

Mock Set B – Paper 1

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1)

Higher tier • Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

(a) Work out $4\frac{1}{6} - 1\frac{3}{4}$

Give your answer as a mixed number.

Write both as improper fractions: $4\frac{1}{6} = \frac{25}{6}$ and $1\frac{3}{4} = \frac{7}{4}$

The lowest common denominator of 6 and 4 is 12.

$$\frac{25}{6} = \frac{50}{12} \quad \text{and} \quad \frac{7}{4} = \frac{21}{12}$$

$$\frac{50}{12} - \frac{21}{12} = \frac{29}{12}$$

$$29 \div 12 = 2 \text{ remainder } 5, \text{ so } \frac{29}{12} = 2\frac{5}{12}$$

Answer: $2\frac{5}{12}$

Tip: If you subtract the whole numbers and the fractions separately, $\frac{1}{6} - \frac{3}{4}$ is negative:
 $3 - \frac{7}{12} = 2\frac{5}{12}$. Converting to improper fractions first avoids this.

Marks: 1 mark for both fractions written over a common denominator, such as $\frac{50}{12}$ and $\frac{21}{12}$.
1 mark for $2\frac{5}{12}$.

(b) Show that $2\frac{5}{8} \div 1\frac{2}{5} = 1\frac{7}{8}$

Write both as improper fractions: $2\frac{5}{8} = \frac{21}{8}$ and $1\frac{2}{5} = \frac{7}{5}$

To divide, multiply by the reciprocal of the second fraction: $\frac{21}{8} \div \frac{7}{5} = \frac{21}{8} \times \frac{5}{7}$

Cancel 21 and 7 (divide both by 7): $\frac{3}{8} \times \frac{5}{1} = \frac{15}{8}$

$$\frac{15}{8} = 1\frac{7}{8} \quad \checkmark$$

Answer: $\frac{21}{8} \times \frac{5}{7} = \frac{105}{56} = \frac{15}{8} = 1\frac{7}{8}$

Tip: In a “show that” question, every step must be written down, including the simplifying: $\frac{105}{56} = \frac{15}{8} = 1\frac{7}{8}$.

Marks: 1 mark for $\frac{21}{8}$ and $\frac{7}{5}$ with the second fraction turned upside down and multiplied. 1 mark for simplifying to $\frac{15}{8}$ and writing it as $1\frac{7}{8}$.

2 (2 marks)

Write 980 as a product of powers of its prime factors.

Divide by the smallest primes in turn.

$$980 \div 2 = 490$$

$$490 \div 2 = 245$$

$$245 \div 5 = 49$$

$$49 \div 7 = 7, \text{ and } 7 \div 7 = 1$$

$$\text{So } 980 = 2 \times 2 \times 5 \times 7 \times 7$$

With powers: $980 = 2^2 \times 5 \times 7^2$

Answer: $2^2 \times 5 \times 7^2$

Tip: Keep dividing until every factor is prime: 49 is not prime (7×7). The question asks for powers, so write 2^2 and 7^2 .

Marks: 1 mark for a correct start, such as $980 = 2 \times 490$ and $490 = 2 \times 245$. 1 mark for $2^2 \times 5 \times 7^2$.

3 (3 marks)

Here are the first five terms of an arithmetic sequence.

47 41 35 29 23

- (a) Find an expression, in terms of n , for the n th term of this sequence.

The terms go down by 6 each time, so the n th term contains $-6n$.

$-6n$ gives $-6, -12, -18, \dots$

To get from -6 to the first term, 47, add 53.

n th term = $53 - 6n$

Check: $n = 3$ gives $53 - 18 = 35$ ✓

Answer: $53 - 6n$

Tip: The sequence is going down, so the number in front of n is negative. Writing $6n + 53$ or $6n - 41$ is a common mistake.

Marks: 1 mark for $-6n$ in an expression. 1 mark for $53 - 6n$.

- (b) Musa says that -100 is a term in this sequence. Is Musa correct?

Set the n th term equal to -100 : $53 - 6n = -100$

$6n = 153$

$n = 153 \div 6 = 25.5$

n must be a whole number, so -100 is **not** a term.

(The terms near -100 are the 25th term, -97 , and the 26th term, -103 .)

Answer: No. $53 - 6n = -100$ gives $n = 25.5$, which is not a whole number.

Tip: Show the calculation. Saying “No, because it is negative” is not a reason: lots of the terms are negative.

4 (4 marks)

A biased spinner lands on A with probability 0.3 and on B with probability 0.25. P(C) is four times P(D).

- (a) Work out the probability that the spinner will land on D.

The four probabilities add up to 1.

$$P(C) + P(D) = 1 - 0.3 - 0.25 = 0.45$$

P(C) is four times P(D), so share 0.45 in the ratio 4:1: that is 5 equal parts.

$$P(D) = 0.45 \div 5 = 0.09$$

$$P(C) = 4 \times 0.09 = 0.36$$

Letter	A	B	C	D
Probability	0.3	0.25	0.36	0.09

Answer: 0.09

Tip: “Four times” means 4 parts and 1 part: divide by 5, not by 4.

Marks: 1 mark for $1 - 0.3 - 0.25 = 0.45$. 1 mark for 0.09.

- (b) Malik spins the spinner 200 times.
Work out an estimate for how many more times the spinner will land on C than on D.

$$P(C) = 4 \times 0.09 = 0.36$$

$$\text{Expected number of C: } 0.36 \times 200 = 72$$

$$\text{Expected number of D: } 0.09 \times 200 = 18$$

$$72 - 18 = 54$$

$$(Or: (0.36 - 0.09) \times 200 = 0.27 \times 200 = 54.)$$

Answer: 54

Tip: An estimate of the number of times = probability $\times$ number of spins. Then find the difference.

Marks: 1 mark for 0.36×200 or 0.09×200 (72 or 18). 1 mark for 54.

5 (3 marks)

A recipe for 15 flapjacks uses 225 g oats, 150 g butter, 90 g sugar and 105 g syrup. Darius has 0.9 kg oats, 650 g butter, 450 g sugar and 500 g syrup. Work out the greatest number of flapjacks he can make.

Change 0.9 kg to grams: $0.9 \times 1000 = 900$ g of oats.

Work out how many batches of 15 each ingredient is enough for.

Oats: $900 \div 225 = 4$ batches, so $4 \times 15 = 60$ flapjacks.

Butter: $650 \div 150 = 4\frac{1}{3}$ batches, so 65 flapjacks.

Sugar: $450 \div 90 = 5$ batches, so 75 flapjacks.

Syrup: $500 \div 105$ is more than 4 batches ($4 \times 105 = 420$), so more than 60 flapjacks.

The oats run out first, so the greatest number is 60.

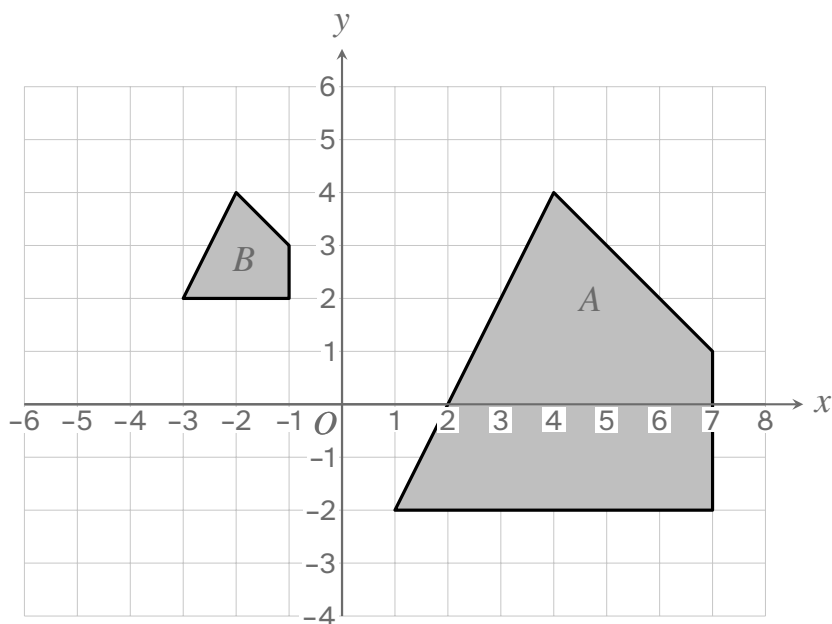
Answer: 60

Tip: Check the units first: 0.9 kg is 900 g. The ingredient that makes the **fewest** flapjacks decides the answer.

Marks: 1 mark for working out the number of flapjacks for one ingredient. 1 mark for doing this for all four ingredients (or checking that there is enough of the others for 60). 1 mark for 60.

6 (2 marks)

Shapes A and B are drawn on the grid.

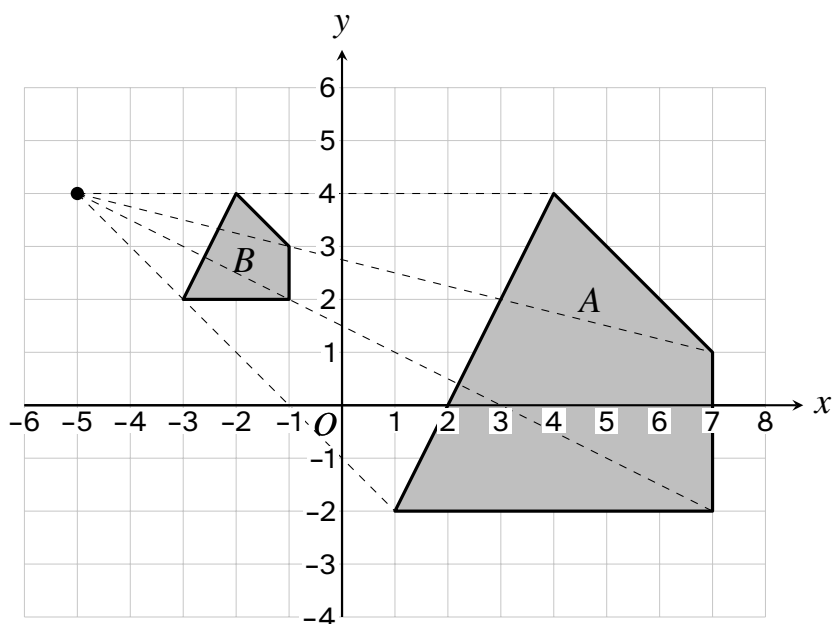


Describe fully the single transformation that maps shape A onto shape B .

Shape B is the same shape as A but smaller, so it is an enlargement.

Each side of B is $\frac{1}{3}$ of the length of the matching side of A (eg the bottom edge: 2 squares instead of 6), so the scale factor is $\frac{1}{3}$.

Draw lines through matching corners. They meet at the centre of enlargement, $(-5, 4)$.



Enlargement, scale factor $\frac{1}{3}$, centre $(-5, 4)$

Answer: Enlargement, scale factor $\frac{1}{3}$, centre $(-5, 4)$

Tip: Describe it fully: the word enlargement, the scale factor and the centre. A scale factor between 0 and 1 makes the shape smaller; it is not -3 or 3 .

Marks: 2 marks for enlargement, scale factor $\frac{1}{3}$ and centre $(-5, 4)$. 1 mark for an enlargement with the correct scale factor or the correct centre.

7 (5 marks)

A craft kit has red, white and blue beads. red : white = 4: 9 and red : blue = 6: 5.

- (a) Find the ratio number of red beads : number of white beads : number of blue beads

Red appears in both ratios, so make the red numbers the same.

The lowest common multiple of 4 and 6 is 12.

red : white = 4: 9 = 12: 27 (multiply by 3)

red : blue = 6: 5 = 12: 10 (multiply by 2)

So red : white : blue = 12: 27: 10

Answer: 12: 27: 10

Tip: Do not just write the numbers next to each other (4: 9: 5). The red parts must be equal before you can combine the ratios.

Marks: 1 mark for making the red parts equal, such as 12: 27 and 12: 10. 1 mark for 12: 27: 10.

- (b) In one craft kit there are 85 more white beads than blue beads.
Work out the total number of beads in this craft kit.

White – blue = 27 – 10 = 17 parts.

17 parts = 85 beads, so 1 part = $85 \div 17 = 5$ beads.

Total number of parts = 12 + 27 + 10 = 49

Total number of beads = $49 \times 5 = 245$

Check: red 60, white 135, blue 50; $135 - 50 = 85$ ✓

Answer: 245

Tip: The 85 beads match the **difference** between the white and blue parts (17 parts), not the total.

Marks: 1 mark for $27 - 10 = 17$ parts. 1 mark for $85 \div 17 = 5$. 1 mark for 245.

8 (4 marks)

PEQ and $RFGS$ are parallel straight lines.

EFG is a triangle.

All angles are measured in degrees.

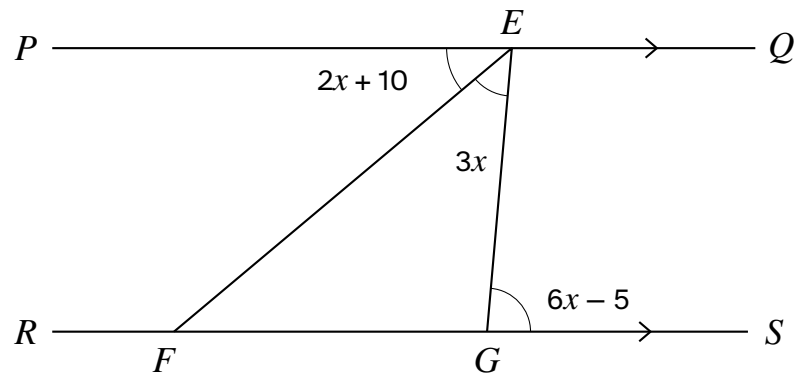


Diagram **NOT** accurately drawn

Work out the size of angle EGF .

You must give a reason for each stage of your working.

Angle $EFG = 2x + 10$ (alternate angles are equal).

Angle $EGF = 180 - (6x - 5) = 185 - 6x$ (angles on a straight line add up to 180°).

Angles in triangle EFG add up to 180° : $(2x + 10) + 3x + (185 - 6x) = 180$

$195 - x = 180$, so $x = 15$

Angle $EGF = 185 - 6 \times 15 = 185 - 90 = 95^\circ$

Check: $EFG = 40^\circ$, $FEG = 45^\circ$, $EGF = 95^\circ$, and $40 + 45 + 95 = 180$ ✓

Another way: angle $QEG = 180 - (2x + 10) - 3x = 170 - 5x$ (angles on a straight line), and $QEG + EGS = 180$ (co-interior angles), so $170 - 5x + 6x - 5 = 180$.

Or: angle EGS is an exterior angle of triangle EFG , so $6x - 5 = (2x + 10) + 3x$ (an exterior angle of a triangle is equal to the sum of the two interior opposite angles).

Answer: 95°

Tip: Finding $x = 15$ is not the end: the question asks for angle EGF . Use the full names of the angle facts: “alternate angles are equal”, not “Z angles”.

Marks: 1 mark for one angle written in terms of x , such as angle $EFG = 2x + 10$. 1 mark for a correct equation in x . 1 mark for 95° . 1 mark for correct reasons.

9 (3 marks)

A bookcase stands on the horizontal floor of a library.
The base of the bookcase is a rectangle 1.1 m by 40 cm.
The bookcase exerts a force of 924 newtons on the floor.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the floor due to the bookcase, in newtons/m².

Write both lengths in metres: 40 cm = 0.4 m

Area of the base = $1.1 \times 0.4 = 0.44 \text{ m}^2$

$$\text{Pressure} = \frac{\text{force}}{\text{area}} = \frac{924}{0.44}$$

Multiply top and bottom by 100: $\frac{92\,400}{44} = 2100$

Answer: 2100 newtons/m²

Tip: Change 40 cm into metres before you multiply. Using $1.1 \times 40 = 44$ gives an answer 100 times too small.

Marks: 1 mark for the area in m² (0.44). 1 mark for $924 \div 0.44$. 1 mark for 2100.

10 (3 marks)

The mean mass of 12 pumpkins is 4.5 kg. The mean mass of the 4 heaviest is 7.2 kg. Leah says the mean mass of the other 8 is less than 3 kg. Is she correct?

Total mass of all 12 pumpkins = $12 \times 4.5 = 54$ kg

Total mass of the 4 heaviest = $4 \times 7.2 = 28.8$ kg

Total mass of the other 8 = $54 - 28.8 = 25.2$ kg

Mean of the other 8 = $25.2 \div 8 = 3.15$ kg

3.15 kg is **more** than 3 kg, so Leah is **not** correct.

Answer: No. The mean of the other 8 is $25.2 \div 8 = 3.15$ kg, which is more than 3 kg.

Tip: Work with totals: mean $\times$ number = total. You cannot subtract the means ($4.5 - 7.2$) or average them.

Marks: 1 mark for 54 or 28.8. 1 mark for $(54 - 28.8) \div 8$. 1 mark for “No” with 3.15 (or with 25.2 compared with 24).

11 (2 marks)

Gemma buys a bike in a sale.

All the prices in the sale are 35% less than the normal prices.

Gemma pays £286 for the bike.

Work out the normal price of the bike.

The sale price is $100\% - 35\% = 65\%$ of the normal price.

$65\% = £286$

$5\% = 286 \div 13 = £22$

$100\% = 22 \times 20 = £440$

Check: 35% of 440 is 154, and $440 - 154 = 286$ ✓

Answer: £440

Tip: £286 is 65%, not 100%. Adding 35% of £286 gives the wrong answer (£386.10).

Marks: 1 mark for using 286 as 65%, such as $286 \div 65 \times 100$. 1 mark for £440.

12 (5 marks)

The table gives information about the heights, h cm, of 120 saplings in a tree nursery.

Height (h cm)	Frequency
$0 < h \leq 10$	12
$10 < h \leq 25$	30
$25 < h \leq 40$	36
$40 < h \leq 70$	30
$70 < h \leq 90$	12

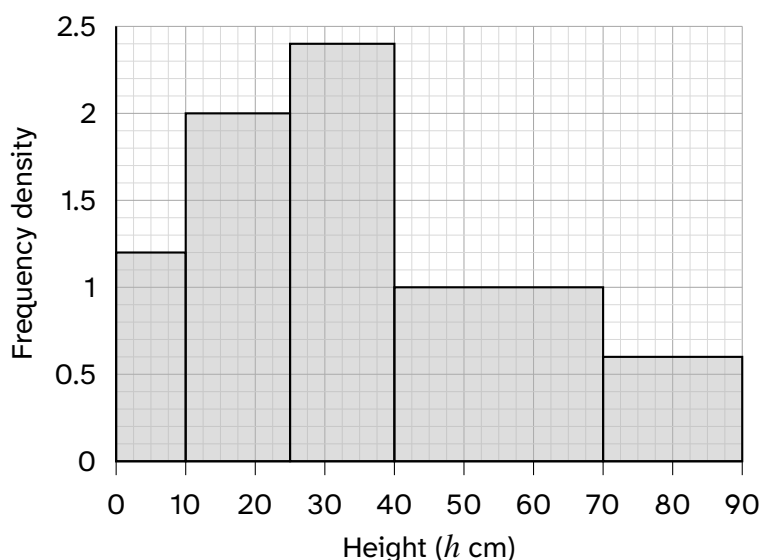
- (a) On the grid, draw a histogram for the information in the table.

Frequency density = frequency $\div$ class width.

Height (h cm)	Frequency	Class width	Frequency density
$0 < h \leq 10$	12	10	1.2
$10 < h \leq 25$	30	15	2
$25 < h \leq 40$	36	15	2.4
$40 < h \leq 70$	30	30	1
$70 < h \leq 90$	12	20	0.6

Choose a scale for the vertical axis: the largest density is 2.4, so go up to 2.5.

Draw each bar across its whole class, with no gaps.



Answer: Bars from 0-10 height 1.2, 10-25 height 2, 25-40 height 2.4, 40-70 height 1, 70-90 height 0.6

Tip: The heights are frequency densities, not frequencies. Label the vertical axis with a scale, and make each bar exactly as wide as its class.

Marks: 3 marks for a fully correct histogram. 2 marks for 4 correct bars. 1 mark for 2 correct bars of different widths, or for all the frequency densities.

- (b) Work out an estimate for the number of saplings with a height between 30 cm and 60 cm.

30 to 40 is 10 cm of the 15 cm wide class $25 < h \leq 40$: $\frac{10}{15} \times 36 = 24$

40 to 60 is 20 cm of the 30 cm wide class $40 < h \leq 70$: $\frac{20}{30} \times 30 = 20$

(Or use the bars: $2.4 \times 10 = 24$ and $1 \times 20 = 20$.)

Estimate = $24 + 20 = 44$

Answer: 44

Tip: Use the fraction of each class that is between 30 and 60. The 25 to 40 class only counts from 30 upwards.

Marks: 1 mark for 24 or 20. 1 mark for 44.

13 (3 marks)

- (a) Write down the value of $\left(\frac{2}{3}\right)^{-1}$

A power of -1 means the reciprocal: turn the fraction upside down.

$$\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$$

Answer: $\frac{3}{2}$

Tip: A negative power does not make the number negative: $\left(\frac{2}{3}\right)^{-1}$ is not $-\frac{2}{3}$.

- (b) Work out the value of $81^{-\frac{3}{4}}$

The negative power means the reciprocal: $81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}}$

The power $\frac{1}{4}$ means the fourth root: $\sqrt[4]{81} = 3$ (because $3 \times 3 \times 3 \times 3 = 81$)

$$81^{\frac{3}{4}} = 3^3 = 27$$

$$\text{So } 81^{-\frac{3}{4}} = \frac{1}{27}$$

Answer: $\frac{1}{27}$

Tip: Do the root first ($\sqrt[4]{81} = 3$), then the power ($3^3 = 27$). Then the minus sign turns it into $\frac{1}{27}$.

Marks: 1 mark for the fourth root, 3, or for 27 or the reciprocal. 1 mark for $\frac{1}{27}$.

14 (2 marks)

14 members each play each of the other members exactly once. Sean says there will be 182 games. Is he correct?

Each of the 14 members plays 13 games: $14 \times 13 = 182$

But each game has two players, so this counts every game twice.

$$\text{Number of games} = 182 \div 2 = 91$$

(Or: $13 + 12 + 11 + \dots + 2 + 1 = 91$.)

So Sean is **not** correct: there will be 91 games.

Answer: No. $14 \times 13 = 182$ counts every game twice, so there are $182 \div 2 = 91$ games.

Tip: When A plays B, that is the same game as B playing A. Halve 14×13 .

Marks: 1 mark for $14 \times 13 \div 2$ or $13 + 12 + \dots + 1$. 1 mark for “No” with 91.

15 (4 marks)

(a) Factorise $5x^2 + 13x - 6$

$5x^2$ must come from $5x \times x$: $(5x \dots)(x \dots)$

The numbers must multiply to -6 and give $+13x$ in the middle.

Try $(5x - 2)(x + 3)$: $5x \times 3 = 15x$ and $-2 \times x = -2x$; $15x - 2x = 13x$ ✓

$$5x^2 + 13x - 6 = (5x - 2)(x + 3)$$

Answer: $(5x - 2)(x + 3)$

Tip: Expand your brackets to check. $(5x + 3)(x - 2)$ gives $-7x$, not $+13x$.

Marks: 1 mark for two brackets that give two of the three terms. 1 mark for $(5x - 2)(x + 3)$.

(b) Hence, solve $5x^2 + 13x - 6 > 0$

$(5x - 2)(x + 3) = 0$ when $x = \frac{2}{5}$ or $x = -3$ (the critical values).

The graph of $y = 5x^2 + 13x - 6$ is a U shape, so it is above the x -axis outside the critical values.

So $5x^2 + 13x - 6 > 0$ when $x < -3$ or $x > \frac{2}{5}$

Answer: $x < -3$ or $x > \frac{2}{5}$

Tip: Sketch the curve. It is a U shape, so the parts above the x -axis (> 0) are the two “outside” regions.

Marks: 1 mark for the critical values -3 and $\frac{2}{5}$. 1 mark for $x < -3$ or $x > \frac{2}{5}$.

16 (4 marks)

For $x > 0$, the functions f and g are such that

$$f(x) = \frac{18}{x+3} \qquad g(x) = 4x - 1$$

(a) Work out $gf(6)$

$gf(6)$ means put 6 into f first, then put the answer into g .

$$f(6) = \frac{18}{6+3} = \frac{18}{9} = 2$$

$$g(2) = 4 \times 2 - 1 = 7$$

Answer: 7

Tip: Work from the inside out: f first, then g . Doing g first gives $\frac{18}{26}$.

Marks: 1 mark for $f(6) = 2$. 1 mark for 7.

(b) Find $f^{-1}(x)$

Write $y = \frac{18}{x+3}$

Multiply by $(x+3)$: $y(x+3) = 18$

Divide by y : $x+3 = \frac{18}{y}$

Subtract 3: $x = \frac{18}{y} - 3$

Swap the letters: $f^{-1}(x) = \frac{18}{x} - 3$

Check: $f(6) = 2$ and $f^{-1}(2) = 9 - 3 = 6$ ✓

Answer: $f^{-1}(x) = \frac{18}{x} - 3$ (or $\frac{18-3x}{x}$)

Tip: The inverse is not the reciprocal $\frac{x+3}{18}$. Rearrange $y = \frac{18}{x+3}$ to make x the subject.

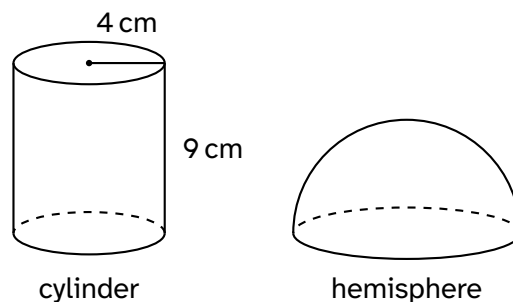
Marks: 1 mark for a correct first step, such as $y(x+3) = 18$. 1 mark for $\frac{18}{x} - 3$ (or $\frac{18-3x}{x}$).

17 (4 marks)

A solid cylinder has radius 4 cm and height 9 cm.

A solid hemisphere has the same volume as the cylinder.

Volume of a sphere = $\frac{4}{3}\pi r^3$
Surface area of a sphere = $4\pi r^2$



*Diagram **NOT** accurately drawn*

Work out the total surface area of the hemisphere.

Give your answer in terms of π .

You must show all your working.

Volume of the cylinder = $\pi r^2 h = \pi \times 4^2 \times 9 = 144\pi \text{ cm}^3$

Volume of a hemisphere of radius $r = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$

$\frac{2}{3}\pi r^3 = 144\pi$

$r^3 = 144 \times \frac{3}{2} = 216$

$$r = \sqrt[3]{216} = 6 \text{ cm}$$

The hemisphere has a curved surface (half a sphere) and a flat circular face.

$$\text{Curved surface} = \frac{1}{2} \times 4\pi \times 6^2 = 72\pi$$

$$\text{Flat face} = \pi \times 6^2 = 36\pi$$

$$\text{Total surface area} = 72\pi + 36\pi = 108\pi \text{ cm}^2$$

Answer: $108\pi \text{ cm}^2$

Tip: A solid hemisphere has two surfaces: the curved half-sphere **and** the flat circle. Leaving out the circle gives 72π .

Marks: 1 mark for 144π and setting it equal to $\frac{2}{3}\pi r^3$. 1 mark for $r = 6$. 1 mark for $\frac{1}{2} \times 4\pi \times 6^2 + \pi \times 6^2$. 1 mark for 108π .

18 (5 marks)

AB is parallel to DE . ACE and BCD are straight lines. $AB = 15 \text{ cm}$, $DE = 10 \text{ cm}$, $BD = 20 \text{ cm}$, $AE = 35 \text{ cm}$.

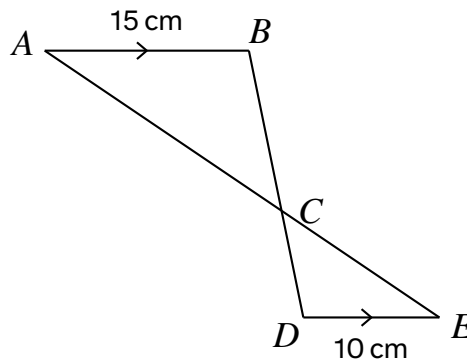


Diagram **NOT** accurately drawn

(a) Prove that triangle ABC is similar to triangle EDC .

Angle BAC = angle DEC (alternate angles are equal, because AB is parallel to DE).

Angle ABC = angle EDC (alternate angles are equal).

Angle ACB = angle ECD (vertically opposite angles are equal).

All three pairs of angles are equal, so triangles ABC and EDC are similar.

Answer: Angle BAC = angle DEC and angle ABC = angle EDC (alternate angles), angle ACB = angle ECD (vertically opposite), so the triangles are similar (AAA).

Tip: Give a reason for every pair of angles. Do not use SAS or ASA here: those prove congruence, and these triangles are not the same size.

Marks: 1 mark for one pair of equal angles with a correct reason. 1 mark for a complete proof: three pairs (or two pairs) of equal angles, all with reasons, and the conclusion.

- (b) $BD = 20$ cm $AE = 35$ cm

Work out the perimeter of triangle CDE .

You must show all your working.

The triangles are similar, so $CE:CA = CD:CB = DE:AB = 10:15 = 2:3$

AE is split at C in the ratio 3:2: $CE = \frac{2}{5} \times 35 = 14$ cm

BD is split at C in the ratio 3:2: $CD = \frac{2}{5} \times 20 = 8$ cm

Perimeter of triangle $CDE = 8 + 14 + 10 = 32$ cm

Answer: 32 cm

Tip: 35 cm is the **whole** line AE . It is split 3:2 (into 21 and 14), so CE is $\frac{2}{5}$ of 35, not $\frac{2}{3}$ of 35.

Marks: 1 mark for using the ratio 2:3 (or $\frac{2}{5}$) to split AE or BD . 1 mark for $CE = 14$ and $CD = 8$. 1 mark for 32.

19 (4 marks)

- (a) Show that $(2\sqrt{3} - 1)^2$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

$$(2\sqrt{3} - 1)^2 = (2\sqrt{3} - 1)(2\sqrt{3} - 1)$$

$$= 2\sqrt{3} \times 2\sqrt{3} - 2\sqrt{3} - 2\sqrt{3} + 1$$

$$2\sqrt{3} \times 2\sqrt{3} = 4 \times 3 = 12$$

$$= 12 - 4\sqrt{3} + 1$$

$$= 13 - 4\sqrt{3}, \text{ so } a = 13 \text{ and } b = -4$$

Answer: $12 - 2\sqrt{3} - 2\sqrt{3} + 1 = 13 - 4\sqrt{3}$

Tip: $(2\sqrt{3})^2$ is $4 \times 3 = 12$, not 6 or 2×3 . And there are two middle terms: $-2\sqrt{3}$ twice.

Marks: 1 mark for expanding with at least 3 of the 4 terms correct. 1 mark for $13 - 4\sqrt{3}$.

- (b) Write $\sqrt{63} + \sqrt{28}$ in the form $k\sqrt{7}$, where k is an integer.

$$63 = 9 \times 7, \text{ so } \sqrt{63} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$$

$$28 = 4 \times 7, \text{ so } \sqrt{28} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$$

$$3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$$

Answer: $5\sqrt{7}$

Tip: $\sqrt{63} + \sqrt{28}$ is not $\sqrt{91}$. Simplify each surd first, then add like terms.

Marks: 1 mark for $3\sqrt{7}$ or $2\sqrt{7}$. 1 mark for $5\sqrt{7}$.

20 (3 marks)

The graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$ is given.

- (a) Heidi says that $\cos 100^\circ$ is greater than $\cos 260^\circ$. Is Heidi correct?

The graph of $y = \cos x^\circ$ is symmetrical about the line $x = 180$.

100 is 80 less than 180, and 260 is 80 more than 180.

So the graph is at the same height at $x = 100$ and $x = 260$: $\cos 100^\circ = \cos 260^\circ$ (both are about -0.17).

Heidi is **not** correct: they are equal.

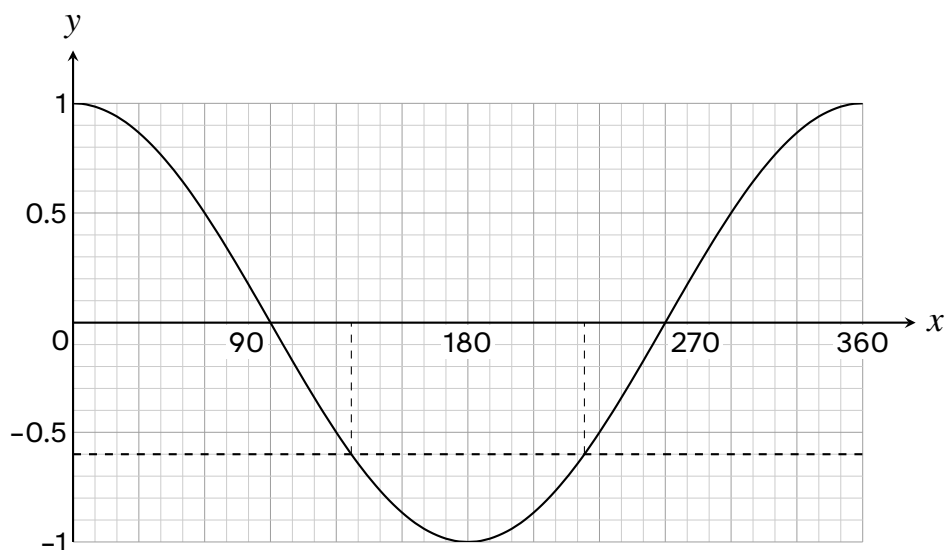
Answer: No. The graph is symmetrical about $x = 180$, so $\cos 100^\circ = \cos 260^\circ$ (both about -0.17).

Tip: Use the symmetry of the graph: 100 and 260 are the same distance either side of 180.

- (b) Use the graph to find estimates of the solutions of $5 \cos x^\circ + 3 = 0$ for $0 \leq x \leq 360$

Rearrange: $5 \cos x^\circ = -3$, so $\cos x^\circ = -0.6$

Draw the line $y = -0.6$ and read off the x -values where it crosses the curve.



$x \approx 127$ and $x \approx 233$

Check with symmetry: the graph is symmetrical about $x = 180$, and $127 + 233 = 360$ ✓

Answer: $x \approx 127$ and $x \approx 233$

Tip: Rearrange first so the left-hand side is $\cos x^\circ$. Then there are two solutions: one each side of 180.

Marks: 1 mark for $\cos x^\circ = -0.6$ (or the line $y = -0.6$) or one correct solution. 1 mark for both solutions.

21 (3 marks)

The first three terms of a sequence are $1 + \sqrt{2}$, $1 + 2\sqrt{2}$, $-1 + 5\sqrt{2}$. Show that this is a geometric sequence.

A sequence is geometric if each term is the term before multiplied by the same number (the common ratio).

Ratio of the first two terms: $\frac{1 + 2\sqrt{2}}{1 + \sqrt{2}}$

Rationalise: multiply the top and bottom by $\sqrt{2} - 1$.

$$\text{Top: } (1 + 2\sqrt{2})(\sqrt{2} - 1) = \sqrt{2} - 1 + 4 - 2\sqrt{2} = 3 - \sqrt{2}$$

$$\text{Bottom: } (1 + \sqrt{2})(\sqrt{2} - 1) = 2 - 1 = 1$$

So the ratio is $3 - \sqrt{2}$.

$$\text{Multiply the second term by this ratio: } (1 + 2\sqrt{2})(3 - \sqrt{2}) = 3 - \sqrt{2} + 6\sqrt{2} - 4 = -1 + 5\sqrt{2}$$

This is the third term, so each term is $(3 - \sqrt{2})$ times the one before: the sequence is geometric.

Another way: in a geometric sequence, $(2\text{nd term})^2 = 1\text{st term} \times 3\text{rd term}$.

$$(1 + 2\sqrt{2})^2 = 9 + 4\sqrt{2} \quad \text{and} \quad (1 + \sqrt{2})(-1 + 5\sqrt{2}) = -1 + 5\sqrt{2} - \sqrt{2} + 10 = 9 + 4\sqrt{2} \quad \checkmark$$

Answer: Common ratio $\frac{1 + 2\sqrt{2}}{1 + \sqrt{2}} = 3 - \sqrt{2}$, and $(1 + 2\sqrt{2})(3 - \sqrt{2}) = -1 + 5\sqrt{2}$, the third term, so the ratio is the same each time.

Tip: The terms look as if they go up by $\sqrt{2}$, but the third term is not $1 + 3\sqrt{2}$. To show a sequence is geometric, show that the ratio between terms stays the same.

Marks: 1 mark for the ratio of two terms written as a fraction, such as $\frac{1+2\sqrt{2}}{1+\sqrt{2}}$. 1 mark for rationalising to get $3 - \sqrt{2}$. 1 mark for showing that the same ratio gives the third term, with a conclusion.

22 (3 marks)

The diagram shows triangle ABC .

D is the point on BC such that AD is perpendicular to BC .

$AB = 8$ cm angle $ABC = 60^\circ$ angle $ACB = 45^\circ$

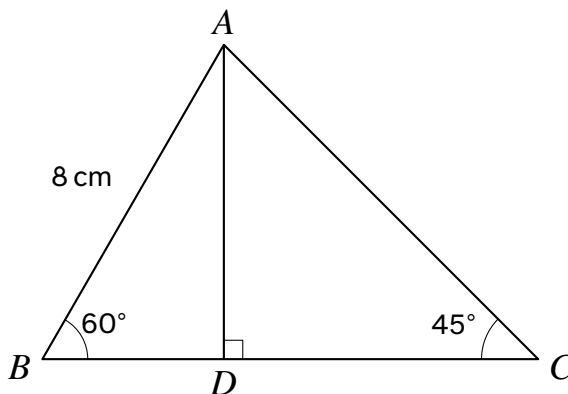


Diagram **NOT** accurately drawn

Show that the length of BC can be written as $4(1 + \sqrt{3})$ cm

In triangle ABD : $BD = 8 \cos 60^\circ = 8 \times \frac{1}{2} = 4$ cm

$$AD = 8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \text{ cm}$$

In triangle ADC : angle $DAC = 180 - 90 - 45 = 45^\circ$, so the triangle is isosceles and $DC = AD = 4\sqrt{3}$ cm.

$$\text{(Or: } DC = \frac{AD}{\tan 45^\circ} = \frac{4\sqrt{3}}{1} = 4\sqrt{3}.)$$

$$BC = BD + DC = 4 + 4\sqrt{3} = 4(1 + \sqrt{3}) \text{ cm}$$

Answer: $BD = 8 \cos 60^\circ = 4$, $AD = 8 \sin 60^\circ = 4\sqrt{3}$, $DC = AD = 4\sqrt{3}$ ($\tan 45^\circ = 1$),
so $BC = 4 + 4\sqrt{3} = 4(1 + \sqrt{3})$

Tip: Learn the exact values: $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 45^\circ = 1$. A calculator answer like 10.93 does not show the result.

Marks: 1 mark for $BD = 4$ or $AD = 4\sqrt{3}$ using exact values. 1 mark for $DC = 4\sqrt{3}$ (from $\tan 45^\circ = 1$ or isosceles). 1 mark for showing $BC = 4 + 4\sqrt{3} = 4(1 + \sqrt{3})$.

23 (5 marks)

The circle **C** has equation $x^2 + y^2 = 130$

The line **L** has equation $x - 2y = 5$

L intersects **C** at the points **A** and **B**.

O is the origin.

Work out the area of triangle **OAB**.

You must show all your working.

From the line: $x = 2y + 5$

Substitute into the circle: $(2y + 5)^2 + y^2 = 130$

$$4y^2 + 20y + 25 + y^2 = 130$$

$$5y^2 + 20y - 105 = 0$$

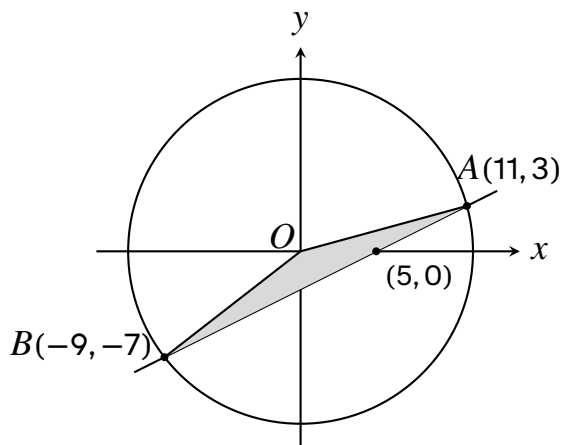
$$\text{Divide by 5: } y^2 + 4y - 21 = 0$$

$$(y + 7)(y - 3) = 0, \text{ so } y = 3 \text{ or } y = -7$$

$$y = 3: x = 2 \times 3 + 5 = 11; \quad y = -7: x = 2 \times (-7) + 5 = -9$$

So **A** and **B** are (11, 3) and (-9, -7). Check: $121 + 9 = 130$ and $81 + 49 = 130$ ✓

L crosses the x -axis where $y = 0$: $x = 5$, at the point **X**(5, 0). This point is between **A** and **B**.



Split triangle **OAB** along **OX** (base 5) into two triangles with heights 3 (above) and 7 (below).

$$\text{Area} = \frac{1}{2} \times 5 \times 3 + \frac{1}{2} \times 5 \times 7 = 7.5 + 17.5 = 25$$

Answer: 25

Tip: Solve the equations together to find A and B . For the area, split the triangle along an axis: the line meets the x -axis at $(5, 0)$, which gives a base of 5.

Marks: 1 mark for substituting $x = 2y + 5$ (or $y = \frac{x-5}{2}$) into $x^2 + y^2 = 130$. 1 mark for a correct quadratic, such as $y^2 + 4y - 21 = 0$, solved. 1 mark for both points, $(11, 3)$ and $(-9, -7)$. 1 mark for a complete method for the area. 1 mark for 25.
