = \sqrt{128} = 8\sqrt{2}$

$AM = \frac{1}{2}AC = 4\sqrt{2}$, so $AM^2 = 32$

Triangle VMA has a right angle at M : $VA^2 = AM^2 + VM^2 = 32 + 7^2 = 32 + 49 = 81$

$$VA = \sqrt{81} = 9$$

Answer: 9 cm

Tip: M is halfway along the diagonal AC , not halfway along a side. Work with $AM^2 = 32$ rather than a decimal: the surds disappear when you square.

Marks: 1 mark for $AM^2 = 32$ (or $AC = 8\sqrt{2}$, or $AC^2 = 128$). 1 mark for $32 + 7^2$. 1 mark for 9.

17 (4 marks)

The curve $y = x^3 - 3x^2 - 2x + 5$ passes through P , where $x = -1$. Show that the equation of the normal to the curve at P is $x + 7y - 20 = 0$

y -coordinate of P : $(-1)^3 - 3(-1)^2 - 2(-1) + 5 = -1 - 3 + 2 + 5 = 3$, so P is $(-1, 3)$

$$\frac{dy}{dx} = 3x^2 - 6x - 2$$

At $x = -1$: $3(1) - 6(-1) - 2 = 3 + 6 - 2 = 7$, so the gradient of the tangent is 7

The normal is perpendicular to the tangent, so its gradient is $-\frac{1}{7}$

$$y - 3 = -\frac{1}{7}(x + 1)$$

Multiply by 7: $7y - 21 = -x - 1$

$x + 7y - 20 = 0$ as required

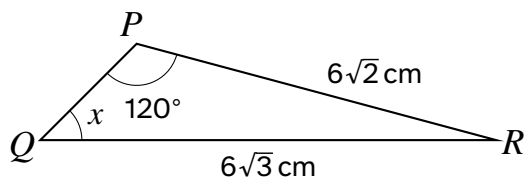
Answer: $P(-1, 3)$, gradient of tangent 7, gradient of normal $-\frac{1}{7}$: $y - 3 = -\frac{1}{7}(x + 1)$, so $7y - 21 = -x - 1$ and $x + 7y - 20 = 0$

Tip: The normal uses the negative reciprocal of the tangent gradient. Put $x = -1$ in brackets when substituting: $-3(-1)^2 = -3$ and $-2(-1) = +2$.

Marks: 1 mark for $y = 3$ at P . 1 mark for $\frac{dy}{dx} = 3x^2 - 6x - 2$. 1 mark for the normal gradient $-\frac{1}{7}$ used in a line through $(-1, 3)$. 1 mark for rearranging fully to $x + 7y - 20 = 0$.

18 (3 marks)

PQR is a triangle.



Not drawn accurately

Work out the value of x .

You must show your working.

$QR = 6\sqrt{3}$ is opposite the 120° angle at P , and $PR = 6\sqrt{2}$ is opposite the angle x at Q .

Sine rule: $\frac{\sin x}{6\sqrt{2}} = \frac{\sin 120^\circ}{6\sqrt{3}}$

The sine graph is symmetrical about 90° , so $\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\sin x = \frac{6\sqrt{2} \times \frac{\sqrt{3}}{2}}{6\sqrt{3}} = \frac{\sqrt{2}}{2}$$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$, so $x = 45$ (x cannot be 135 because the triangle already has a 120° angle).

Answer: $x = 45$

Tip: Match each side with the angle opposite it. $\sin 120^\circ = \sin 60^\circ$ because of the symmetry of the sine graph. $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are the same number, $\sin 45^\circ$.

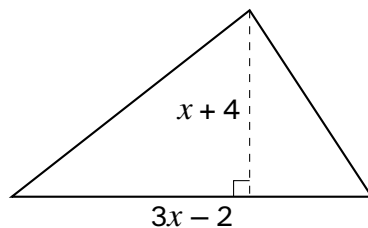
Marks: 1 mark for a correct sine rule. 1 mark for $\sin 120^\circ = \frac{\sqrt{3}}{2}$. 1 mark for 45 (from $\sin x = \frac{\sqrt{2}}{2}$).

19 (5 marks)

The diagram shows a triangle.

The base of the triangle is $(3x - 2)$ cm.

The perpendicular height of the triangle is $(x + 4)$ cm.



Not drawn accurately

A triangle has base $(3x - 2)$ cm and perpendicular height $(x + 4)$ cm. Its area is less than 12 cm^2 . Work out the set of values of x for which this is possible.

$$\text{Area} = \frac{1}{2}(3x - 2)(x + 4), \text{ so } \frac{1}{2}(3x - 2)(x + 4) < 12$$

$$(3x - 2)(x + 4) < 24$$

$$3x^2 + 12x - 2x - 8 < 24$$

$$3x^2 + 10x - 32 < 0$$

$$\text{Factorise: } (3x + 16)(x - 2) < 0$$

$$\text{Critical values: } x = -\frac{16}{3} \text{ and } x = 2$$

The graph of $y = 3x^2 + 10x - 32$ is U-shaped, so it is below 0 between the roots:

$$-\frac{16}{3} < x < 2$$

Every length must be positive: $3x - 2 > 0$, so $x > \frac{2}{3}$ (and $x + 4 > 0$ gives $x > -4$, which is already true)

Combine: $\frac{2}{3} < x < 2$

Answer: $\frac{2}{3} < x < 2$

Tip: The area condition only gives the upper limit. The lower limit comes from the lengths: $3x - 2$ must be positive, so $x > \frac{2}{3}$. Many students lose this mark every year.

Marks: 1 mark for $\frac{1}{2}(3x - 2)(x + 4) < 12$. 1 mark for $3x^2 + 10x - 32 < 0$. 1 mark for factorising (or the critical values $-\frac{16}{3}$ and 2). 1 mark for $x > \frac{2}{3}$ from $3x - 2 > 0$. 1 mark for $\frac{2}{3} < x < 2$.

20 (5 marks)

A curve has equation

$$y = x^3 + 12x^2 + 63x - 4$$

(a) Show that y is an increasing function for all values of x .

$$\frac{dy}{dx} = 3x^2 + 24x + 63$$

Take out the common factor 3: $\frac{dy}{dx} = 3(x^2 + 8x + 21)$

Complete the square: $x^2 + 8x + 21 = (x + 4)^2 - 16 + 21 = (x + 4)^2 + 5$

$$\text{So } \frac{dy}{dx} = 3[(x + 4)^2 + 5] = 3(x + 4)^2 + 15$$

$(x + 4)^2 \geq 0$ for every value of x , so $\frac{dy}{dx} \geq 15 > 0$

The gradient is always positive, so y is an increasing function for all values of x .

Answer: $\frac{dy}{dx} = 3x^2 + 24x + 63 = 3(x + 4)^2 + 15$; $(x + 4)^2 \geq 0$, so $\frac{dy}{dx} \geq 15 > 0$ for all x , so y is increasing

Tip: Increasing for all x means $\frac{dy}{dx} > 0$ for all x . Take out the factor 3 first, then complete the square, and finish with a sentence about the gradient.

Marks: 1 mark for $\frac{dy}{dx} = 3x^2 + 24x + 63$. 1 mark for starting to complete the square, eg $3(x + 4)^2$. 1 mark for $3(x + 4)^2 + 15$ (or $3[(x + 4)^2 + 5]$). 1 mark for explaining why $\frac{dy}{dx}$ is always positive and concluding.

(b) Work out the least value of the gradient of the curve.

From part (a): $\frac{dy}{dx} = 3(x + 4)^2 + 15$

$(x + 4)^2$ is smallest (zero) when $x = -4$, so the least gradient is $3 \times 0 + 15 = 15$

Answer: 15

Tip: Use the completed-square form from part (a): the smallest value of $3(x + 4)^2 + 15$ is when the bracket is zero.

21 (6 marks)

A circle has centre $(3, 0)$ and radius $\sqrt{17}$. The line $x + 3y = -8$ meets the circle at two points. Work out the coordinates of these points.

Equation of the circle: $(x - 3)^2 + y^2 = 17$

From the line: $x = -8 - 3y$

Substitute: $(-8 - 3y - 3)^2 + y^2 = 17$, so $(-11 - 3y)^2 + y^2 = 17$

Expand: $121 + 66y + 9y^2 + y^2 = 17$

$10y^2 + 66y + 104 = 0$, so $5y^2 + 33y + 52 = 0$

Factorise: $(5y + 13)(y + 4) = 0$

$y = -\frac{13}{5}$ or $y = -4$

When $y = -4$: $x = -8 + 12 = 4$

When $y = -\frac{13}{5}$: $x = -8 + \frac{39}{5} = -\frac{1}{5}$

Answer: $(4, -4)$ and $\left(-\frac{1}{5}, -\frac{13}{5}\right)$

Tip: Write the circle's equation first: the radius is $\sqrt{17}$, so the right-hand side is 17. Making x the subject of the line avoids fractions. Expand $(-11 - 3y)^2$ as $(-11 - 3y)(-11 - 3y)$, not $121 + 9y^2$.

Marks: 1 mark for substituting the line into $(x - 3)^2 + y^2 = 17$. 1 mark for expanding the squared bracket correctly. 1 mark for a correct quadratic equal to 0. 1 mark for factorising (or the formula). 1 mark for both y values (or both x values). 1 mark for both points paired correctly.

22 (3 marks)

Solve $4 \cos^2 x = 3$ for $0^\circ \leq x \leq 360^\circ$

$$\cos^2 x = \frac{3}{4}$$

Square root, taking both signs: $\cos x = \frac{\sqrt{3}}{2}$ or $\cos x = -\frac{\sqrt{3}}{2}$

$$\cos x = \frac{\sqrt{3}}{2}: x = 30^\circ \text{ or } x = 360 - 30 = 330^\circ$$

$$\cos x = -\frac{\sqrt{3}}{2}: x = 180 - 30 = 150^\circ \text{ or } x = 180 + 30 = 210^\circ$$

Answer: $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

Tip: When you square root, remember the negative root: $\cos x = -\frac{\sqrt{3}}{2}$ gives two more angles. Use the graph of $y = \cos x$ (symmetrical about 180°) to find them.

Marks: 1 mark for $\cos x = \pm \frac{\sqrt{3}}{2}$. 1 mark for two correct angles. 1 mark for all four angles and no others.
