

Mock Set B – Paper 3

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1)

Higher tier • Paper 3 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Lighthouse *A* flashes every 42 seconds.

Lighthouse *B* flashes every 54 seconds.

At 9 pm the two lighthouses flash at the same time.

Amelia says,

“They will next flash at the same time before 9.05 pm.”

Is Amelia correct?

You must show how you get your answer.

The lighthouses flash together again after a number of seconds that is a multiple of 42 **and** of 54, so find the LCM.

$$42 = 2 \times 3 \times 7 \quad 54 = 2 \times 3^3$$

$$\text{LCM} = 2 \times 3^3 \times 7 = 378 \text{ seconds}$$

$$378 \text{ seconds} = 6 \text{ minutes } 18 \text{ seconds } (6 \times 60 = 360, \text{ and } 378 - 360 = 18)$$

So they next flash together at 6 minutes 18 seconds past 9 pm.

This is **after** 9.05 pm, so Amelia is **not** correct.

Answer: No. The LCM of 42 and 54 is 378 seconds = 6 min 18 s, so they next flash together at 9.06 pm and 18 seconds, which is after 9.05 pm.

Tip: “At the same time again” means a common multiple, so find the LCM, not the HCF.

Change 378 seconds into minutes and seconds before you compare: it is not 3.78 minutes.

Marks: 1 mark for a correct method to find the LCM (prime factors or lists of multiples). 1 mark for 378 seconds. 1 mark for 'No' with the time or 6 minutes 18 seconds compared with 5 minutes.

2 (4 marks)

- (a) Simplify $(t^{-2})^{-4}$

For a power of a power, multiply the indices.

$$-2 \times -4 = 8$$

$$(t^{-2})^{-4} = t^8$$

Answer: t^8

Tip: -2×-4 is $+8$. Adding the indices (t^{-6}) is the rule for multiplying, not for a power of a power.

- (b) Expand $5p^2(2p - 3q)$

Multiply each term in the bracket by $5p^2$.

$$5p^2 \times 2p = 10p^3$$

$$5p^2 \times (-3q) = -15p^2q$$

$$5p^2(2p - 3q) = 10p^3 - 15p^2q$$

Answer: $10p^3 - 15p^2q$

Tip: $p^2 \times p = p^3$: add the powers when you multiply.

- (c) Factorise fully $21c^2d + 35cd^3$

The highest common factor of 21 and 35 is 7.

Both terms contain at least one c and at least one d .

So the highest common factor of the two terms is $7cd$.

$$21c^2d \div 7cd = 3c \quad \text{and} \quad 35cd^3 \div 7cd = 5d^2$$

$$21c^2d + 35cd^3 = 7cd(3c + 5d^2)$$

Answer: $7cd(3c + 5d^2)$

Tip: “Fully” means take out every common factor: the number **and** the letters. Expand your answer to check it.

Marks: 2 marks for a fully correct answer. 1 mark for a correct partial factorisation, such as $7c(3cd + 5d^3)$ or $cd(21c + 35d^2)$.

3 (2 marks)

When Shanice truncates the number w to 1 decimal place, she gets 26.8
Complete the error interval for w .

Truncating means chopping off the digits after the first decimal place, without rounding.

Any number from 26.8 up to (but not including) 26.9 truncates to 26.8

For example 26.8, 26.83 and 26.8999 all truncate to 26.8, but 26.9 does not.

Answer: $26.8 \leq w < 26.9$

Tip: Truncation is not rounding. If 26.8 had been rounded, the interval would be $26.75 \leq w < 26.85$, but a truncated value can only be bigger than or equal to the number shown.

Marks: 1 mark for 26.8. 1 mark for 26.9.

4 (3 marks)

The scale drawing shows a rectangular garden $ABCD$ (scale: 1 cm represents 1 m).
Hugo is going to put a bird table in the garden.

The bird table must be closer to AB than to AD , and less than 5 m from C .

On the diagram, shade the region where Hugo can put the bird table.
Label the region R .

You must show all your construction lines.

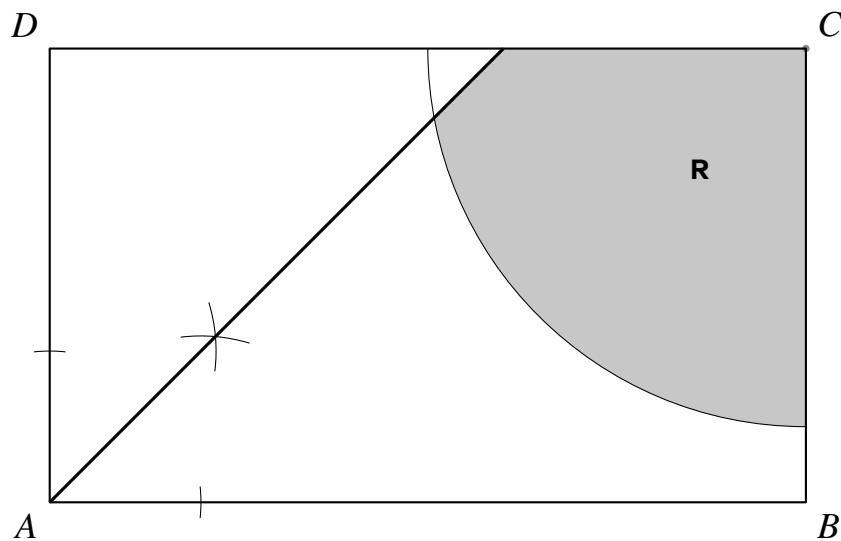
“Closer to AB than to AD ”: points that are the same distance from AB and AD lie on the bisector of angle DAB .

Construct the bisector of angle DAB with compasses: arcs from A cutting AB and AD , then equal arcs from those two points to meet. Draw the line from A through where they meet.

Points below the bisector are closer to AB .

“Less than 5 m from C ”: with the scale 1 cm to 1 m, draw an arc of radius 5 cm with centre C . Points inside the arc are less than 5 m from C .

The bird table must be inside the arc **and** below the bisector, so shade the region where these overlap.



Scale: 1 cm represents 1 m

Answer: Region below the bisector of angle DAB and inside the arc of radius 5 cm, centre C , shaded and labelled R

Tip: Leave all your construction arcs on the diagram. Do not rub them out: they show the examiner your method.

Marks: 1 mark for the angle bisector with construction arcs. 1 mark for an arc of radius 5 cm with centre C . 1 mark for the correct region shaded.

5 (4 marks)

Solder is an alloy of tin and lead.

mass of tin : mass of lead = 3:2

The density of tin is 7.3 g/cm^3

The density of lead is 11.3 g/cm^3

A reel of solder wire contains 14 cm^3 of solder.

Work out the mass of the solder on the reel.

Give your answer correct to 3 significant figures.

You must show all your working.

Think about a piece of solder with mass 5 g: it has 3 g of tin and 2 g of lead.

Volume = mass $\div$ density

Volume of 3 g of tin = $3 \div 7.3 = 0.41095 \dots \text{ cm}^3$

Volume of 2 g of lead = $2 \div 11.3 = 0.17699 \dots \text{ cm}^3$

So 5 g of solder has a volume of $0.41095 \dots + 0.17699 \dots = 0.58795 \dots \text{ cm}^3$

Density of the solder = $5 \div 0.58795 \dots = 8.5041 \dots \text{ g/cm}^3$

Mass of the solder on the reel = density $\times$ volume = $8.5041 \dots \times 14 = 119.05 \dots \text{ g}$

Answer: 119 g

Tip: Do not take the mean of the two densities. The metals are mixed in a ratio by **mass**, so work out the volume that each mass takes up.

Marks: 1 mark for the volume of some tin or some lead (mass $\div$ density). 1 mark for the density of the solder (or an equation for the mass). 1 mark for multiplying by 14. 1 mark for 119.

6 (4 marks)

Yasmin wants to buy 48 phone cases for her market stall.
She can buy them from a supplier in Ireland or from a supplier in England.

Supplier in Ireland	Supplier in England
Pack of 6 phone cases for €27.30	Pack of 8 phone cases for £31.40
Delivery charge €12 for each order	Free delivery

The exchange rate is £1 = €1.18

Yasmin wants to buy the 48 phone cases as cheaply as possible.

Which supplier should she use?

You must show how you get your answer.

Ireland: $48 \div 6 = 8$ packs. Cost = $8 \times 27.30 + 12 = €230.40$

Change to pounds: $230.40 \div 1.18 = £195.25$ (to the nearest penny)

England: $48 \div 8 = 6$ packs. Cost = $6 \times 31.40 = £188.40$

£188.40 is less than £195.25, so the supplier in **England** is cheaper (by £6.85).

Answer: The supplier in England: £188.40, compared with €230.40 = £195.25 from the supplier in Ireland.

Tip: Include the delivery charge. Without it the Irish supplier looks cheaper ($€218.40 = £185.08$), which is the wrong decision. To change euros into pounds, **divide** by 1.18.

Marks: 1 mark for the total cost from one supplier. 1 mark for a correct currency conversion. 1 mark for both totals in the same currency. 1 mark for England with the figures.

7 (4 marks)

Parveen plays basketball.

She takes two free throws.

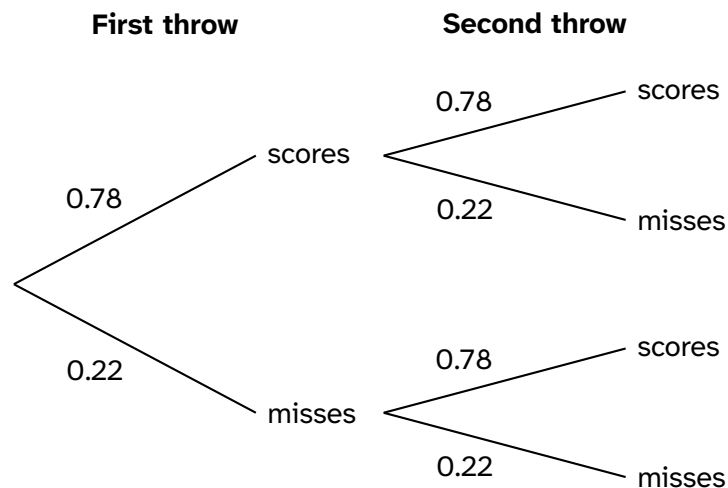
The probability that she scores with a free throw is 0.78

The two throws are independent.

(a) Complete the probability tree diagram.

Each pair of branches adds up to 1: $1 - 0.78 = 0.22$

The throws are independent, so the second throw has the same probabilities whatever happened on the first throw.



Answer: 0.22 on the first throw; 0.78, 0.22, 0.78, 0.22 on the second throw

Marks: 1 mark for 0.22 on the first throw. 1 mark for all four second-throw probabilities correct.

(b) Ayesha says,

“The probability that Parveen scores with exactly one of her two throws is $0.78 \times 0.22 = 0.1716$ ”

Is Ayesha correct?

You must give a reason for your answer.

Parveen can score with exactly one throw in two ways:

score then miss: $0.78 \times 0.22 = 0.1716$

miss then score: $0.22 \times 0.78 = 0.1716$

$P(\text{exactly one}) = 0.1716 + 0.1716 = 0.3432$

Ayesha has only included one of the two ways, so she is **not** correct.

Answer: No. She has missed the case ‘miss then score’. The probability is $2 \times 0.1716 = 0.3432$

Tip: “Exactly one” can happen in more than one order. Follow every path on the tree that gives the outcome you want, then add.

Marks: 1 mark for including both ways (score–miss and miss–score). 1 mark for ‘No’ with 0.3432 or a correct reason.

8 (4 marks)

The diagram shows a triangle and a rectangle.

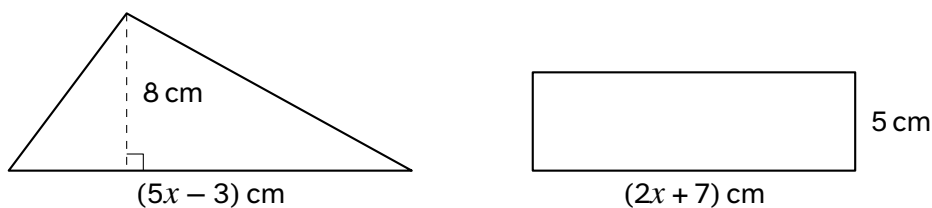


Diagram **NOT** accurately drawn

The area of the triangle is equal to the area of the rectangle.

Work out the perimeter of the rectangle.

You must show all your working.

$$\text{Area of the triangle} = \frac{1}{2} \times (5x - 3) \times 8 = 4(5x - 3) = 20x - 12$$

$$\text{Area of the rectangle} = 5(2x + 7) = 10x + 35$$

$$\text{The areas are equal: } 20x - 12 = 10x + 35$$

$$10x = 47, \text{ so } x = 4.7$$

$$\text{Length of the rectangle} = 2 \times 4.7 + 7 = 16.4 \text{ cm}$$

$$\text{Perimeter} = 2 \times 16.4 + 2 \times 5 = 42.8 \text{ cm}$$

Answer: 42.8 cm

Tip: Use the **perpendicular** height and remember the $\frac{1}{2}$ for the triangle. After finding x , read the question again: it asks for the perimeter, not x .

Marks: 1 mark for the area of the triangle in terms of x . 1 mark for an equation with the two areas. 1 mark for $x = 4.7$. 1 mark for 42.8.

9 (1 mark)

Qasim simplifies $4(x^2 + 2x - 3) - 3(x^2 - x + 2)$

His working is shown below.

$$\begin{aligned}4x^2 + 8x - 12 - 3x^2 + 3x + 6 \\= x^2 + 11x - 6\end{aligned}$$

His final expression is not correct.

Explain the mistake that Qasim has made.

Look at each term in Qasim's first line of working.

$$4(x^2 + 2x - 3) = 4x^2 + 8x - 12 \quad (\text{correct})$$

$$-3(x^2 - x + 2) = -3x^2 + 3x - 6, \text{ because } -3 \times +2 = -6$$

Qasim has written +6 instead of -6.

The correct answer is $x^2 + 11x - 18$.

Answer: He worked out -3×2 as +6. It should be -6 (so the answer should be $x^2 + 11x - 18$).

Tip: A minus sign in front of a bracket changes the sign of **every** term inside it. Check each term, including the last one.

10 (2 marks)

7 identical printing presses can print all the copies of a newspaper in 6 hours.

The newspaper wants all the copies printed in 4 hours 50 minutes.

All the presses work at the same rate.

Work out the least number of these printing presses needed.

7 presses take 6 hours, so 1 press would take $7 \times 6 = 42$ hours.

$$4 \text{ hours } 50 \text{ minutes} = 4\frac{50}{60} = 4.833 \dots \text{ hours}$$

$$\text{Number of presses} = 42 \div 4.833 \dots = 8.68 \dots$$

8 presses would not be enough, so round **up**: 9 presses are needed.

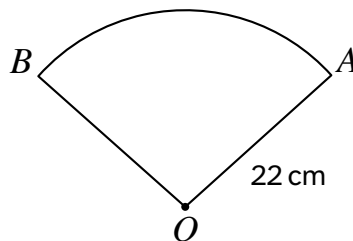
Answer: 9

Tip: 4 hours 50 minutes is not 4.5 hours. And you cannot have 8.68 presses: 8 would be too slow, so you need 9.

Marks: 1 mark for $7 \times 6 \div 4$ hours 50 minutes (8.68...). 1 mark for 9.

11 (3 marks)

The diagram shows a paper fan in the shape of a sector OAB of a circle, centre O .
The radius of the circle is 22 cm.
The perimeter of the fan is 81 cm.



*Diagram **NOT** accurately drawn*

Work out the size of angle AOB .
Give your answer correct to 1 decimal place.

Perimeter = arc AB + the two radii

$$\text{Arc } AB = 81 - 22 - 22 = 37 \text{ cm}$$

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r, \text{ so } \frac{\theta}{360} \times 2 \times \pi \times 22 = 37$$

$$\theta = \frac{37 \times 360}{2 \times \pi \times 22} = 96.361...$$

Answer: 96.4°

Tip: The perimeter of a sector includes the two straight edges. Subtract both radii before you use the arc length formula.

Marks: 1 mark for the arc length 37. 1 mark for a correct method for the angle using 37. 1 mark for 96.4.

12 (3 marks)

Solve the simultaneous equations

$$4x - 3y = 20$$

$$6x + 5y = 11$$

Make the coefficients of y the same size: multiply the first equation by 5 and the second by 3.

$$20x - 15y = 100$$

$$18x + 15y = 33$$

The signs of $15y$ are different, so add: $38x = 133$

$$x = 133 \div 38 = 3.5$$

Substitute into $4x - 3y = 20$: $14 - 3y = 20$

$$-3y = 6, \text{ so } y = -2$$

Check in $6x + 5y = 11$: $21 - 10 = 11$ ✓

Answer: $x = 3.5$, $y = -2$

Tip: When the signs of the matching terms are different, **add** the equations. Check your answers in the equation you did not substitute into.

Marks: 1 mark for making the coefficients of x or y the same and adding or subtracting. 1 mark for substituting to find the other value. 1 mark for both answers.

13 (2 marks)

x and y are integers.

$$-4 < 3x + 2 < 20$$

$$4 < y \leq 7$$

$$x + 2y = 13$$

Work out all the possible values of x .

Solve $-4 < 3x + 2 < 20$: subtract 2: $-6 < 3x < 18$, divide by 3: $-2 < x < 6$

So x could be $-1, 0, 1, 2, 3, 4$ or 5

$x + 2y = 13$, so $y = \frac{13 - x}{2}$. y is an integer, so $13 - x$ must be even, so x must be odd:
 $-1, 1, 3$ or 5

$x = -1$: $y = 7$ ✓ $x = 1$: $y = 6$ ✓ $x = 3$: $y = 5$ ✓ $x = 5$: $y = 4$ ✗ (y must be more than 4)

Answer: $-1, 1, 3$

Tip: Check every condition for each value. $4 < y$ means $y = 4$ is not allowed, but $y \leq 7$ means $y = 7$ is allowed. Do not forget the negative value.

Marks: 1 mark for $-2 < x < 6$ or a correct pair of values of x and y . 1 mark for $-1, 1$ and 3 only.

14 (3 marks)

Prove algebraically that $0.53\dot{6}$ can be written as $\frac{161}{300}$

$$\text{Let } x = 0.53\dot{6} = 0.53666 \dots$$

$$100x = 53.666 \dots$$

$$1000x = 536.666 \dots$$

$$\text{Subtract: } 1000x - 100x = 536.666 \dots - 53.666 \dots$$

$$900x = 483$$

$$x = \frac{483}{900} = \frac{161}{300} \text{ (dividing the top and bottom by 3)}$$

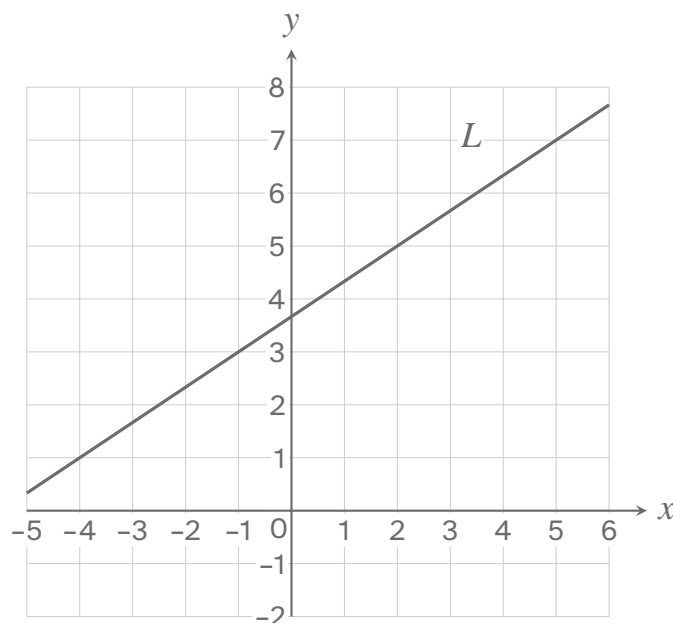
Answer: $1000x - 100x = 536.\dot{6} - 53.\dot{6}$, so $900x = 483$ and $x = \frac{483}{900} = \frac{161}{300}$

Tip: Choose two multiples of x whose decimal parts are identical (.666 ...) so that they cancel. Write the dots or "..." to show the decimals recur.

Marks: 1 mark for two multiples of x with the same recurring part (such as $100x$ and $1000x$). 1 mark for subtracting to get $900x = 483$. 1 mark for simplifying to $161/300$.

15 (3 marks)

Line L is shown on the grid.



Work out an equation for L .

Pick two points on L where it crosses the grid exactly: $(-4, 1)$ and $(2, 5)$.

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{5 - 1}{2 - (-4)} = \frac{4}{6} = \frac{2}{3}$$

$$y = \frac{2}{3}x + c. \text{ Substitute } (2, 5): 5 = \frac{4}{3} + c, \text{ so } c = \frac{11}{3}$$

$$y = \frac{2}{3}x + \frac{11}{3} \text{ (or } 3y = 2x + 11)$$

Check with $(-1, 3)$: $\frac{2}{3} \times (-1) + \frac{11}{3} = \frac{9}{3} = 3$ ✓

Answer: $y = \frac{2}{3}x + \frac{11}{3}$ (or $3y = 2x + 11$)

Tip: L does not cross the y -axis at a whole number, so you cannot read off c . Use a point that is exactly on the line to work it out.

Marks: 1 mark for the gradient $2/3$. 1 mark for substituting a point into $y = (\text{their gradient})x + c$.
1 mark for a correct equation.

16 (4 marks)

Make p the subject of the formula $m = \frac{\sqrt{p} + 5}{2\sqrt{p} - 3}$

Multiply both sides by $(2\sqrt{p} - 3)$: $m(2\sqrt{p} - 3) = \sqrt{p} + 5$

Expand: $2m\sqrt{p} - 3m = \sqrt{p} + 5$

Collect the $\sqrt{p}$ terms on one side: $2m\sqrt{p} - \sqrt{p} = 3m + 5$

Factorise: $\sqrt{p}(2m - 1) = 3m + 5$

Divide: $\sqrt{p} = \frac{3m + 5}{2m - 1}$

Square both sides: $p = \left(\frac{3m + 5}{2m - 1}\right)^2$

Answer: $p = \left(\frac{3m + 5}{2m - 1}\right)^2$

Tip: Treat $\sqrt{p}$ as a single letter until the very end. Square only when $\sqrt{p}$ is on its own.

Marks: 1 mark for multiplying out the fraction. 1 mark for getting the $\sqrt{p}$ terms on one side and the other terms on the other. 1 mark for factorising. 1 mark for the final answer.

17 (4 marks)

Chloe's grandmother invested some money in a savings bond for Chloe.

The bond paid 3.8% per year compound interest.

At the end of 3 years, the bond was worth £5368.26

Chloe says,

“The bond earned more than £575 in interest.”

Is Chloe correct?

You must show all your working.

An increase of 3.8% a year is a multiplier of 1.038 each year.

After 3 years: original amount $\times 1.038^3 = 5368.26$

Original amount = $5368.26 \div 1.038^3 = 5368.26 \div 1.11838 \dots = £4800.00$

Interest = $5368.26 - 4800 = £568.26$

£568.26 is less than £575, so Chloe is **not** correct.

Answer: No. The bond started at £4800, so the interest was £568.26, which is less than £575

Tip: To go back to the original amount, **divide** by the multiplier. Taking off 3.8% three times ($\times 0.962^3$) does not get you back to the start.

Marks: 1 mark for the multiplier 1.038^3 . 1 mark for dividing 5368.26 by 1.038^3 . 1 mark for £4800 or £568.26. 1 mark for 'No' with £568.26.

18 (4 marks)

The diagram shows a shade sail ABC in the shape of a triangle.

$AB = 5.2$ m and $AC = 6.3$ m

The perimeter of the shade sail is 16.9 m.

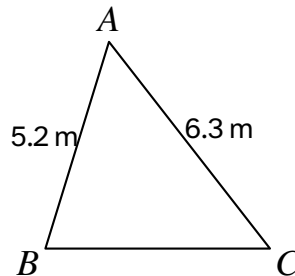


Diagram **NOT** accurately drawn

Work out the area of the shade sail.

Give your answer correct to 3 significant figures.

You must show all your working.

$$BC = 16.9 - 5.2 - 6.3 = 5.4 \text{ m}$$

Use the cosine rule to find angle B (between AB and BC , opposite AC):

$$\cos B = \frac{5.2^2 + 5.4^2 - 6.3^2}{2 \times 5.2 \times 5.4} = \frac{16.51}{56.16} = 0.29398 \dots$$

$$B = 72.90 \dots^\circ$$

$$\text{Area} = \frac{1}{2}ac \sin B = \frac{1}{2} \times 5.2 \times 5.4 \times \sin 72.90 \dots^\circ = 13.419 \dots \text{ m}^2$$

Answer: 13.4 m²

Tip: In $\frac{1}{2}ab \sin C$, the angle C must be the angle **between** the sides a and b . Keep the unrounded angle in your calculator.

Marks: 1 mark for $BC = 5.4$. 1 mark for using the cosine rule to find an angle. 1 mark for $\frac{1}{2}ab \sin C$ with the angle between the two sides. 1 mark for 13.4.

19 (4 marks)

A, B, C and D are points on a circle, centre O .

AC is a diameter of the circle.

DCE is a straight line.

Angle $BCE = 64^\circ$

angle BAC : angle $CAD = 5:3$

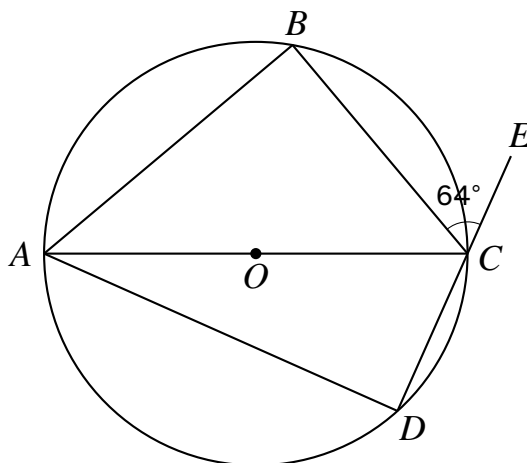


Diagram **NOT** accurately drawn

Work out the size of angle ACD .

Give a reason for each stage of your working.

Angle $BCD = 180^\circ - 64^\circ = 116^\circ$ (angles on a straight line add up to 180°)

Angle $BAD = 180^\circ - 116^\circ = 64^\circ$ (opposite angles of a cyclic quadrilateral add up to 180°)

Angle BAC : angle $CAD = 5:3$, so angle $CAD = \frac{3}{8} \times 64^\circ = 24^\circ$

Angle $ADC = 90^\circ$ (the angle in a semicircle is 90° , because AC is a diameter)

Angle $ACD = 180^\circ - 90^\circ - 24^\circ = 66^\circ$ (angles in a triangle add up to 180°)

Answer: 66°

Tip: Use the words of the theorem in your reasons, for example “opposite angles of a cyclic quadrilateral add up to 180° ”. Just writing “cyclic quadrilateral” is not enough.

Marks: 1 mark for angle $BAD = 64^\circ$ (or angle $BCD = 116^\circ$). 1 mark for angle $CAD = 24^\circ$. 1 mark for 66° . 1 mark for all the reasons.

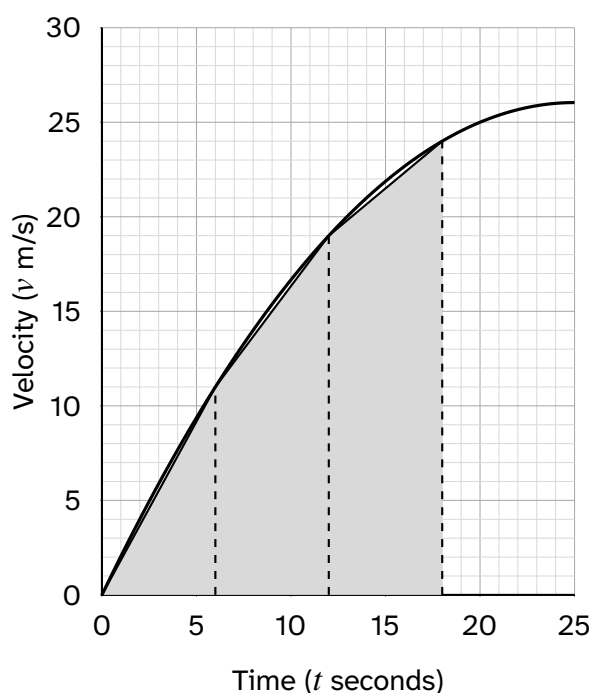
20 (4 marks)

Pedro rides a jet ski. He starts from rest. The graph in the question shows his velocity, v metres per second, t seconds after he starts.

- (a) Use 3 strips of equal width to work out an estimate for the distance Pedro travels in the first 18 seconds.

3 strips of equal width from 0 to 18 seconds: each strip is 6 seconds wide.

Read the velocities: at $t = 0$, $v = 0$; at $t = 6$, $v = 11$; at $t = 12$, $v = 19$; at $t = 18$, $v = 24$



Triangle: $\frac{1}{2} \times 6 \times 11 = 33$

Trapezium: $\frac{1}{2} \times (11 + 19) \times 6 = 90$

Trapezium: $\frac{1}{2} \times (19 + 24) \times 6 = 129$

Distance $\approx 33 + 90 + 129 = 252$ m

Answer: 252 m

Tip: Read the velocities at 6, 12 and 18 seconds carefully: one small square is 1 m/s. The first strip is a triangle because the jet ski starts from rest.

Marks: 1 mark for the area of one strip. 1 mark for adding the areas of all three strips. 1 mark for 252 (or a value from 244 to 260).

- (b) Is your answer to part (a) an underestimate or an overestimate of the actual distance?

Give a reason for your answer.

The curve bends so that it is above the straight tops of the strips.

So the strips leave out some of the area under the curve.

The estimate is an **underestimate**.

Answer: Underestimate, because the straight tops of the strips are below the curve, so the strips miss some of the area under it.

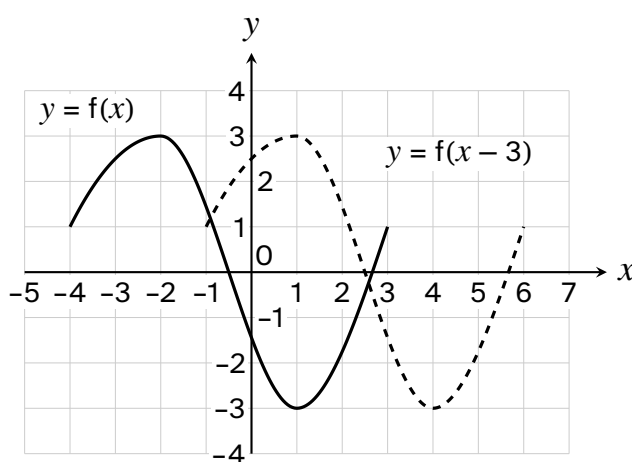
21 (2 marks)

The graph of $y = f(x)$ is drawn on the grid in the question.

- (a) On the same grid, sketch the graph of $y = f(x - 3)$

$y = f(x - 3)$ is the graph of $y = f(x)$ translated 3 units to the **right**.

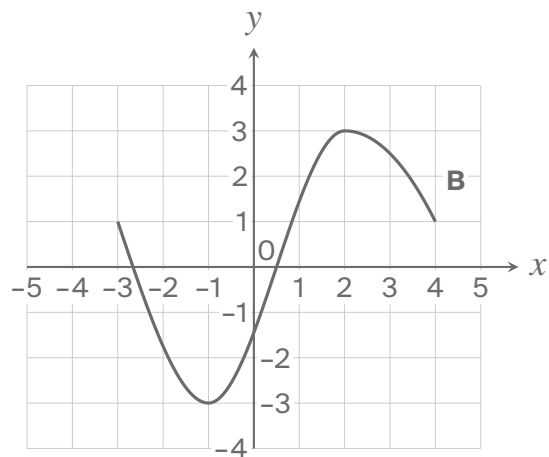
Each point moves 3 to the right: $(-4, 1) \rightarrow (-1, 1)$, $(-2, 3) \rightarrow (1, 3)$, $(1, -3) \rightarrow (4, -3)$, $(3, 1) \rightarrow (6, 1)$



Answer: The curve translated 3 units to the right: maximum at $(1, 3)$, minimum at $(4, -3)$, through $(-1, 1)$ and $(6, 1)$

Tip: $f(x - 3)$ moves the graph 3 to the **right**, not to the left. A change inside the bracket does the opposite of what the sign suggests.

- (b) Graph **B**, shown on the grid below, is the image of the graph of $y = f(x)$ after a reflection.



Write down an equation of graph **B**.

Graph **B** is the graph of $y = f(x)$ reflected in the y -axis: the maximum $(-2, 3)$ has moved to $(2, 3)$ and the minimum $(1, -3)$ to $(-1, -3)$.

Each x -coordinate changes sign and each y -coordinate stays the same.

A reflection in the y -axis is $y = f(-x)$.

Answer: $y = f(-x)$

Tip: $y = -f(x)$ is a reflection in the x -axis (the graph turns upside down). $y = f(-x)$ is a reflection in the y -axis.

22 (5 marks)

$OABC$ is a trapezium.

$$\overrightarrow{OA} = 3\mathbf{a} \quad \overrightarrow{CB} = 2\mathbf{a} \quad \overrightarrow{OC} = 2\mathbf{b}$$

M is the point on OA such that $OM:MA = 2:1$

N is the point on AB such that $AN:NB = 1:2$

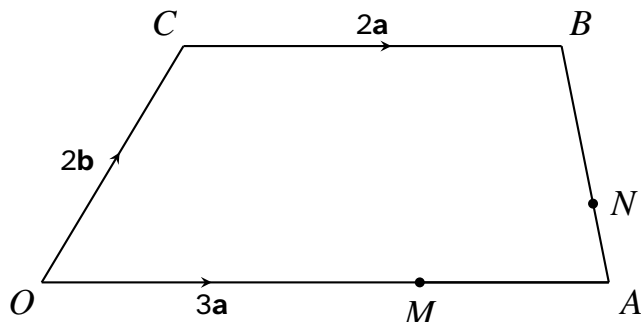


Diagram **NOT** accurately drawn

Use a vector method to prove that MN is parallel to OB .

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CB} = -3\mathbf{a} + 2\mathbf{b} + 2\mathbf{a} = -\mathbf{a} + 2\mathbf{b}$$

$$AN:NB = 1:2, \text{ so } \overrightarrow{AN} = \frac{1}{3}\overrightarrow{AB} = -\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$$

$$OM:MA = 2:1, \text{ so } \overrightarrow{MA} = \frac{1}{3} \times 3\mathbf{a} = \mathbf{a}$$

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN} = \mathbf{a} - \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = \frac{2}{3}(\mathbf{a} + \mathbf{b})$$

$$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB} = 2\mathbf{b} + 2\mathbf{a} = 2(\mathbf{a} + \mathbf{b})$$

$$\text{So } \overrightarrow{MN} = \frac{1}{3}\overrightarrow{OB}$$

$\overrightarrow{MN}$ is a multiple of $\overrightarrow{OB}$, so MN is parallel to OB .

Answer: $\overrightarrow{MN} = \frac{2}{3}(\mathbf{a} + \mathbf{b})$ and $\overrightarrow{OB} = 2(\mathbf{a} + \mathbf{b})$, so $\overrightarrow{MN} = \frac{1}{3}\overrightarrow{OB}$ and MN is parallel to OB .

Tip: To prove two lines are parallel, show that one vector is a number times the other. Take out a common factor, such as $(\mathbf{a} + \mathbf{b})$, to make this clear.

Marks: 1 mark for $\overrightarrow{AB}$. 1 mark for $\overrightarrow{AN}$ (or $\overrightarrow{MA} = \mathbf{a}$ with a route to N). 1 mark for a correct route for $\overrightarrow{MN}$. 1 mark for $\overrightarrow{MN} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$. 1 mark for comparing with $\overrightarrow{OB} = 2\mathbf{a} + 2\mathbf{b}$ and concluding.

23 (3 marks)

Prove algebraically that the product of any two consecutive even numbers is always 1 less than an odd square number.

Let the two consecutive even numbers be $2n$ and $2n + 2$, where n is an integer.

Product: $2n(2n + 2) = 4n^2 + 4n$

$$(2n + 1)^2 = 4n^2 + 4n + 1, \text{ so } 4n^2 + 4n = (2n + 1)^2 - 1$$

$2n$ is even, so $2n + 1$ is odd, and $(2n + 1)^2$ is an odd square number.

So the product is always 1 less than an odd square number.

Answer: $2n(2n + 2) = 4n^2 + 4n = (2n + 1)^2 - 1$, and $2n + 1$ is odd, so the product is 1 less than an odd square number.

Tip: Consecutive even numbers go up in 2s: $2n$ and $2n + 2$, not n and $n + 1$. Checking a few numbers ($4 \times 6 = 24 = 25 - 1$) is not a proof.

Marks: 1 mark for $2n$ and $2n + 2$. 1 mark for $4n^2 + 4n$. 1 mark for writing it as $(2n + 1)^2 - 1$ with a conclusion that $(2n + 1)^2$ is an odd square.

24 (5 marks)

A florist has a bucket that contains only red tulips and yellow tulips.

There are 7 yellow tulips in the bucket.

Mohammed takes at random two tulips from the bucket.

The probability that the two tulips are the same colour is $\frac{7}{15}$

Work out the number of red tulips in the bucket.

You must show all your working.

Let n be the number of red tulips. There are $n + 7$ tulips in total.

$$P(\text{both red}) = \frac{n}{n+7} \times \frac{n-1}{n+6} \quad P(\text{both yellow}) = \frac{7}{n+7} \times \frac{6}{n+6}$$

$$P(\text{same colour}) = \frac{n(n-1) + 42}{(n+7)(n+6)} = \frac{7}{15}$$

$$\text{Cross-multiply: } 15(n^2 - n + 42) = 7(n^2 + 13n + 42)$$

$$15n^2 - 15n + 630 = 7n^2 + 91n + 294$$

$$8n^2 - 106n + 336 = 0, \text{ and dividing by 2: } 4n^2 - 53n + 168 = 0$$

Factorise: $(4n - 21)(n - 8) = 0$, so $n = 5.25$ or $n = 8$

The number of tulips must be a whole number, so $n = 8$.

$$\text{Check: } \frac{8}{15} \times \frac{7}{14} + \frac{7}{15} \times \frac{6}{14} = \frac{56+42}{210} = \frac{98}{210} = \frac{7}{15} \quad \checkmark$$

Answer: 8

Tip: The tulips are not put back, so the second fraction has one fewer on the top **and** the bottom. "Same colour" means both red **or** both yellow: add the two probabilities.

Marks: 1 mark for the probability of two of the same colour (both red or both yellow) in terms of n . 1 mark for a correct equation for $P(\text{same colour})$. 1 mark for a 3-term quadratic. 1 mark for solving it. 1 mark for 8.
