

Mock Set A – Paper 1

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

The functions f and g are given by

$$f(x) = 11 - 4x \quad \text{and} \quad g(x) = x^2 - 6$$

(a) Work out $fg(-2)$

$fg(-2)$ means do g first, then f .

$$g(-2) = (-2)^2 - 6 = 4 - 6 = -2$$

$$f(-2) = 11 - 4 \times (-2) = 11 + 8 = 19$$

Answer: 19

Tip: Work from the right: g acts on -2 first. $(-2)^2 = 4$, not -4 . Then take care with $-4 \times (-2) = +8$.

Marks: 1 mark for $g(-2) = -2$. 1 mark for 19.

(b) Work out $f^{-1}(x)$

Write $y = 11 - 4x$ and make x the subject.

Add $4x$ and subtract y : $4x = 11 - y$

Divide by 4: $x = \frac{11 - y}{4}$

Write it in terms of x : $f^{-1}(x) = \frac{11-x}{4}$

Answer: $f^{-1}(x) = \frac{11-x}{4}$

Tip: Take care with the sign when you divide by a negative. $\frac{x-11}{4}$ is a common wrong answer. Check: $f(1) = 7$, so $f^{-1}(7)$ should be 1.

Marks: 1 mark for a correct first step, e.g. $4x = 11 - y$ or $x = 11 - 4y$. 1 mark for $\frac{11-x}{4}$.

2 (2 marks)

Factorise fully $50x^3y - 8xy^3$

Both terms have a factor of 2, x and y : $50x^3y - 8xy^3 = 2xy(25x^2 - 4y^2)$

$25x^2 - 4y^2 = (5x)^2 - (2y)^2$ is a difference of two squares.

$$(5x)^2 - (2y)^2 = (5x - 2y)(5x + 2y)$$

$$\text{So } 50x^3y - 8xy^3 = 2xy(5x - 2y)(5x + 2y)$$

Answer: $2xy(5x - 2y)(5x + 2y)$

Tip: “Fully” is a clue that there is more to do after taking out the common factor. Always check whether the bracket left is a difference of two squares.

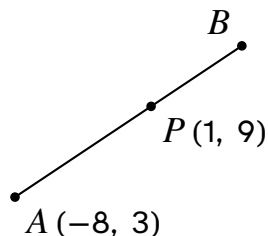
Marks: 1 mark for $2xy(25x^2 - 4y^2)$, or for $(5x - 2y)(5x + 2y)$ with a different common factor. 2 marks for $2xy(5x - 2y)(5x + 2y)$.

3 (3 marks)

A is the point $(-8, 3)$ and P is the point $(1, 9)$

P lies on the line segment AB so that

$$AP:PB = 3:2$$



Not drawn accurately

Work out the coordinates of B .

From A to P : x goes up by $1 - (-8) = 9$ and y goes up by $9 - 3 = 6$

AP is 3 parts of the ratio, so 1 part is a change of $9 \div 3 = 3$ in x and $6 \div 3 = 2$ in y .

PB is 2 parts: a change of $2 \times 3 = 6$ in x and $2 \times 2 = 4$ in y .

B : $(1 + 6, 9 + 4) = (7, 13)$

Answer: $(7, 13)$

Tip: P is between A and B , so B is beyond P . AP is 3 parts and PB is 2 parts, so B is $\frac{2}{3}$ of the step AP further on from P , not another whole step AP (that gives $(10, 15)$).

Marks: 1 mark for the changes from A to P (9 and 6) or one part (3 and 2). 1 mark for a correct method for one coordinate of B . 1 mark for $(7, 13)$.

4 (3 marks)

$$\mathbf{A} = \begin{pmatrix} 4 & a \\ b & 8 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$$

$\mathbf{AB} = 22\mathbf{I}$ where $\mathbf{I}$ is the identity matrix.

Work out the values of a and b .

Multiply row by column:

$$\mathbf{AB} = \begin{pmatrix} 4 \times 4 + a \times 3 & 4 \times (-1) + a \times 2 \\ b \times 4 + 8 \times 3 & b \times (-1) + 8 \times 2 \end{pmatrix} = \begin{pmatrix} 16 + 3a & -4 + 2a \\ 4b + 24 & -b + 16 \end{pmatrix}$$

$$22\mathbf{I} = \begin{pmatrix} 22 & 0 \\ 0 & 22 \end{pmatrix}$$

Top right: $-4 + 2a = 0$, so $a = 2$ (check top left: $16 + 6 = 22$ ✓)

Bottom left: $4b + 24 = 0$, so $b = -6$ (check bottom right: $6 + 16 = 22$ ✓)

Answer: $a = 2$, $b = -6$

Tip: The identity matrix has 1s on the leading diagonal and 0s elsewhere, so $22\mathbf{I}$ has 22 on the diagonal and 0 in the other two places. Use the 0 entries to find a and b quickly, then check with the diagonal.

Marks: 1 mark for multiplying out (at least two entries correct). 1 mark for setting an entry equal to the matching entry of $22\mathbf{I}$. 1 mark for both values.

5 (3 marks)

The n th term of a sequence is $\frac{5n - 2}{4n + 3}$

(a) Write down the limiting value of the sequence as $n \rightarrow \infty$

For large n , the -2 and the $+3$ make very little difference.

The terms get closer and closer to $\frac{5n}{4n} = \frac{5}{4}$

Answer: $\frac{5}{4}$

Tip: Compare the coefficients of n on the top and the bottom.

(b) Show that every term of the sequence is less than the limiting value.

n is a positive integer, so $4n + 3$ is positive.

Subtract the n th term from $\frac{5}{4}$: $\frac{5}{4} - \frac{5n - 2}{4n + 3} = \frac{5(4n + 3) - 4(5n - 2)}{4(4n + 3)}$

$$= \frac{20n + 15 - 20n + 8}{4(4n + 3)} = \frac{23}{4(4n + 3)}$$

23 is positive and $4(4n + 3)$ is positive, so $\frac{5}{4} - \frac{5n - 2}{4n + 3} > 0$ for every value of n .

So every term is less than $\frac{5}{4}$.

Answer: $\frac{5}{4} - \frac{5n-2}{4n+3} = \frac{23}{4(4n+3)} > 0$ because $4n+3 > 0$, so every term is less than $\frac{5}{4}$

Tip: Working out a few terms is not enough: you must show it for every n . Subtract the n th term from the limiting value and show that the answer is always positive. Say why the denominator is positive.

Marks: 1 mark for writing $\frac{5}{4} - \frac{5n-2}{4n+3}$ as a single fraction, $\frac{23}{4(4n+3)}$ (or for $4(5n-2) < 5(4n+3)$ leading to $-8 < 15$). 1 mark for a full argument that says why the result is positive and ends with a conclusion.

6 (3 marks)

$$y = 3x^2 \left(x^4 - 5 + \frac{2}{x} \right)$$

Work out $\frac{dy}{dx}$

Expand first. $3x^2 \times \frac{2}{x} = \frac{6x^2}{x} = 6x$

$$y = 3x^6 - 15x^2 + 6x$$

Differentiate each term: multiply by the power, then take 1 off the power.

$$\frac{dy}{dx} = 18x^5 - 30x + 6$$

Answer: $\frac{dy}{dx} = 18x^5 - 30x + 6$

Tip: You cannot differentiate a product by differentiating each bracket. Expand first. The $6x$ term differentiates to 6, so the answer has a constant term.

Marks: 1 mark for expanding to $3x^6 - 15x^2 + 6x$ (at least two terms correct). 1 mark for differentiating at least two of your terms correctly. 1 mark for $18x^5 - 30x + 6$.

7 (5 marks)

- (a) Simplify fully $\sqrt{5}(\sqrt{180} - \sqrt{45})$

$$\sqrt{180} = \sqrt{36} \times \sqrt{5} = 6\sqrt{5} \text{ and } \sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\text{So the bracket is } 6\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$$

$$\sqrt{5} \times 3\sqrt{5} = 3 \times 5 = 15$$

Answer: 15

Tip: Either simplify the surds in the bracket first, or multiply in: $\sqrt{5} \times \sqrt{180} = \sqrt{900} = 30$.
You cannot subtract under the square root: $\sqrt{180} - \sqrt{45}$ is not $\sqrt{135}$.

Marks: 1 mark for $6\sqrt{5} - 3\sqrt{5}$, or $\sqrt{900} - \sqrt{225}$, or $30 - 15$. 1 mark for 15.

- (b) Rationalise the denominator and simplify fully $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$. Give your answer in the form $p + q\sqrt{7}$ where p and q are integers.

Multiply the top and bottom by $3 + \sqrt{7}$ (change the sign in the denominator).

$$\text{Denominator: } (3 - \sqrt{7})(3 + \sqrt{7}) = 9 - 7 = 2$$

$$\text{Numerator: } (5 + \sqrt{7})(3 + \sqrt{7}) = 15 + 5\sqrt{7} + 3\sqrt{7} + 7 = 22 + 8\sqrt{7}$$

$$\frac{22 + 8\sqrt{7}}{2} = 11 + 4\sqrt{7}$$

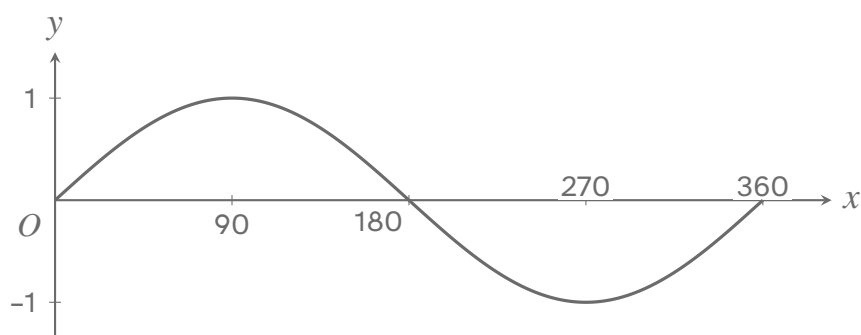
Answer: $11 + 4\sqrt{7}$

Tip: $(3 - \sqrt{7})(3 + \sqrt{7}) = 9 - 7$, not $9 + 7$. Divide **both** terms of the numerator by 2 at the end.

Marks: 1 mark for multiplying the top and bottom by $3 + \sqrt{7}$. 1 mark for $\frac{22 + 8\sqrt{7}}{2}$. 1 mark for $11 + 4\sqrt{7}$.

8 (2 marks)

The graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ is shown.



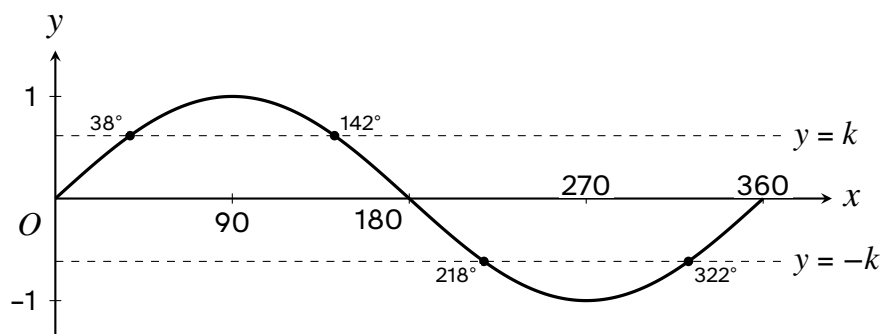
$\sin 38^\circ = k$. Use the graph to find the two solutions of $\sin x = -k$ in the range $0^\circ \leq x \leq 360^\circ$

$\sin x$ is negative for x between 180° and 360° (the curve is below the x -axis).

The curve has rotational symmetry about $(180, 0)$: $\sin(180^\circ + 38^\circ) = -\sin 38^\circ$, so $x = 218^\circ$

The curve is symmetrical about $x = 270$ between 180 and 360 :

$\sin(360^\circ - 38^\circ) = -\sin 38^\circ$, so $x = 322^\circ$



Answer: 218° and 322°

Tip: 142° is a common wrong answer: $\sin 142^\circ = +k$, not $-k$. Mark the line $y = -k$ on the sketch to see where the solutions are.

Marks: 1 mark for each correct angle.

9 (3 marks)

$$a(x+2)^2 - b(x-5) \equiv 3x^2 + 7x + c$$

Work out the values of a , b and c .

Expand the left-hand side: $a(x^2 + 4x + 4) - bx + 5b = ax^2 + 4ax + 4a - bx + 5b$

Compare the x^2 terms: $a = 3$

Compare the x terms: $4a - b = 7$, so $12 - b = 7$ and $b = 5$

Compare the constants: $c = 4a + 5b = 12 + 25 = 37$

Answer: $a = 3$, $b = 5$, $c = 37$

Tip: $-b(x-5) = -bx + 5b$: a negative times a negative gives $+5b$. Expand $(x+2)^2$ as $x^2 + 4x + 4$, not $x^2 + 4$.

Marks: 1 mark for $a = 3$. 1 mark for $b = 5$. 1 mark for $c = 37$.

10 (3 marks)

Simplify fully $\frac{10x^3y^{-1} + 4x^{-2}y^5}{2x^{-3}y^{-2}}$. Give your answer with positive powers only.

Divide each term of the numerator by $2x^{-3}y^{-2}$.

First term: $10 \div 2 = 5$, $x^{3-(-3)} = x^6$, $y^{-1-(-2)} = y^1$, giving $5x^6y$

Second term: $4 \div 2 = 2$, $x^{-2-(-3)} = x^1$, $y^{5-(-2)} = y^7$, giving $2xy^7$

Answer: $5x^6y + 2xy^7$

Answer: $5x^6y + 2xy^7$

Tip: Dividing by a negative power adds to the power: $x^3 \div x^{-3} = x^{3+3} = x^6$. Divide **both** terms of the numerator by the denominator.

Marks: 1 mark for the coefficients 5 and 2. 1 mark for the powers of x (6 and 1). 1 mark for the powers of y (1 and 7).

11 (4 marks)

A circle has centre $(-3, 2)$

The circle passes through the point $(2, -2)$

- (a) Work out the equation of the circle.

$$\begin{aligned}\text{Radius}^2 &= (\text{distance from } (-3, 2) \text{ to } (2, -2))^2 \\ &= (2 - (-3))^2 + (-2 - 2)^2 = 5^2 + (-4)^2 = 25 + 16 = 41 \\ \text{Equation: } &(x + 3)^2 + (y - 2)^2 = 41\end{aligned}$$

Answer: $(x + 3)^2 + (y - 2)^2 = 41$

Tip: The centre $(-3, 2)$ gives $(x + 3)^2$, not $(x - 3)^2$. The right-hand side is $r^2 = 41$; do not square root it.

Marks: 1 mark for $(x + 3)^2 + (y - 2)^2$ or for $r^2 = 41$. 1 mark for the full equation.

- (b) The circle crosses the y -axis at two points.
Work out the exact y -coordinates of these two points.
Give your answers in their simplest surd form.

On the y -axis, $x = 0$.

$$\begin{aligned}(0 + 3)^2 + (y - 2)^2 &= 41 \\ (y - 2)^2 &= 41 - 9 = 32 \\ y - 2 &= \pm\sqrt{32} = \pm 4\sqrt{2} \\ y &= 2 + 4\sqrt{2} \text{ or } y = 2 - 4\sqrt{2}\end{aligned}$$

Answer: $2 + 4\sqrt{2}$ and $2 - 4\sqrt{2}$

Tip: Square rooting gives two answers: $+\sqrt{32}$ and $-\sqrt{32}$. Simplify $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$.

Marks: 1 mark for substituting $x = 0$ to get $(y - 2)^2 = 32$ (or $y^2 - 4y - 28 = 0$). 1 mark for both answers.

12 (4 marks)

The function f is given by

$$\begin{aligned} f(x) &= 2x & 0 \leq x < 2 \\ &= x^2 - 8x + 16 & 2 \leq x < 5 \\ &= 1 & 5 \leq x \leq 8 \end{aligned}$$

Draw the graph of $y = f(x)$ for $0 \leq x \leq 8$

$2x$: a straight line from $(0, 0)$ to $(2, 4)$.

$x^2 - 8x + 16 = (x - 4)^2$: a table of values for $2 \leq x \leq 5$:

x	2	3	4	5
y	4	1	0	1

Plot these and join them with a smooth curve (minimum at $(4, 0)$).

1 : a horizontal line from $(5, 1)$ to $(8, 1)$.

The pieces join up at $(2, 4)$ and $(5, 1)$. Draw the straight pieces with a ruler.



Answer: Line from (0, 0) to (2, 4); smooth curve through (2, 4), (3, 1), (4, 0), (5, 1); line from (5, 1) to (8, 1)

Tip: Use a ruler for the straight pieces, but draw the quadratic piece as a smooth curve, not with straight segments. Each piece must start and stop at the right x -values.

Marks: 1 mark for the line from (0, 0) to (2, 4). 1 mark for plotting the points of the curve. 1 mark for a smooth curve through them. 1 mark for the horizontal line from (5, 1) to (8, 1).

13 (3 marks)

Solve $(2x + 3)^2 \leq x(x - 1) + 19$

Expand both sides: $4x^2 + 12x + 9 \leq x^2 - x + 19$

Make one side 0: $3x^2 + 13x - 10 \leq 0$

Factorise: $(3x - 2)(x + 5) \leq 0$

Critical values: $x = \frac{2}{3}$ and $x = -5$

$y = (3x - 2)(x + 5)$ is a U-shaped curve, so it is below or on the x -axis between the critical values.

$$-5 \leq x \leq \frac{2}{3}$$

Answer: $-5 \leq x \leq \frac{2}{3}$

Tip: Expand $(2x + 3)^2$ as $4x^2 + 12x + 9$: it has a middle term. The answer is an inequality, not two values. Sketch the curve to decide whether you want between or outside the critical values. Check with $x = 0$: $9 \leq 19$ is true, so 0 must be in your answer.

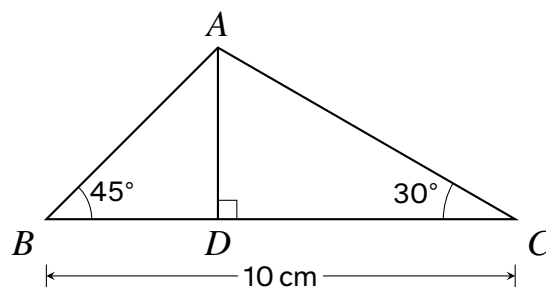
Marks: 1 mark for $3x^2 + 13x - 10$ and $(3x - 2)(x + 5)$. 1 mark for the critical values $\frac{2}{3}$ and -5 . 1 mark for $-5 \leq x \leq \frac{2}{3}$.

14 (3 marks)

BDC is a straight line.

AD is perpendicular to BC .

$BC = 10$ cm



Not drawn accurately

Work out the length AD .

Give your answer in the form $p\sqrt{3} + q$ where p and q are integers.

Let $AD = h$.

Triangle ABD : $\tan 45^\circ = \frac{h}{BD}$ and $\tan 45^\circ = 1$, so $BD = h$

Triangle ADC : $\tan 30^\circ = \frac{h}{DC}$, so $DC = \frac{h}{\tan 30^\circ} = h \div \frac{1}{\sqrt{3}} = h\sqrt{3}$

$BD + DC = BC$: $h + h\sqrt{3} = 10$, so $h(1 + \sqrt{3}) = 10$ and $h = \frac{10}{1 + \sqrt{3}}$

Rationalise: $\frac{10}{1 + \sqrt{3}} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{10(\sqrt{3} - 1)}{3 - 1} = 5(\sqrt{3} - 1)$

$AD = 5\sqrt{3} - 5 \text{ cm}$

Answer: $5\sqrt{3} - 5 \text{ cm}$

Tip: When no length in a triangle is known, call the shared side h and write the other lengths in terms of h . Learn the exact values: $\tan 45^\circ = 1$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$. To rationalise $\frac{10}{1+\sqrt{3}}$, multiply the top and bottom by $\sqrt{3} - 1$.

Marks: 1 mark for BD and DC both in terms of AD (h and $h\sqrt{3}$). 1 mark for $h + h\sqrt{3} = 10$ leading to $h = \frac{10}{1+\sqrt{3}}$. 1 mark for $5\sqrt{3} - 5$.

15 (4 marks)

Make x the subject of $y(3 + x^2) = 4(x^2 - w)$

Expand both brackets: $3y + x^2y = 4x^2 - 4w$

Collect the x^2 terms on one side: $3y + 4w = 4x^2 - x^2y$

Factorise: $3y + 4w = x^2(4 - y)$

Divide: $x^2 = \frac{3y + 4w}{4 - y}$

Square root: $x = \pm \sqrt{\frac{3y + 4w}{4 - y}}$

Answer: $x = \pm \sqrt{\frac{3y + 4w}{4 - y}}$

Tip: When the new subject appears twice, get both terms on the same side and factorise. The square root must cover the whole fraction.

Marks: 1 mark for expanding both brackets. 1 mark for collecting the x^2 terms on one side. 1 mark for factorising and dividing to get $x^2 = \frac{3y + 4w}{4 - y}$. 1 mark for the square root.

16 (5 marks)

$$f(x) = 2x^3 - 3x^2 - 23x + 12$$

- (a) Use the factor theorem to show that $(2x - 1)$ is a factor of $f(x)$.

If $(2x - 1)$ is a factor then $f\left(\frac{1}{2}\right) = 0$, because $2x - 1 = 0$ when $x = \frac{1}{2}$.

$$f\left(\frac{1}{2}\right) = 2 \times \frac{1}{8} - 3 \times \frac{1}{4} - 23 \times \frac{1}{2} + 12$$

$$= \frac{1}{4} - \frac{3}{4} - \frac{23}{2} + 12$$

$$= -\frac{1}{2} - 11\frac{1}{2} + 12 = 0$$

$f\left(\frac{1}{2}\right) = 0$, so $(2x - 1)$ is a factor of $f(x)$.

Answer: $f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{3}{4} - \frac{23}{2} + 12 = 0$, so $(2x - 1)$ is a factor.

Tip: Substitute $x = \frac{1}{2}$ (the value that makes $2x - 1 = 0$), not $x = 1$ or $x = 2$. Show each term, and finish with a statement.

Marks: 1 mark for substituting $x = \frac{1}{2}$. 1 mark for showing it equals 0 and saying $(2x - 1)$ is a factor.

- (b) Hence solve $f(x) = 0$

Write $f(x) = (2x - 1)(x^2 + bx + c)$.

$$x^3: 2x \times x^2 = 2x^3 \quad \checkmark \quad \text{constant: } -1 \times c = 12, \text{ so } c = -12$$

$$x^2: 2b - 1 = -3, \text{ so } b = -1$$

$$f(x) = (2x - 1)(x^2 - x - 12)$$

$$x^2 - x - 12 = (x + 3)(x - 4)$$

$$(2x - 1)(x + 3)(x - 4) = 0$$

$$x = \frac{1}{2}, x = -3 \text{ or } x = 4$$

Answer: $x = \frac{1}{2}, x = -3, x = 4$

Tip: “Solve” means give the values of x , not the factors. Remember to include $x = \frac{1}{2}$ from the factor in part (a).

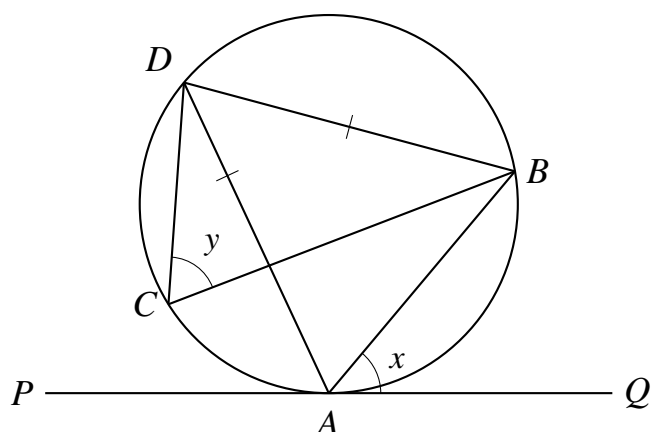
Marks: 1 mark for the quadratic factor $x^2 - x - 12$. 1 mark for factorising it to $(x + 3)(x - 4)$. 1 mark for all three solutions.

17 (4 marks)

A, B, C and D are points on a circle.

PAQ is the tangent to the circle at A .

$DA = DB$



Not drawn accurately

Prove that $x + 2y = 180$

Give a reason for each step of your working.

Angle $ADB = x$

(Alternate segment theorem: the angle between the tangent AQ and the chord AB equals the angle in the alternate segment.)

$DA = DB$, so triangle ABD is isosceles and angle $DAB = \text{angle } DBA$

(Base angles of an isosceles triangle are equal.)

$$\text{Angle } DAB = \frac{180 - x}{2}$$

(Angles in a triangle add up to 180° .)

$$\text{Angle } DCB = \text{angle } DAB, \text{ so } y = \frac{180 - x}{2}$$

(Angles in the same segment are equal: both stand on the chord DB .)

$$\text{Multiply by 2: } 2y = 180 - x, \text{ so } x + 2y = 180$$

Answer: Angle $ADB = x$ (alternate segment theorem); angle $DAB = \text{angle } DBA = \frac{180-x}{2}$ (base angles of an isosceles triangle, angles in a triangle add to 180°); $y = \text{angle } DAB$ (angles in the same segment), so $2y = 180 - x$ and $x + 2y = 180$.

Tip: Name each angle with three letters (angle ADB , not angle D), mark the angles on the diagram, and use the proper names of the theorems. A proof needs a reason for every step.

Marks: 1 mark for angle $ADB = x$. 1 mark for angle DAB (or DBA) = $\frac{180-x}{2}$. 1 mark for $y = \text{angle } DAB$ leading to $x + 2y = 180$. 1 mark for all the reasons.

18 (3 marks)

Write $3x^2 - 9x + 2$ in the form $a(x+p)^2 + q$ where a, p and q are constants.

Take out the factor 3 from the first two terms: $3(x^2 - 3x) + 2$

$$\text{Half of } -3 \text{ is } -\frac{3}{2}: x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$$

$$3\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 2 = 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 2$$

$$-\frac{27}{4} + \frac{8}{4} = -\frac{19}{4}$$

$$3x^2 - 9x + 2 = 3\left(x - \frac{3}{2}\right)^2 - \frac{19}{4}$$

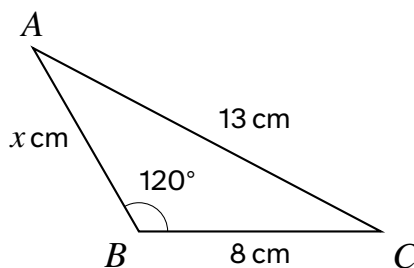
Answer: $3\left(x - \frac{3}{2}\right)^2 - \frac{19}{4}$

Tip: Multiply the $-\frac{9}{4}$ by the 3 outside the bracket: $-\frac{27}{4}$, not $-\frac{9}{4}$. Expand your answer to check it.

Marks: 1 mark for $3\left(x - \frac{3}{2}\right)^2$. 1 mark for $3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 2$ (or equivalent). 1 mark for $-\frac{19}{4}$.

19 (4 marks)

Here is a triangle ABC .



Not drawn accurately

Work out the value of x .

You must show your working.

$$\text{Cosine rule: } AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos 120^\circ$$

$$13^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 120^\circ$$

$$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}, \text{ so } -2 \times x \times 8 \times \left(-\frac{1}{2}\right) = +8x$$

$$169 = x^2 + 64 + 8x$$

$$x^2 + 8x - 105 = 0$$

$$(x + 15)(x - 7) = 0$$

$$x = -15 \text{ or } x = 7$$

x is a length, so it cannot be negative: $x = 7$

Answer: $x = 7$

Tip: $\cos 120^\circ$ is negative, so the $-2bc \cos A$ term becomes $+8x$. Reject $x = -15$: a length cannot be negative.

Marks: 1 mark for the cosine rule with the correct values. 1 mark for $x^2 + 8x - 105 = 0$. 1 mark for solving it. 1 mark for $x = 7$ only.

20 (6 marks)

The equation of a curve is $y = x^4 - 4x^3 - 8x^2$

The curve has three stationary points. Use calculus to find the coordinates of each stationary point and to decide whether it is a maximum or a minimum.

$$\frac{dy}{dx} = 4x^3 - 12x^2 - 16x$$

At a stationary point $\frac{dy}{dx} = 0$: $4x^3 - 12x^2 - 16x = 0$

Take out the common factor $4x$: $4x(x^2 - 3x - 4) = 0$

$4x(x + 1)(x - 4) = 0$, so $x = 0$, $x = -1$ or $x = 4$

$x = 0$: $y = 0$ $x = -1$: $y = 1 + 4 - 8 = -3$ $x = 4$: $y = 256 - 256 - 128 = -128$

$$\frac{d^2y}{dx^2} = 12x^2 - 24x - 16$$

$x = 0$: $\frac{d^2y}{dx^2} = -16 < 0$, so $(0, 0)$ is a maximum.

$x = -1$: $\frac{d^2y}{dx^2} = 12 + 24 - 16 = 20 > 0$, so $(-1, -3)$ is a minimum.

$x = 4$: $\frac{d^2y}{dx^2} = 192 - 96 - 16 = 80 > 0$, so $(4, -128)$ is a minimum.

Answer: $(0, 0)$ maximum; $(-1, -3)$ minimum; $(4, -128)$ minimum

Tip: Never divide by x : that loses the stationary point at $x = 0$. Take out $4x$ as a factor instead. Saying “maximum” or “minimum” is not enough: show the sign of $\frac{d^2y}{dx^2}$ at each point.

Marks: 1 mark for $\frac{dy}{dx}$. 1 mark for setting it equal to 0 and factorising. 1 mark for the three x -values. 1 mark for the three y -values. 1 mark for $\frac{d^2y}{dx^2}$ and substituting. 1 mark for the nature of all three, with the values of $\frac{d^2y}{dx^2}$ shown.

21 (6 marks)

Solve the simultaneous equations $\frac{x}{2} - \frac{y}{3} = 1$ and $x^2 + xy = 16$

Clear the fractions in the first equation: multiply by 6: $3x - 2y = 6$

Rearrange: $2y = 3x - 6$, so $y = \frac{3x - 6}{2}$

Substitute into $x^2 + xy = 16$: $x^2 + x\left(\frac{3x - 6}{2}\right) = 16$

Multiply by 2: $2x^2 + 3x^2 - 6x = 32$

$$5x^2 - 6x - 32 = 0$$

Factorise: $(5x - 16)(x + 2) = 0$

$$x = \frac{16}{5} \text{ or } x = -2$$

$$x = \frac{16}{5}: y = \frac{\frac{48}{5} - 6}{2} = \frac{\frac{18}{5}}{2} = \frac{9}{5}$$

$$x = -2: y = \frac{-6 - 6}{2} = -6$$

Check $(-2, -6)$: $4 + 12 = 16$ ✓

Answer: $x = \frac{16}{5}, y = \frac{9}{5}$ and $x = -2, y = -6$

Tip: Clear the fractions first: multiply every term of $\frac{x}{2} - \frac{y}{3} = 1$ by 6, including the 1. Pair each x with its own y , and use the linear equation to find y .

Marks: 1 mark for rearranging the linear equation. 1 mark for substituting into $x^2 + xy = 16$. 1 mark for $5x^2 - 6x - 32 = 0$. 1 mark for factorising (or the formula). 1 mark for both x -values. 1 mark for both correct pairs.

22 (3 marks)

Prove that $(\sin x - 2 \cos x)^2 + \tan x (4 \cos^2 x - \cos x) \equiv (4 + 3 \sin x)(1 - \sin x)$

Start with the left-hand side.

$$(\sin x - 2 \cos x)^2 = \sin^2 x - 4 \sin x \cos x + 4 \cos^2 x$$

$$\tan x = \frac{\sin x}{\cos x}, \text{ so}$$

$$\tan x (4 \cos^2 x - \cos x) = \frac{\sin x}{\cos x} \times 4 \cos^2 x - \frac{\sin x}{\cos x} \times \cos x = 4 \sin x \cos x - \sin x$$

Left-hand side

$$= \sin^2 x - 4 \sin x \cos x + 4 \cos^2 x + 4 \sin x \cos x - \sin x = \sin^2 x + 4 \cos^2 x - \sin x$$

Use $\cos^2 x = 1 - \sin^2 x$:

$$\sin^2 x + 4(1 - \sin^2 x) - \sin x = \sin^2 x + 4 - 4 \sin^2 x - \sin x = 4 - \sin x - 3 \sin^2 x$$

This is a quadratic in $\sin x$. Factorise: $4 - \sin x - 3 \sin^2 x = (4 + 3 \sin x)(1 - \sin x)$, which is the right-hand side.

$$(\text{Check: } (4 + 3 \sin x)(1 - \sin x) = 4 - 4 \sin x + 3 \sin x - 3 \sin^2 x = 4 - \sin x - 3 \sin^2 x \quad \checkmark)$$

Answer: LHS = $\sin^2 x - 4 \sin x \cos x + 4 \cos^2 x + 4 \sin x \cos x - \sin x = \sin^2 x + 4(1 - \sin^2 x) - \sin x = 4 - \sin x - 3 \sin^2 x = (4 + 3 \sin x)(1 - \sin x) = \text{RHS}$

Tip: $(\sin x - 2 \cos x)^2$ has three terms, including the middle term $-4 \sin x \cos x$. Work from one side only, and bracket $(1 - \sin^2 x)$ when you substitute it. If you reach a quadratic in $\sin x$, factorise it like $4 - s - 3s^2 = (4 + 3s)(1 - s)$.

Marks: 1 mark for expanding the squared bracket. 1 mark for using $\tan x = \frac{\sin x}{\cos x}$ to get $4 \sin x \cos x - \sin x$. 1 mark for using $\cos^2 x = 1 - \sin^2 x$ and factorising to reach the right-hand side with no errors.