

Mock Set A – Paper 2

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Solve $\frac{w+4}{2w-1} = \frac{w-3}{2w+5}$

Multiply both sides by both denominators (cross-multiply) to clear the fractions:

$$(w+4)(2w+5) = (w-3)(2w-1)$$

Expand each side: $2w^2 + 5w + 8w + 20 = 2w^2 - w - 6w + 3$

$$2w^2 + 13w + 20 = 2w^2 - 7w + 3$$

The $2w^2$ terms cancel: $13w + 20 = -7w + 3$

$$20w = -17$$

$$w = -\frac{17}{20} = -0.85$$

Answer: $w = -\frac{17}{20}$ (or -0.85)

Tip: Cross-multiply as **one** equation and keep the brackets. Setting the tops equal and the bottoms equal gives two different equations and is wrong.

Marks: 1 mark for cross-multiplying to $(w+4)(2w+5) = (w-3)(2w-1)$. 1 mark for expanding both sides correctly (or for $20w = -17$). 1 mark for -0.85 .

2 (5 marks)

Line L has equation $5x + 3y = 8$

P is the point $(-4, 6)$

- (a) Work out the equation of the line that is perpendicular to L and passes through P .
Give your answer in the form $y = mx + c$

Rearrange L : $3y = -5x + 8$, so $y = -\frac{5}{3}x + \frac{8}{3}$

Gradient of $L = -\frac{5}{3}$

Perpendicular gradient: flip the fraction and change the sign: $\frac{3}{5}$

Through $P(-4, 6)$: $y - 6 = \frac{3}{5}(x - (-4))$

$$y - 6 = 0.6x + 2.4$$

$$y = 0.6x + 8.4$$

Answer: $y = 0.6x + 8.4$ (or $y = \frac{3}{5}x + \frac{42}{5}$)

Tip: Rearrange to $y = \dots$ before reading the gradient: the gradient of $5x + 3y = 8$ is $-\frac{5}{3}$, not 5 or -5 . For the perpendicular, flip **and** change the sign.

Marks: 1 mark for the gradient of L , $-\frac{5}{3}$. 1 mark for using the perpendicular gradient $\frac{3}{5}$ with the point $(-4, 6)$. 1 mark for $y = 0.6x + 8.4$.

- (b) The line in part (a) crosses the x -axis at Q .
Work out the length PQ .
Give your answer to 3 significant figures.

On the x -axis $y = 0$: $0 = 0.6x + 8.4$, so $x = -14$

Q is $(-14, 0)$ and P is $(-4, 6)$.

Change in x : 10. Change in y : 6.

$$PQ = \sqrt{10^2 + 6^2} = \sqrt{136} = 11.66 \dots$$

$$PQ = 11.7 \text{ (3 s.f.)}$$

Answer: 11.7

Tip: Q is on the x -axis, so put $y = 0$ into your equation. Then use Pythagoras on the changes in x and y .

Marks: 1 mark for $Q(-14, 0)$ or for $\sqrt{10^2 + 6^2}$. 1 mark for 11.7.

3 (3 marks)

Simplify fully $\frac{6x^2 - 11x - 10}{2x^2 + 7x - 30}$

Factorise the numerator: $6 \times (-10) = -60$ and $-15 + 4 = -11$

$$6x^2 - 15x + 4x - 10 = 3x(2x - 5) + 2(2x - 5) = (3x + 2)(2x - 5)$$

Factorise the denominator: $2 \times (-30) = -60$ and $12 - 5 = 7$

$$2x^2 + 12x - 5x - 30 = 2x(x + 6) - 5(x + 6) = (2x - 5)(x + 6)$$

$$\frac{(3x + 2)(2x - 5)}{(2x - 5)(x + 6)}$$

Cancel the common factor $(2x - 5)$: $\frac{3x + 2}{x + 6}$

Answer: $\frac{3x + 2}{x + 6}$

Tip: Factorise first, then cancel whole brackets only. Cancelling single terms (for example the x in $\frac{3x+2}{x+6}$) is wrong, so stop at $\frac{3x+2}{x+6}$.

Marks: 1 mark for factorising the numerator. 1 mark for factorising the denominator. 1 mark for $\frac{3x+2}{x+6}$.

4 (4 marks)

Here are the first four terms of a linear sequence.

$$3p - q \quad 3p + 3q \quad 3p + 7q \quad 3p + 11q$$

- (a) Work out an expression for the n th term of the sequence.

Each term is $4q$ more than the one before, so the common difference is $4q$.

The n th term is first term $+(n - 1) \times$ difference

$$3p - q + (n - 1) \times 4q = 3p - q + 4nq - 4q$$

$$= 3p + 4nq - 5q \quad \text{or} \quad 3p + (4n - 5)q$$

Answer: $3p + (4n - 5)q$ (or $3p + 4nq - 5q$)

Tip: Check your expression by putting in $n = 1$ and $n = 2$: you should get $3p - q$ and $3p + 3q$.

Marks: 1 mark for $4nq$ (the common difference $4q$ used). 1 mark for $3p + 4nq - 5q$ or any equivalent.

- (b) The 5th term of the sequence is 34.5
The 12th term of the sequence is 104.5
Work out the values of p and q .

$$\text{5th term: } 3p + (4 \times 5 - 5)q = 3p + 15q = 34.5$$

$$\text{12th term: } 3p + (4 \times 12 - 5)q = 3p + 43q = 104.5$$

$$\text{Subtract: } 28q = 70, \text{ so } q = 2.5$$

$$3p + 15 \times 2.5 = 34.5 \Rightarrow 3p = -3, \text{ so } p = -1$$

Answer: $p = -1, q = 2.5$

Tip: From the 5th term to the 12th term is 7 steps of $4q$, so $28q = 104.5 - 34.5$. This is a quick way to find q .

Marks: 1 mark for two correct equations (or for $28q = 70$). 1 mark for both values.

5 (3 marks)

The equation of a curve is $y = x^5 + kx^3$ where k is a constant. The gradient of the curve at the point where $x = -2$ is 50. Work out the value of k .

Differentiate: $\frac{dy}{dx} = 5x^4 + 3kx^2$

Substitute $x = -2$ into $\frac{dy}{dx}$ (not into y): $5 \times (-2)^4 + 3k \times (-2)^2 = 50$

$(-2)^4 = 16$ and $(-2)^2 = 4$: $80 + 12k = 50$

$12k = -30$

$k = -2.5$

Answer: $k = -2.5$

Tip: The gradient is $\frac{dy}{dx}$, so substitute into the derivative, not into the equation of the curve. Use brackets for $(-2)^4$ and $(-2)^2$: both are positive.

Marks: 1 mark for $5x^4$ or $3kx^2$. 1 mark for $80 + 12k = 50$. 1 mark for -2.5 .

6 (3 marks)

A 4-digit locker code uses the digits 0 to 9 with no digit used more than once. As a 4-digit number the code is even and greater than 4000. How many different codes are possible?

First digit: 4, 5, 6, 7, 8 or 9. Last digit: 0, 2, 4, 6 or 8.

If the first digit is even, it cannot also be the last digit, so split into two cases.

First digit 4, 6 or 8 (3 choices): last digit is even but not the first digit, so 4 choices.

Middle two digits: 8 choices, then 7 choices. $3 \times 4 \times 8 \times 7 = 672$

First digit 5, 7 or 9 (3 choices): last digit 0, 2, 4, 6 or 8, so 5 choices.

Middle two digits: 8, then 7. $3 \times 5 \times 8 \times 7 = 840$

Total: $672 + 840 = 1512$

Answer: 1512

Tip: $6 \times 5 \times 8 \times 7 = 1680$ is wrong: it counts codes such as 4824 where the first and last digits are the same. When two restricted places can share a digit, split into cases and **add**.

Marks: 3 marks for 1512. Otherwise 1 mark for one case correct (672 or 840), or 2 marks for both cases.

7 (4 marks)

- (a) Circle the matrix that represents a reflection in the line $y = -x$

Options: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Reflecting in $y = -x$ sends $(1, 0)$ to $(0, -1)$ and $(0, 1)$ to $(-1, 0)$.

These images are the columns of the matrix: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Answer: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Tip: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the reflection in $y = x$. Check the images of $(1, 0)$ and $(0, 1)$ to tell them apart.

- (b) A shape is reflected in the line $y = -x$ and the image is then reflected in the x -axis. Use matrix multiplication to work out the single equivalent transformation, and describe it fully.

Reflection in $y = -x$: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Reflection in the x -axis: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The reflection in the x -axis happens second, so its matrix goes on the **left**:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ sends $(1, 0)$ to $(0, 1)$ and $(0, 1)$ to $(-1, 0)$: a quarter turn anticlockwise about O .

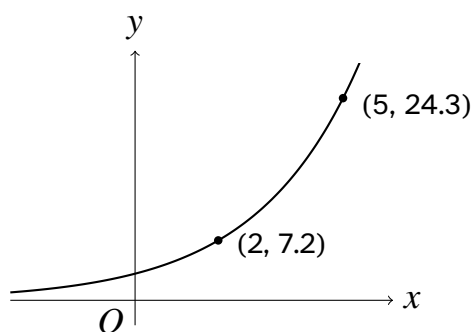
Answer: Rotation, 90° anticlockwise, about the origin (matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$)

Tip: The transformation done second goes on the left. Multiplying the other way round gives $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, a rotation 90° **clockwise**. Always give the centre: “about the origin”.

Marks: 1 mark for multiplying the two correct matrices in the right order. 1 mark for $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. 1 mark for a full description: rotation, 90° anticlockwise, about the origin.

8 (5 marks)

Here is a sketch of the curve $y = a \times b^x$ where a and b are positive constants. The points $(2, 7.2)$ and $(5, 24.3)$ lie on the curve.



Not drawn accurately

- (a) Work out the values of a and b .

Substitute each point into $y = a \times b^x$:

$$(2, 7.2): ab^2 = 7.2 \quad (5, 24.3): ab^5 = 24.3$$

Divide the second equation by the first: $b^3 = \frac{24.3}{7.2} = 3.375$

$$b = \sqrt[3]{3.375} = 1.5$$

$$a \times 1.5^2 = 7.2, \text{ so } a = \frac{7.2}{2.25} = 3.2$$

Answer: $a = 3.2$, $b = 1.5$

Tip: Dividing one equation by the other cancels a . Remember $a \times b^x$ means a times (b^x) , not $(ab)^x$.

Marks: 1 mark for both equations $ab^2 = 7.2$ and $ab^5 = 24.3$. 1 mark for $b^3 = 3.375$ (or $b = 1.5$). 1 mark for both values.

- (b) The function f is given by $f(x) = a \times b^x$ with domain $-2 \leq x \leq 4$. Work out the range of f .

$b > 1$, so f is increasing: the smallest value is at $x = -2$ and the largest at $x = 4$.

$$f(-2) = 3.2 \times 1.5^{-2} = \frac{3.2}{2.25} = 1.422 \dots$$

$$f(4) = 3.2 \times 1.5^4 = 3.2 \times 5.0625 = 16.2$$

Range: $1.42 \leq f(x) \leq 16.2$

Answer: $1.42 \leq f(x) \leq 16.2$ (lower end $\frac{64}{45}$)

Tip: The range is the set of output values, so write it with $f(x)$ (or y), not x . $1.5^{-2} = \frac{1}{2.25}$, so it divides.

Marks: 1 mark for one end correct. 1 mark for the full range written with $f(x)$.

9 (3 marks)

n is a positive integer. $2n + 1$ and $2n + 3$ are consecutive odd numbers. Prove that the difference between their squares is always 4 times the even number between them.

$$(2n + 3)^2 = 4n^2 + 12n + 9$$

$$(2n + 1)^2 = 4n^2 + 4n + 1$$

Difference (put the second square in a bracket):

$$(4n^2 + 12n + 9) - (4n^2 + 4n + 1) = 8n + 8$$

The even number between $2n + 1$ and $2n + 3$ is $2n + 2$.

$$4 \times (2n + 2) = 8n + 8$$

So the difference between the squares is always 4 times the even number between them.

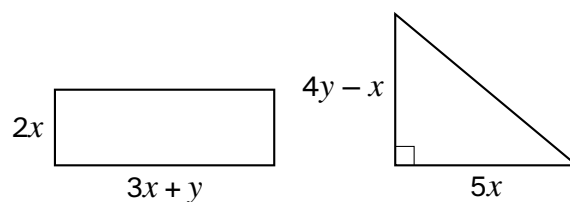
Answer: $(2n + 3)^2 - (2n + 1)^2 = 8n + 8 = 4(2n + 2)$, and $2n + 2$ is the even number between them

Tip: Put the second expansion in a bracket before subtracting, so that every sign changes. Finish with a sentence that says what you have proved.

Marks: 1 mark for expanding one of the squares correctly. 1 mark for the difference $8n + 8$. 1 mark for $8n + 8 = 4(2n + 2)$ with $2n + 2$ identified as the even number between them.

10 (3 marks)

The diagram shows a rectangle and a right-angled triangle. All lengths are in centimetres.



Not drawn accurately

Rectangle: sides $2x$ and $3x + y$. Right-angled triangle: shorter sides $5x$ and $4y - x$. Area of rectangle : area of triangle = 4:5. Work out $x:y$ in its simplest form.

Area of rectangle: $2x(3x + y) = 6x^2 + 2xy$

Area of triangle: $\frac{1}{2} \times 5x \times (4y - x) = 10xy - 2.5x^2$

$\frac{\text{rectangle}}{\text{triangle}} = \frac{4}{5}$, so $5(6x^2 + 2xy) = 4(10xy - 2.5x^2)$

$$30x^2 + 10xy = 40xy - 10x^2$$

$$40x^2 = 30xy$$

Divide by $10x$ (allowed, as $x > 0$): $4x = 3y$

$$\frac{x}{y} = \frac{3}{4}, \text{ so } x:y = 3:4$$

Answer: 3:4

Tip: Cross-multiply the ratio the right way round: if rectangle:triangle = 4:5 then $5 \times \text{rectangle} = 4 \times \text{triangle}$. Remember the $\frac{1}{2}$ in the area of the triangle.

Marks: 1 mark for a correct area of either shape. 1 mark for a correct equation linking the areas with 4 and 5. 1 mark for 3:4.

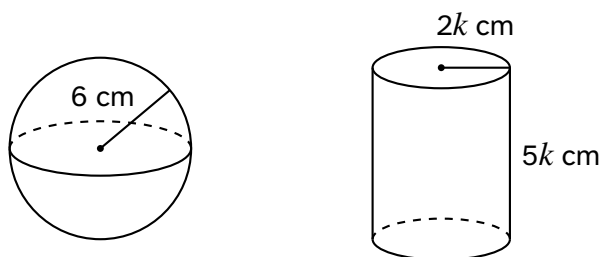
11 (3 marks)

A solid metal ball is a sphere of radius 6 cm.

A closed cylindrical tin has radius $2k$ cm and height $5k$ cm.

Surface area of a sphere = $4\pi r^2$ where r is the radius

The total surface area of the tin is equal to the surface area of the ball.



Not drawn accurately

Work out the value of k .

Give your answer to 3 significant figures.

$$\text{Surface area of the ball: } 4 \times \pi \times 6^2 = 144\pi \text{ cm}^2$$

The tin is closed, so it has two circular ends and a curved surface.

$$\text{Two ends: } 2 \times \pi \times (2k)^2 = 8\pi k^2 \quad \text{Curved surface: } 2\pi \times 2k \times 5k = 20\pi k^2$$

$$\text{Total for the tin: } 28\pi k^2$$

$$28\pi k^2 = 144\pi \Rightarrow k^2 = \frac{144}{28} = 5.142 \dots$$

$$k = \sqrt{5.142 \dots} = 2.267 \dots = 2.27 \text{ (3 s.f.)}$$

Answer: $k = 2.27$

Tip: Square the whole radius: $(2k)^2 = 4k^2$, not $2k^2$. A closed cylinder has two circular ends as well as the curved surface.

Marks: 1 mark for 144π or the tin's total surface area $28\pi k^2$. 1 mark for the equation $28\pi k^2 = 144\pi$ (or $k^2 = \frac{36}{7}$). 1 mark for 2.27.

12 (5 marks)

Lorcan cycles 21 km from his home to a village at an average speed of v km/h. He then cycles home along the same route at an average speed that is 2.5 km/h less. The journey home takes 15 minutes longer than the journey to the village.

(a) Show that $2v^2 - 5v - 420 = 0$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Time to the village: } \frac{21}{v} \text{ hours. Time home: } \frac{21}{v - 2.5} \text{ hours.}$$

$$15 \text{ minutes} = 0.25 \text{ hours, so } \frac{21}{v - 2.5} - \frac{21}{v} = 0.25$$

$$\text{Multiply every term by } 4v(v - 2.5): 84v - 84(v - 2.5) = v(v - 2.5)$$

$$84v - 84v + 210 = v^2 - 2.5v$$

$$0 = v^2 - 2.5v - 210$$

$$\text{Multiply by 2: } 2v^2 - 5v - 420 = 0$$

Answer: $\frac{21}{v - 2.5} - \frac{21}{v} = 0.25$ leads to $2v^2 - 5v - 420 = 0$

Tip: Change 15 minutes to 0.25 hours (not 0.15) because the speeds are in km/h. In a "show that", write every step: do not jump to the given equation.

Marks: 1 mark for the equation $\frac{21}{v - 2.5} - \frac{21}{v} = 0.25$ (or equivalent). 1 mark for clearing the fractions correctly to reach $2v^2 - 5v - 420 = 0$.

- (b) Work out the value of v .
Give your answer to 3 significant figures.

Use the quadratic formula with $a = 2$, $b = -5$, $c = -420$:

$$v = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-420)}}{2 \times 2} = \frac{5 \pm \sqrt{3385}}{4}$$

$$v = 15.795 \dots \text{ or } v = -13.295 \dots$$

A speed must be positive (and more than 2.5 km/h), so $v = 15.8$ (3 s.f.)

Answer: $v = 15.8$

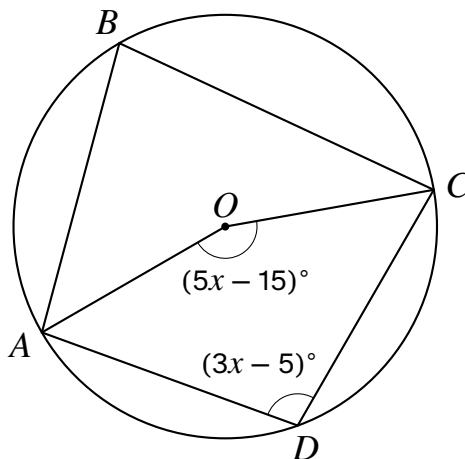
Tip: Use brackets for negative numbers on your calculator: $(-5)^2 = 25$. Reject the negative solution: a speed cannot be negative.

Marks: 1 mark for substituting correctly into the quadratic formula. 1 mark for 15.79 ... (or both solutions). 1 mark for 15.8 only.

13 (4 marks)

A , B , C and D are points on a circle, centre O .

Angle $AOC = (5x - 15)^\circ$ and angle $ADC = (3x - 5)^\circ$



Not drawn accurately

Work out the size of angle ABC .

Angle at the centre is twice the angle at the circumference: angle $ABC = \frac{1}{2}(5x - 15)$

Opposite angles of a cyclic quadrilateral add up to 180° : angle ABC + angle $ADC = 180$

$$\frac{1}{2}(5x - 15) + 3x - 5 = 180$$

Multiply by 2: $5x - 15 + 6x - 10 = 360$

$$11x = 385, \text{ so } x = 35$$

$$\text{Angle } ABC = \frac{1}{2}(5 \times 35 - 15) = \frac{1}{2} \times 160 = 80^\circ$$

Check: angle $ADC = 3 \times 35 - 5 = 100^\circ$ and $80 + 100 = 180$ ✓

Answer: 80°

Tip: Angle ABC is at the circumference and stands on the same arc as the marked angle at O , so it is half of it. Read the question: it asks for an angle, not the value of x .

Marks: 1 mark for angle $ABC = \frac{1}{2}(5x - 15)$. 1 mark for a correct equation in x . 1 mark for $x = 35$. 1 mark for 80° .

14 (4 marks)

a is a constant.

In the expansion of $(4 + ax)^5$ the term in x^2 is $1000x^2$

Work out the possible values of a .

Row 5 of Pascal's triangle: 1 5 10 10 5 1

The x^2 term uses the third coefficient, 10: $10 \times 4^3 \times (ax)^2$

$$= 10 \times 64 \times a^2 x^2 = 640a^2 x^2$$

$$640a^2 = 1000$$

$$a^2 = \frac{1000}{640} = 1.5625$$

$$a = \pm\sqrt{1.5625} = \pm 1.25$$

Answer: $a = 1.25$ or $a = -1.25$

Tip: Square the whole of ax : $(ax)^2 = a^2x^2$. The power of 4 goes down as the power of x goes up, so the x^2 term has 4^3 . Square rooting gives two answers: $\pm$.

Marks: 1 mark for the coefficient 10 (or 5C_2). 1 mark for $10 \times 4^3 \times a^2$ (or $640a^2$). 1 mark for $a^2 = 1.5625$. 1 mark for both values.

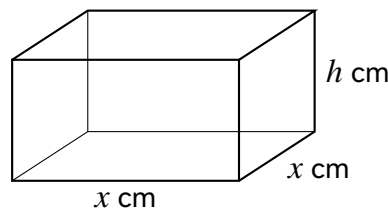
15 (6 marks)

An open-topped gift box is made from card.

The box is a cuboid with a square base of side x cm and height h cm.

The total area of the card is 1500 cm^2

Assume there is no overlap and no card is wasted.



Not drawn accurately

- (a) The volume of the box is $V \text{ cm}^3$

Show that $V = 375x - 0.25x^3$

Area of card: base x^2 plus four sides, each xh : $x^2 + 4xh = 1500$

Make h the subject: $h = \frac{1500 - x^2}{4x}$

$$V = x^2h = x^2 \times \frac{1500 - x^2}{4x} = \frac{x(1500 - x^2)}{4}$$

$$V = 375x - 0.25x^3$$

Answer: $x^2 + 4xh = 1500$, so $h = \frac{1500-x^2}{4x}$ and $V = x^2h = 375x - 0.25x^3$

Tip: The box has no lid, so the area of card is one base and four sides: $x^2 + 4xh$, not $2x^2 + 4xh$.

Marks: 1 mark for $x^2 + 4xh = 1500$. 1 mark for substituting h into $V = x^2h$ and simplifying to $375x - 0.25x^3$.

- (b) Use calculus to work out the maximum volume of the box.
You must show that your answer is a maximum.
Give your answer to 3 significant figures.

$$\frac{dV}{dx} = 375 - 0.75x^2$$

$$\text{At a maximum } \frac{dV}{dx} = 0: 0.75x^2 = 375, x^2 = 500$$

$$x = \sqrt{500} = 22.36 \dots \quad (x \text{ is a length, so } x > 0)$$

$$V = 375 \times 22.36 \dots - 0.25 \times 22.36 \dots^3 = 5590.16 \dots$$

$$\frac{d^2V}{dx^2} = -1.5x = -1.5 \times 22.36 \dots = -33.5 \dots < 0, \text{ so this is a maximum.}$$

Maximum volume = 5590 cm^3 (3 s.f.)

Answer: 5590 cm^3 (a maximum, as $\frac{d^2V}{dx^2} = -33.5 \dots < 0$)

Tip: Many students stop at $x = 22.4$. The question asks for the maximum **volume**, so substitute x back into V . You must test the nature too: a negative second derivative means a maximum.

Marks: 1 mark for $\frac{dV}{dx} = 375 - 0.75x^2$. 1 mark for setting it equal to 0 and getting $x = 22.36 \dots$ 1 mark for 5590. 1 mark for showing it is a maximum, e.g. $\frac{d^2V}{dx^2} < 0$.

16 (5 marks)

Solve the simultaneous equations $2a + b + 4c = 6$, $6a + 5b - c = -1$, $4a - 3b + 3c = 22$.

Label the equations (1) $2a + b + 4c = 6$, (2) $6a + 5b - c = -1$, (3) $4a - 3b + 3c = 22$

Eliminate a . (2) $- 3 \times$ (1): $6a + 5b - c - (6a + 3b + 12c) = -1 - 18$, so $2b - 13c = -19$
(4)

(3) $- 2 \times$ (1): $4a - 3b + 3c - (4a + 2b + 8c) = 22 - 12$, so $-5b - 5c = 10$, i.e. $b + c = -2$
(5)

From (5): $b = -2 - c$. Substitute into (4): $2(-2 - c) - 13c = -19$

$-4 - 15c = -19$, so $c = 1$

$b = -2 - 1 = -3$

In (1): $2a - 3 + 4 = 6$, so $2a = 5$, $a = 2.5$

Check in (3): $4(2.5) - 3(-3) + 3(1) = 10 + 9 + 3 = 22$ ✓

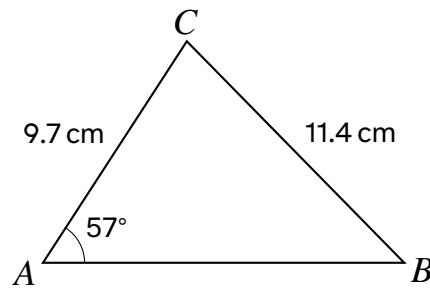
Answer: $a = 2.5$, $b = -3$, $c = 1$

Tip: Number your equations and say which ones you are combining. Eliminate the **same** letter twice, then solve the two new equations. Check your answers in the equation you did not use last.

Marks: 1 mark for eliminating one letter from a pair of equations. 1 mark for eliminating the same letter from a different pair. 1 mark for solving these to find one value. 1 mark for two correct values. 1 mark for all three.

17 (4 marks)

Here is triangle ABC .



Not drawn accurately

Work out the area of triangle ABC .

Give your answer to 3 significant figures.

Use the sine rule to find angle B (opposite $AC = 9.7$): $\frac{\sin B}{9.7} = \frac{\sin 57^\circ}{11.4}$

$$\sin B = \frac{9.7 \times \sin 57^\circ}{11.4} = 0.7136 \dots$$

$$B = \sin^{-1}(0.7136 \dots) = 45.52 \dots^\circ \quad (B \text{ is acute because } AC < BC, \text{ so } B < 57^\circ)$$

$$\text{Angle } C = 180 - 57 - 45.52 \dots = 77.47 \dots^\circ$$

$$\text{Area} = \frac{1}{2} \times 9.7 \times 11.4 \times \sin 77.47 \dots^\circ = 53.973 \dots$$

$$\text{Area} = 54.0 \text{ cm}^2 \quad (3 \text{ s.f.})$$

Answer: 54.0 cm²

Tip: 57° is not between the two given sides, so you cannot use $\frac{1}{2}ab \sin C$ with it straight away. Find the angle between 9.7 and 11.4 first. Keep full calculator values until the end.

Marks: 1 mark for a correct sine rule for angle B . 1 mark for $B = 45.5^\circ$. 1 mark for $\frac{1}{2} \times 9.7 \times 11.4 \times \sin C$ with $C = 180 - 57 - B$. 1 mark for 54.0.

18 (4 marks)

$$\text{Solve } (25^x)^{x+2} = 5 \times 125^x$$

Write everything as a power of 5: $25 = 5^2$, $125 = 5^3$, $5 = 5^1$

Left: $(25^x)^{x+2} = (5^{2x})^{x+2} = 5^{2x(x+2)} = 5^{2x^2+4x}$

Right: $5 \times 125^x = 5^1 \times 5^{3x} = 5^{3x+1}$

The bases are equal, so the powers are equal: $2x^2 + 4x = 3x + 1$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = \frac{1}{2} \text{ or } x = -1$$

Answer: $x = \frac{1}{2}$ or $x = -1$

Tip: For a power of a power, multiply the indices: $(5^{2x})^{x+2} = 5^{2x(x+2)}$. Do not forget that $5 = 5^1$, so the right-hand side is 5^{3x+1} .

Marks: 1 mark for writing both sides as powers of 5. 1 mark for $2x(x+2) = 3x+1$. 1 mark for solving $2x^2 + x - 1 = 0$. 1 mark for both answers.

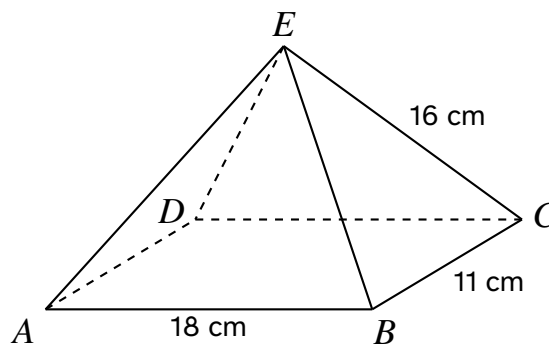
19 (5 marks)

$ABCDE$ is a pyramid.

The base, $ABCD$, is a horizontal rectangle.

E is vertically above the centre of the base.

$AB = 18$ cm $BC = 11$ cm Each sloping edge is 16 cm



Not drawn accurately

Work out the size of the angle between the face EBC and the base $ABCD$.

Let X be the centre of the base and M the midpoint of BC .

Half the diagonal: $AX = \frac{1}{2}\sqrt{18^2 + 11^2} = \frac{1}{2}\sqrt{445} = 10.547 \dots$ cm

$$\text{Height: } EX = \sqrt{16^2 - 10.547...^2} = \sqrt{256 - 111.25} = \sqrt{144.75} = 12.031... \text{ cm}$$

XM goes from the centre to the middle of BC , so $XM = \frac{1}{2} \times 18 = 9 \text{ cm}$

EM is perpendicular to BC (triangle EBC is isosceles), so the angle between the face and the base is angle EMX .

$$\tan EMX = \frac{EX}{XM} = \frac{12.031...}{9}$$

$$\text{Angle } EMX = \tan^{-1}(1.336...) = 53.20...^\circ = 53.2^\circ$$

Answer: 53.2°

Tip: The angle between a face and the base is in the right-angled triangle EXM , where M is the midpoint of the edge. Use half of AB (9 cm), not half of BC , for XM . Draw this triangle separately.

Marks: 1 mark for half the diagonal (10.5). 1 mark for the height (12.0). 1 mark for using 9 cm from the centre to the middle of BC . 1 mark for a correct trig ratio for the angle. 1 mark for 53.2° .

20 (4 marks)

Solve $3 \sin^2 x + 4 = 9 \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place.

Use $\cos^2 x = 1 - \sin^2 x$ to get an equation in $\sin x$ only:

$$3 \sin^2 x + 4 = 9(1 - \sin^2 x) = 9 - 9 \sin^2 x$$

$$12 \sin^2 x = 5, \text{ so } \sin^2 x = \frac{5}{12}$$

$$\sin x = \pm \sqrt{\frac{5}{12}} = \pm 0.6454...$$

$$\sin x = 0.6454...: x = 40.20...^\circ \text{ and } 180 - 40.20... = 139.79...^\circ$$

$$\sin x = -0.6454...: x = 180 + 40.20... = 220.20...^\circ \text{ and } 360 - 40.20... = 319.79...^\circ$$

$$x = 40.2^\circ, 139.8^\circ, 220.2^\circ, 319.8^\circ$$

Answer: $40.2^\circ, 139.8^\circ, 220.2^\circ, 319.8^\circ$

Tip: When you square root, remember the negative root: $\sin x = -0.645$ gives two more angles. Sketch $y = \sin x$ from 0° to 360° to find every solution.

Marks: 1 mark for replacing $\cos^2 x$ by $1 - \sin^2 x$. 1 mark for $\sin^2 x = \frac{5}{12}$ (or $\sin x = \pm 0.645$). 1 mark for two correct angles. 1 mark for all four and no others.
