

Mock Set A – Paper 3

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1)

Higher tier • Paper 3 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

$$V = 2u^2 - 5u$$

Joel works out the value of V when $u = -4$

Here is his working.

$$V = 2 \times -4^2 - 5 \times -4$$

$$V = 2 \times -16 + 20$$

$$V = -12$$

Joel's answer is wrong.

(a) What mistake has Joel made?

In Joel's second line he has written -4^2 as -16 .

But $u = -4$, so $u^2 = (-4)^2 = -4 \times -4 = +16$

Joel squared 4 and then made the answer negative, instead of squaring -4 .

Answer: He worked out $(-4)^2$ as -16 . It should be $+16$.

Tip: A negative number squared is always positive. Put negative numbers in brackets when you substitute: $2 \times (-4)^2$.

(b) Work out the correct value of V when $u = -4$

$$V = 2 \times (-4)^2 - 5 \times (-4)$$

$$= 2 \times 16 + 20$$

$$= 32 + 20 = 52$$

Answer: $V = 52$

Tip: $-5 \times -4 = +20$: two negatives multiplied give a positive.

2 (2 marks)

In a sailing race, the boats sail around two buoys, A and B .

The bearing of B from A is 124°

Work out the bearing of A from B .

Draw a north line at A and at B . The two north lines are parallel.

The bearing back from B to A points in the opposite direction, so add 180° .

$$124^\circ + 180^\circ = 304^\circ$$

Answer: 304°

Tip: A sketch with a north line at each buoy helps. A back bearing always differs from the original bearing by 180° .

Marks: 1 mark for $124 + 180$ or a correct angle (such as 56°) found using the north lines. 1 mark for 304° .

3 (3 marks)

Idris goes on a journey.

First he travels 51 km by ferry at an average speed of 34 km/h.

He then travels straight on by coach for 2 hours 40 minutes at an average speed of 57 km/h.

Work out Idris's average speed for the whole journey.

Give your answer correct to 1 decimal place.

$$\text{Ferry time} = \frac{51}{34} = 1.5 \text{ hours}$$

$$2 \text{ hours } 40 \text{ minutes} = 2\frac{40}{60} = 2.666 \dots \text{ hours}$$

$$\text{Coach distance} = 57 \times 2.666 \dots = 152 \text{ km}$$

Total distance = $51 + 152 = 203$ km

Total time = $1.5 + 2.666 \dots = 4.1666 \dots$ hours

Average speed = $\frac{203}{4.1666 \dots} = 48.72$ km/h

Answer: 48.7 km/h

Tip: 2 hours 40 minutes is not 2.4 hours: 40 minutes = $\frac{40}{60}$ of an hour. Average speed is total distance $\div$ total time, not the mean of the two speeds.

Marks: 1 mark for the ferry time or the coach distance. 1 mark for total distance $\div$ total time. 1 mark for 48.7.

4 (3 marks)

Bethan runs a market stall.

She buys 150 bunches of flowers for a total of £315

She sells 128 of the bunches for £3.75 each.

She sells the rest of the bunches at a reduced price of £1.50 each.

Work out Bethan's percentage profit.

Give your answer correct to 1 decimal place.

Money from the full-price bunches: $128 \times 3.75 = £480$

Number sold at the reduced price: $150 - 128 = 22$

Money from these: $22 \times 1.50 = £33$

Total income = $480 + 33 = £513$

Profit = $513 - 315 = £198$

Percentage profit = $\frac{198}{315} \times 100 = 62.857 \dots \%$

Answer: 62.9%

Tip: Percentage profit is always worked out as a percentage of what Bethan **paid** (£315), not of what she took (£513).

Marks: 1 mark for working out the total income (£513). 1 mark for profit $\div$ 315 $\times$ 100. 1 mark for 62.9.

5 (4 marks)

A theatre has 640 seats.

For the Saturday matinee, $\frac{5}{8}$ of the seats are sold at the full price of £28.50 each.

Then $\frac{3}{4}$ of the remaining seats are sold at the concession price of £17 each.

Of the seats still left, 25% are given free to a local school and the rest are sold on the day for £12 each.

Work out the total income from ticket sales for the Saturday matinee.

You must show all your working.

Full price seats: $\frac{5}{8} \times 640 = 400$

Remaining seats: $640 - 400 = 240$

Concession seats: $\frac{3}{4} \times 240 = 180$

Seats still left: $240 - 180 = 60$

Free seats: 25% of 60 = 15, so $60 - 15 = 45$ are sold on the day.

Income: $400 \times 28.50 = £11\,400$

$$180 \times 17 = £3060$$

$$45 \times 12 = £540$$

Total = $11\,400 + 3060 + 540 = £15\,000$

Answer: £15 000

Tip: “ $\frac{3}{4}$ of the remaining seats” means $\frac{3}{4}$ of 240, not $\frac{3}{4}$ of 640. Read each step carefully to see what it is a fraction of.

Marks: 1 mark for 400 full-price seats (or 240 remaining). 1 mark for 180 concession seats. 1 mark for 45 seats sold on the day. 1 mark for £15 000.

6 (3 marks)

(a) Change 0.42 m^2 into cm^2

$$1 \text{ m} = 100 \text{ cm, so } 1 \text{ m}^2 = 100 \times 100 = 10\,000 \text{ cm}^2$$

$$0.42 \times 10\,000 = 4200$$

Answer: 4200 cm^2

Tip: Square the length factor for area units: multiply by 10 000, not 100.

- (b) Esme's drone has a top speed of 19 metres per second.
Marcus's drone has a top speed of 65 kilometres per hour.
Whose drone has the greater top speed?
You must show how you get your answer.

Change Esme's speed into km/h.

In one hour there are $60 \times 60 = 3600$ seconds.

Distance in one hour = $19 \times 3600 = 68\,400\text{ m} = 68.4\text{ km}$

So Esme's drone has a top speed of 68.4 km/h.

68.4 km/h is more than 65 km/h, so **Esme's** drone has the greater top speed.

Answer: Esme's drone ($19\text{ m/s} = 68.4\text{ km/h}$, which is more than 65 km/h)

Tip: Compare the speeds in the same units. To change m/s to km/h, multiply by 3600 and divide by 1000 (so multiply by 3.6).

Marks: 1 mark for changing one speed into the units of the other (68.4 km/h or 18.05... m/s). 1 mark for Esme's drone with the two speeds compared in the same units.

7 (4 marks)

Amira, Josh and Kezia are cousins.
Josh's age is 5 years more than twice Amira's age.
Kezia is 4 years younger than Josh.
In 4 years' time, the total of their three ages will be 78 years.
Work out the age of the oldest cousin now.

Let Amira's age now be x .

Josh: $2x + 5$ Kezia: $2x + 5 - 4 = 2x + 1$

Total of their ages now: $x + (2x + 5) + (2x + 1) = 5x + 6$

In 4 years each of them is 4 years older, so the total goes up by $3 \times 4 = 12$.

$$5x + 6 + 12 = 78$$

$$5x = 60, \text{ so } x = 12$$

Amira 12, Josh $2 \times 12 + 5 = 29$, Kezia 25

The oldest is Josh, aged 29.

Answer: 29 years

Tip: In 4 years' time **all three** cousins are 4 years older, so the total increases by 12, not 4.

Marks: 1 mark for expressions for Josh and Kezia. 1 mark for a correct equation. 1 mark for $x = 12$. 1 mark for 29.

8 (3 marks)

PQR and PRS are right-angled triangles.

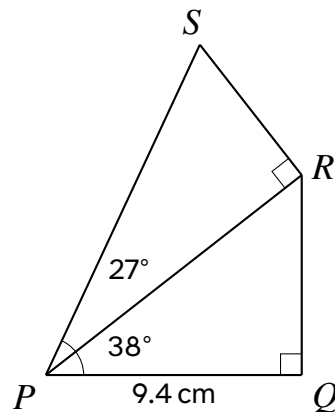


Diagram **NOT** accurately drawn

Work out the length of RS .

Give your answer correct to 3 significant figures.

In triangle PQR , PQ is adjacent to the 38° angle and PR is the hypotenuse.

$$\cos 38^\circ = \frac{9.4}{PR}, \text{ so } PR = \frac{9.4}{\cos 38^\circ} = 11.928 \dots \text{ cm}$$

In triangle PRS , PR is adjacent to the 27° angle and RS is opposite it.

$$\tan 27^\circ = \frac{RS}{PR}, \text{ so } RS = 11.928 \dots \times \tan 27^\circ = 6.078 \dots \text{ cm}$$

Answer: 6.08 cm

Tip: Keep the full calculator value of PR for the next step. Rounding PR to 11.9 gives 6.06, which is not accurate enough.

Marks: 1 mark for working out PR . 1 mark for a correct method for RS using your PR . 1 mark for 6.08.

9 (2 marks)

Each member of a gym is given a membership code.

The code is made of 2 letters followed by 2 digits, for example KP47

The letters can be any of the 26 letters of the alphabet and can be the same.

The two digits must be different.

Work out how many different codes are possible.

There are 26 choices for the first letter and 26 for the second.

There are 10 choices for the first digit, then only 9 for the second (it must be different).

Number of codes = $26 \times 26 \times 10 \times 9 = 60\,840$

Answer: 60 840

Tip: Multiply the numbers of choices for each position (the product rule). Do not add them.

Marks: 1 mark for multiplying the numbers of choices with at least three correct. 1 mark for 60 840.

10 (5 marks)

The table gives information about the masses, in kg, of 80 cabin bags.

Mass (m kg)	$0 < m \leq 2$	$2 < m \leq 4$	$4 < m \leq 6$	$6 < m \leq 8$	$8 < m \leq 10$	$10 < m \leq 12$
Frequency	9	21	26	14	7	3
Cumulative frequency						

- (a) Complete the cumulative frequency row in the table.

Keep a running total of the frequencies, working along the table:

$$9, \quad 9 + 21 = 30, \quad 30 + 26 = 56, \quad 56 + 14 = 70, \quad 70 + 7 = 77, \quad 77 + 3 = 80$$

Mass (m kg)	$0 < m \leq 2$	$2 < m \leq 4$	$4 < m \leq 6$	$6 < m \leq 8$	$8 < m \leq 10$	$10 < m \leq 12$
Frequency	9	21	26	14	7	3
Cumulative frequency	9	30	56	70	77	80

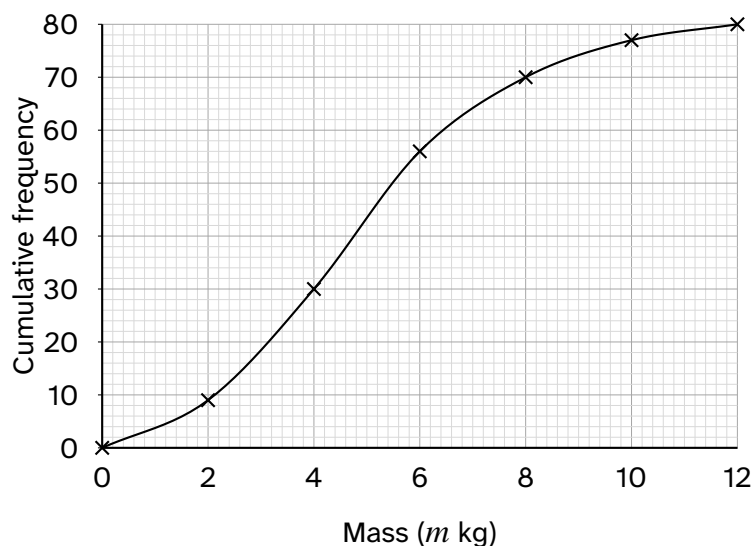
Answer: 9, 30, 56, 70, 77, 80

- (b) On the grid, draw a cumulative frequency graph for your table.

Plot each cumulative frequency at the **upper** end of its class: (2, 9), (4, 30), (6, 56), (8, 70), (10, 77), (12, 80)

No bag weighs 0 kg or less, so also plot (0, 0).

Join the points with a smooth curve (or with straight lines).



Answer: Points plotted at (2, 9), (4, 30), (6, 56), (8, 70), (10, 77), (12, 80), joined with a curve or straight lines

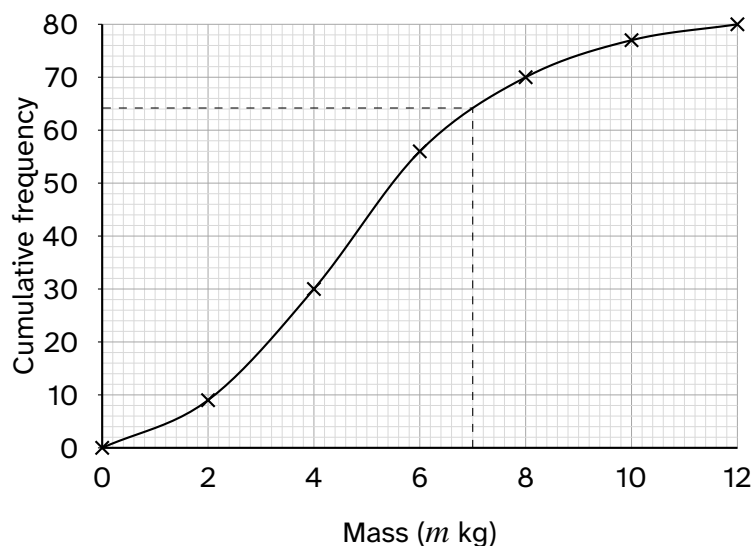
Tip: Plot at the upper class boundaries (2, 4, 6, ...), not at the midpoints.

Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a fully correct graph.

- (c) The airline's weight limit for a cabin bag is 7 kg.
Sofia says, "More than a quarter of these bags are over the limit."
Is Sofia correct?
You must show how you get your answer.

A quarter of 80 bags is 20 bags.

Read up from 7 kg to the curve and across: about 64 bags weigh 7 kg or less.



So about $80 - 64 = 16$ bags weigh more than 7 kg.

16 is less than 20, so Sofia is **not** correct.

Answer: No. About 16 bags weigh more than 7 kg, which is fewer than 20 (a quarter of 80).

Tip: The graph shows how many bags weigh 7 kg **or less**. Subtract from 80 to find how many weigh more.

Marks: 1 mark for reading the graph at 7 kg (or at a cumulative frequency of 60). 1 mark for a correct conclusion supported by figures.

11 (4 marks)

A farmer has a water tank for his animals.

The tank is in the shape of a cylinder.

The tank has radius 60 cm and height 1.5 m.

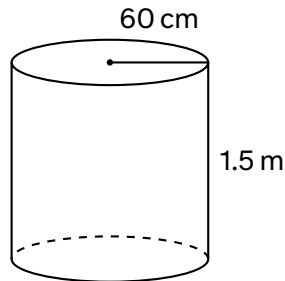


Diagram **NOT** accurately drawn

The tank is empty.

Water flows into the tank at a constant rate of 18 litres per minute.

Work out how long it takes to fill the tank to $\frac{2}{3}$ of its height.

Give your answer in minutes and seconds, correct to the nearest second.

Work in cm: 1.5 m = 150 cm

$\frac{2}{3}$ of the height = 100 cm

Volume of water = $\pi \times 60^2 \times 100 = 1130\,973.3 \dots \text{cm}^3$

1 litre = 1000 cm^3 , so this is 1130.973 ... litres.

Time = $1130.973 \dots \div 18 = 62.831 \dots$ minutes

$0.831 \dots \times 60 = 49.9 \dots$ seconds, which is 50 seconds to the nearest second.

Answer: 62 minutes 50 seconds

Tip: 62.83 minutes is **not** 62 minutes 83 seconds. Multiply the decimal part by 60 to change it to seconds.

Marks: 1 mark for a correct volume (full or two-thirds). 1 mark for changing to litres. 1 mark for dividing by 18. 1 mark for 62 minutes 50 seconds.

12 (3 marks)

Show that $(3x - 1)(x + 2)(x - 5)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

Expand two of the brackets first: $(x + 2)(x - 5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$

Multiply by $(3x - 1)$: $3x(x^2 - 3x - 10) - 1(x^2 - 3x - 10)$

$$= 3x^3 - 9x^2 - 30x - x^2 + 3x + 10$$

$$= 3x^3 - 10x^2 - 27x + 10$$

This is in the form $ax^3 + bx^2 + cx + d$ with $a = 3, b = -10, c = -27, d = 10$, which are all integers.

Answer: $3x^3 - 10x^2 - 27x + 10$

Tip: Write out every term before collecting like terms, and take care with $-1 \times -10 = +10$.

Marks: 1 mark for expanding two brackets correctly. 1 mark for multiplying all of it by the third bracket (6 terms, at least half of them correct). 1 mark for $3x^3 - 10x^2 - 27x + 10$.

13 (3 marks)

8 volunteers take 2 hours 15 minutes to clear the litter from a beach.

- (a) How long would it take 5 volunteers to clear the litter from the same beach?
Give your answer in hours and minutes.

$$2 \text{ hours } 15 \text{ minutes} = 135 \text{ minutes}$$

With 8 volunteers it takes 135 minutes, so 1 volunteer would take $8 \times 135 = 1080$ minutes.

$$\text{With 5 volunteers: } 1080 \div 5 = 216 \text{ minutes}$$

$$216 \text{ minutes} = 3 \text{ hours } 36 \text{ minutes}$$

Answer: 3 hours 36 minutes

Tip: Fewer volunteers take **longer**. This is inverse proportion, so check your answer is more than 2 hours 15 minutes.

Marks: 1 mark for 135×8 (or 2.25×8) or $135 \times 8 \div 5$. 1 mark for 3 hours 36 minutes.

- (b) State one assumption you made in working out your answer to part (a).

I assumed that all the volunteers work at the same rate (and keep working at that rate the whole time).

Answer: All the volunteers work at the same rate.

14 (4 marks)

An aquarium tank is in the shape of a cuboid.

The length of the base of the tank is 90 cm, correct to the nearest centimetre.

The width of the base of the tank is 38 cm, correct to the nearest centimetre.

The capacity of the tank is 150 litres, correct to the nearest 10 litres.

Nisha says, "The height of the tank must be less than 46 cm."

By considering bounds, determine whether Nisha is correct.

You must show all your working.

Bounds: length 89.5 to 90.5 cm, width 37.5 to 38.5 cm, capacity 145 to 155 litres.

$$\text{Height} = \frac{\text{volume}}{\text{length} \times \text{width}}$$

For the biggest possible height, use the biggest volume and divide by the smallest length and width.

$$155 \text{ litres} = 155\,000 \text{ cm}^3$$

$$\text{Upper bound of the height} = \frac{155\,000}{89.5 \times 37.5} = \frac{155\,000}{3356.25} = 46.18 \dots \text{ cm}$$

46.18 ... is more than 46, so the height of the tank could be more than 46 cm.

Nisha is **not** correct.

Answer: Nisha is not correct: the upper bound of the height is 46.18 ... cm, which is more than 46 cm, so the height could be more than 46 cm.

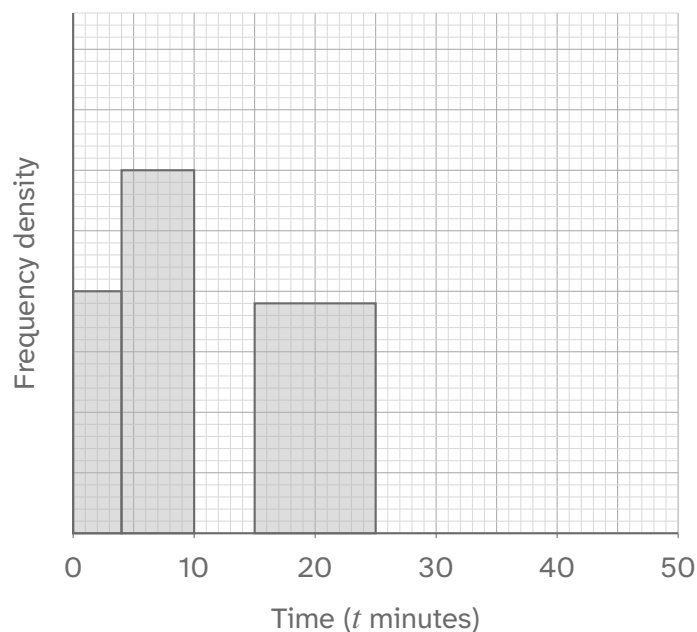
Tip: Dividing by a smaller number gives a bigger answer, so the upper bound of a division uses the **lower** bounds of the numbers you divide by. Working out $150\,000 \div (90 \times 38) = 43.9$ does not answer the question: you need the biggest height the tank could have.

Marks: 1 mark for one correct bound. 1 mark for upper bound of volume $\div$ (lower bound of length $\times$ lower bound of width), with litres changed to cm^3 . 1 mark for 46.18... 1 mark for saying Nisha is not correct because 46.18... is more than 46.

15 (5 marks)

The incomplete table and the incomplete histogram give information about the times, in minutes, that 150 customers spent in a bookshop.

Time (t minutes)	Frequency
$0 < t \leq 4$	16
$4 < t \leq 10$	
$10 < t \leq 15$	40
$15 < t \leq 25$	38
$25 < t \leq 45$	20



- (a) Use the histogram to complete the table.

The table gives 16 customers for $0 < t \leq 4$. That bar is 4 minutes wide, so its frequency density is $16 \div 4 = 4$.

This bar is 20 small squares high, so each small square on the vertical axis is 0.2.

The bar for $4 < t \leq 10$ is 30 small squares high, so its frequency density is $30 \times 0.2 = 6$.

Frequency = frequency density $\times$ class width = $6 \times 6 = 36$

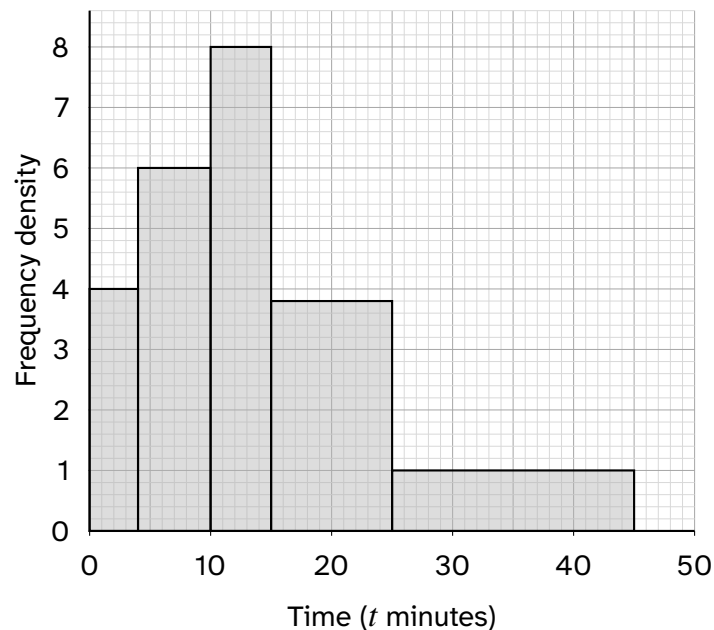
Answer: 36

- (b) Use the table to complete the histogram.

Frequency density = frequency $\div$ class width

$$10 < t \leq 15: 40 \div 5 = 8$$

$$25 < t \leq 45: 20 \div 20 = 1$$



Answer: Bars drawn for $10 < t \leq 15$ at height 8 and for $25 < t \leq 45$ at height 1 (on the scale from the $0 < t \leq 4$ bar)

Tip: The height of each bar is the frequency density, not the frequency. Use the first bar to work out the scale on the vertical axis.

Marks: 1 mark for one correct bar. 1 mark for both bars correct.

- (c) Work out an estimate for the number of customers who spent between 8 minutes and 20 minutes in the bookshop.

From 8 to 10 minutes: $\frac{2}{6}$ of the 36 customers in $4 < t \leq 10 = 12$ (or $2 \times 6 = 12$)

From 10 to 15 minutes: all 40 customers

From 15 to 20 minutes: $\frac{5}{10}$ of the 38 customers in $15 < t \leq 25 = 19$ (or $5 \times 3.8 = 19$)

$$\text{Estimate} = 12 + 40 + 19 = 71$$

Answer: 71

Tip: Use area: frequency = frequency density $\times$ width, for just the part of each bar inside the interval.

Marks: 1 mark for one correct partial frequency (12 or 19). 1 mark for 71.

16 (3 marks)

The number of birds of one species on an island is recorded at the start of each year.

P_n is the number of birds n years after the start of 2025

$P_0 = 3200$ and $P_{n+1} = kP_n$ where k is a constant.

$P_2 = 2738$

(a) Work out the value of k .

$$P_1 = kP_0 \text{ and } P_2 = kP_1 = k \times kP_0 = k^2P_0$$

$$3200k^2 = 2738$$

$$k^2 = \frac{2738}{3200} = 0.855625$$

$$k = \sqrt{0.855625} = 0.925 \text{ (} k \text{ is positive, as the number of birds is positive)}$$

Answer: $k = 0.925$

Tip: Two years have passed, so the multiplier is used twice: $P_2 = k^2P_0$. Dividing 2738 by 3200 gives k^2 , not k .

Marks: 1 mark for $3200k^2 = 2738$ or $2738 \div 3200$. 1 mark for 0.925.

(b) Write down what your value of k tells you about the number of birds on the island.

$$k = 0.925 = 1 - 0.075$$

So the number of birds goes down by 7.5% each year.

Answer: The number of birds decreases by 7.5% each year.

17 (6 marks)

- (a) Show that the equation $x^3 - 5x - 3 = 0$ has a solution between $x = 2$ and $x = 3$

$$\text{Let } f(x) = x^3 - 5x - 3$$

$$f(2) = 8 - 10 - 3 = -5$$

$$f(3) = 27 - 15 - 3 = 9$$

$f(2)$ is negative and $f(3)$ is positive. The sign changes and the graph is continuous, so there is a solution between $x = 2$ and $x = 3$.

Answer: $f(2) = -5 < 0$ and $f(3) = 9 > 0$: there is a change of sign, so there is a solution between 2 and 3.

Tip: Finding the two values is not enough: you must say that the sign changes, and so there is a solution between 2 and 3.

Marks: 1 mark for substituting 2 and 3. 1 mark for -5 and 9 with a conclusion about the change of sign.

- (b) Show that the equation $x^3 - 5x - 3 = 0$ can be rearranged to give $x = \sqrt[3]{5x + 3}$

$$\text{Add } 5x + 3 \text{ to both sides: } x^3 = 5x + 3$$

$$\text{Take the cube root of both sides: } x = \sqrt[3]{5x + 3}$$

Answer: $x^3 = 5x + 3$, so $x = \sqrt[3]{5x + 3}$

Tip: Show the middle step ($x^3 = 5x + 3$). Writing down only the final equation, which is given, scores nothing.

- (c) Starting with $x_0 = 2.5$, use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 3}$ three times to find an estimate for a solution of $x^3 - 5x - 3 = 0$

$$x_1 = \sqrt[3]{5 \times 2.5 + 3} = \sqrt[3]{15.5} = 2.49332 \dots$$

$$x_2 = \sqrt[3]{5 \times 2.49332 \dots + 3} = 2.49152 \dots$$

$$x_3 = \sqrt[3]{5 \times 2.49152 \dots + 3} = 2.49104 \dots$$

Answer: $x_3 = 2.491$ (accept 2.491...)

Tip: Type $\sqrt[3]{5 \times 2.5 + 3}$, press =, then type $\sqrt[3]{5 \text{ ANS} + 3}$ and press = twice more.

Marks: 1 mark for $x_1 = 2.493 \dots$ 1 mark for working out x_2 and x_3 . 1 mark for 2.491.

18 (4 marks)

A bucket is in the shape of a frustum of a cone.

The frustum is made by removing a small cone from a large cone.

The top of the bucket is a circle of radius 15 cm.

The base of the bucket is a circle of radius 9 cm.

The vertical height of the bucket is 20 cm.

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

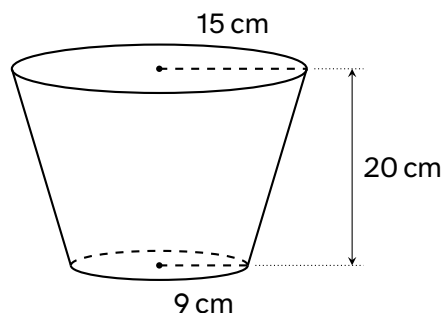


Diagram **NOT** accurately drawn

Work out the capacity of the bucket.

Give your answer in litres, correct to 3 significant figures.

You must show all your working.

Complete the cone: the bucket is a large cone with a small cone removed from its bottom.

Let the height of the large cone be H cm. The small cone has height $H - 20$ cm.

The two cones are similar, so $\frac{H - 20}{H} = \frac{9}{15}$

$$15(H - 20) = 9H, \text{ so } 15H - 300 = 9H, 6H = 300, H = 50$$

Large cone: radius 15 cm, height 50 cm. Small cone: radius 9 cm, height $50 - 20 = 30$ cm.

$$\text{Volume of the large cone} = \frac{1}{3} \times \pi \times 15^2 \times 50 = 3750\pi \text{ cm}^3$$

$$\text{Volume of the small cone} = \frac{1}{3} \times \pi \times 9^2 \times 30 = 810\pi \text{ cm}^3$$

$$\text{Volume of the bucket} = 3750\pi - 810\pi = 2940\pi = 9236.2 \dots \text{ cm}^3$$

1 litre = 1000 cm^3 , so the capacity is $9236.2 \dots \div 1000 = 9.236 \dots$ litres.

Answer: 9.24 litres

Tip: The cone formula needs the height of a whole cone, not the 20 cm height of the bucket. Use similar triangles: the radius goes down by 6 cm over 20 cm of height, so the 9 cm radius needs another 30 cm to reach the point.

Marks: 1 mark for finding the height of the large cone (50 cm) or the small cone (30 cm). 1 mark for the volume of either cone. 1 mark for large cone – small cone, changed to litres. 1 mark for 9.24.

19 (3 marks)

$ABCD$ is a square.

P is a point on AB and Q is a point on AD such that $AP = AQ$.

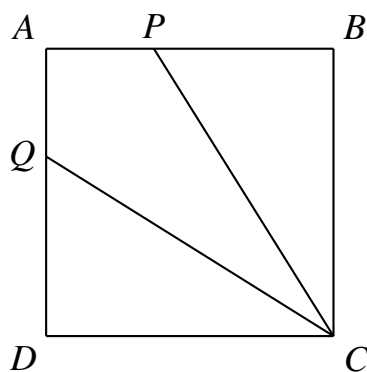


Diagram **NOT** accurately drawn

Prove that triangle CBP is congruent to triangle CDQ .

$CB = CD$ (the sides of a square are equal)

Angle $CBP = \text{angle } CDQ = 90^\circ$ (the angles of a square are all 90°)

$AB = AD$ (sides of the square) and $AP = AQ$ (given),

$$\text{so } BP = AB - AP = AD - AQ = DQ$$

Two sides and the included angle are equal, so triangles CBP and CDQ are congruent (SAS).

Answer: $CB = CD$, angle $CBP = \text{angle } CDQ = 90^\circ$, $BP = DQ$, so the triangles are congruent (SAS).

Tip: You must explain **why** $BP = DQ$: it is not given in the question. Finish by naming the congruence condition (SAS).

Marks: 1 mark for one pair of equal sides or angles with a reason. 1 mark for all three pairs with reasons, including why $BP = DQ$. 1 mark for the conclusion with SAS.

20 (4 marks)

Solve $\frac{7}{x+2} + \frac{2}{x-3} = 2$

Multiply every term by $(x+2)(x-3)$: $7(x-3) + 2(x+2) = 2(x+2)(x-3)$

Expand: $7x - 21 + 2x + 4 = 2(x^2 - x - 6)$

$$9x - 17 = 2x^2 - 2x - 12$$

Rearrange: $0 = 2x^2 - 11x + 5$

Factorise: $(2x - 1)(x - 5) = 0$

$$x = \frac{1}{2} \text{ or } x = 5$$

Answer: $x = \frac{1}{2}$ or $x = 5$

Tip: Remember to multiply the 2 on the right-hand side by $(x+2)(x-3)$ as well.

Marks: 1 mark for multiplying by $(x+2)(x-3)$. 1 mark for $2x^2 - 11x + 5 = 0$. 1 mark for a correct method to solve it. 1 mark for both solutions.

21 (4 marks)

$VABCD$ is a pyramid.

The base $ABCD$ is a horizontal rectangle with $AB = 14$ cm and $BC = 9$ cm.

V is vertically above the centre of the rectangle.

The volume of the pyramid is 630 cm^3

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{vertical height}$$

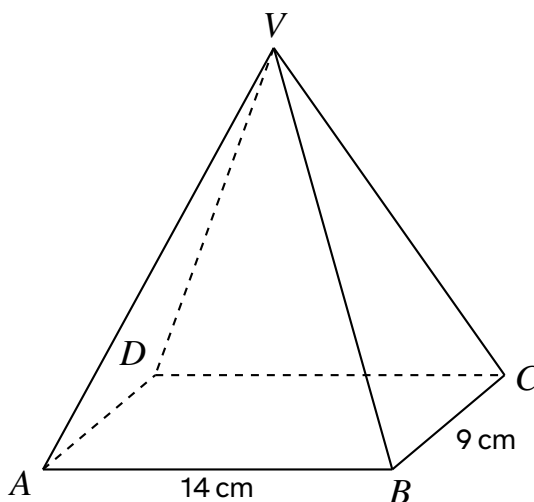


Diagram **NOT** accurately drawn

Work out the size of the angle between VA and the base $ABCD$.

Give your answer correct to 1 decimal place.

Let M be the centre of the rectangle, so V is vertically above M and VM is the vertical height.

$$\text{Area of the base} = 14 \times 9 = 126 \text{ cm}^2$$

$$\frac{1}{3} \times 126 \times VM = 630, \text{ so } 42 \times VM = 630 \text{ and } VM = 15 \text{ cm}$$

$$AC = \sqrt{14^2 + 9^2} = \sqrt{277} = 16.643 \dots \text{ cm, so } AM = \frac{1}{2} \times 16.643 \dots = 8.321 \dots \text{ cm}$$

The angle we need is angle VAM , in the right-angled triangle VAM .

$$\tan VAM = \frac{VM}{AM} = \frac{15}{8.321 \dots} = 1.802 \dots$$

$$\text{Angle } VAM = \tan^{-1}(1.802 \dots) = 60.97 \dots^\circ$$

Answer: 61.0°

Tip: The angle with the base is between VA and the line AM on the base directly below it. Halve the diagonal AC , and use the height of the pyramid, not a sloping edge.

Marks: 1 mark for the vertical height (15 cm) from the volume. 1 mark for half the diagonal of the base (8.32... cm). 1 mark for a correct trig statement for angle VAM . 1 mark for 61.0.

22 (6 marks)

L is the straight line with equation $x + y = 5$

C is the curve with equation $y = 2x^2 - 3x - 4$

L and C intersect at the points A and B .

Work out the length of AB .

Give your answer correct to 3 significant figures.

You must show all your working.

From the equation of L : $y = 5 - x$

Substitute into the equation of C : $5 - x = 2x^2 - 3x - 4$

Rearrange: $2x^2 - 2x - 9 = 0$

This does not factorise, so use the quadratic formula with $a = 2$, $b = -2$, $c = -9$:

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 2 \times (-9)}}{2 \times 2} = \frac{2 \pm \sqrt{76}}{4}$$

$$x = 2.6794 \dots \text{ or } x = -1.6794 \dots$$

$$y = 5 - x: y = 2.3205 \dots \text{ or } y = 6.6794 \dots$$

So A and B are (2.679 ..., 2.320 ...) and (-1.679 ..., 6.679 ...).

Difference in x : $2.6794 \dots - (-1.6794 \dots) = 4.3588 \dots$ Difference in y :

$$6.6794 \dots - 2.3205 \dots = 4.3588 \dots$$

By Pythagoras: $AB = \sqrt{4.3588 \dots^2 + 4.3588 \dots^2} = \sqrt{38} = 6.164 \dots$

Answer: 6.16

Tip: Find both points first, keeping the full calculator values, then use Pythagoras on the differences in x and in y . Do not stop once you have the coordinates.

Marks: 1 mark for substituting to get one equation in x . 1 mark for $2x^2 - 2x - 9 = 0$. 1 mark for using the quadratic formula correctly. 1 mark for both x values. 1 mark for using Pythagoras with the two points. 1 mark for 6.16.
