

Mock Set A – Paper 1

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1)

Higher tier • Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Work out $6.12 \div 0.36$

Multiply both numbers by 100 so that you divide by a whole number.

$$6.12 \div 0.36 = 612 \div 36$$

Long division: $36 \times 10 = 360$, $612 - 360 = 252$

$$36 \times 7 = 252$$

$$\text{So } 612 \div 36 = 10 + 7 = 17$$

Answer: 17

Tip: Multiply **both** numbers by the same power of 10. Moving the decimal point in only one of them gives 1.7 or 170.

Marks: 1 mark for a complete method, such as $612 \div 36$ worked out. 1 mark for 17.

2 (3 marks)

Work out $1\frac{3}{4} \times 2\frac{2}{9}$

Give your answer as a mixed number in its simplest form.

Write both numbers as improper fractions.

$$1\frac{3}{4} = \frac{7}{4} \quad \text{and} \quad 2\frac{2}{9} = \frac{20}{9}$$

$$\frac{7}{4} \times \frac{20}{9} = \frac{140}{36}$$

Simplify (divide top and bottom by 4): $\frac{140}{36} = \frac{35}{9}$

$$35 \div 9 = 3 \text{ remainder } 8, \text{ so } \frac{35}{9} = 3\frac{8}{9}$$

Answer: $3\frac{8}{9}$

Tip: Do not multiply the whole numbers and the fractions separately (1×2 and $\frac{3}{4} \times \frac{2}{9}$): that gives the wrong answer.

Marks: 1 mark for both improper fractions. 1 mark for $\frac{140}{36}$ or $\frac{35}{9}$. 1 mark for the answer as a mixed number in its simplest form.

3 (2 marks)

The price of a family ticket increased by 20% to £57.60

Jodie says the price before the increase was £46.08

Is Jodie correct?

After a 20% increase, the new price is 120% of the old price.

$$120\% = £57.60$$

$$10\% = 57.60 \div 12 = £4.80$$

$$100\% = 4.80 \times 10 = £48$$

The price before the increase was £48, not £46.08, so Jodie is **not** correct.

£46.08 is what you get if you take 20% of £57.60 off £57.60. That is wrong: the 20% was worked out on the **old** price.

Check: 20% of £48 is £9.60, and $48 + 9.60 = £57.60$ ✓

Answer: No. The price before was £48 ($57.60 \div 1.2$), not £46.08

Tip: Do not take 20% off £57.60. The 20% was added to the **old** price, so £57.60 is 120%, not 100%.

Marks: 1 mark for $57.60 \div 1.2$ (or another correct method, such as 46.08×1.2). 1 mark for “No” with £48 (or with $55.296 \neq 57.60$).

4 (5 marks)

A youth orchestra has 168 members.

$\frac{3}{7}$ of the members play a string instrument.

Each of the other members plays either a brass instrument or a woodwind instrument.

number who play brass : number who play woodwind = 3:5

45% of the members who play woodwind are in their first year with the orchestra.

Work out the number of woodwind players who are in their first year with the orchestra.

String players: $\frac{3}{7}$ of 168 = $168 \div 7 \times 3 = 72$

Other members: $168 - 72 = 96$

Brass : woodwind = 3:5, so there are $3 + 5 = 8$ parts.

One part = $96 \div 8 = 12$

Woodwind players: $12 \times 5 = 60$

10% of 60 = 6, 40% = 24, 5% = 3

45% of 60 = $24 + 3 = 27$

Answer: 27

Tip: Share only the members who do **not** play strings (96) in the ratio 3:5, not all 168.

Marks: 1 mark for 72 (or 96). 1 mark for $96 \div 8$. 1 mark for 60 woodwind players. 1 mark for a method to find 45% of 60. 1 mark for 27.

5 (4 marks)

The diagram shows a regular octagon $ABCDEFGH$ and a square $ABPQ$.
The straight line CP is drawn.

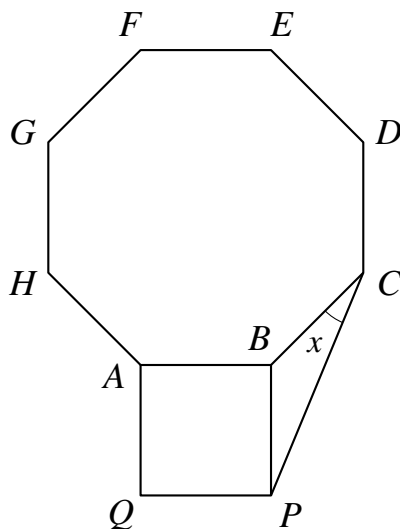


Diagram **NOT** accurately drawn

Work out the size of the angle marked x .

Give a reason for each stage of your working.

Exterior angle of a regular octagon $= 360 \div 8 = 45^\circ$ (exterior angles of a polygon add up to 360°).

Interior angle $= 180 - 45 = 135^\circ$ (angles on a straight line add up to 180°).

Angle $ABP = 90^\circ$ (angles in a square are 90°).

Angle $PBC = 360 - 135 - 90 = 135^\circ$ (angles around a point add up to 360°).

$BC = AB$ (the octagon is regular) and $BP = AB$ (sides of a square are equal), so $BC = BP$.

So triangle BCP is isosceles and angle $BCP = \text{angle } BPC$ (base angles of an isosceles triangle are equal).

$$x = (180 - 135) \div 2 = 22.5^\circ \text{ (angles in a triangle add up to } 180^\circ\text{).}$$

Answer: 22.5°

Tip: Look for the equal sides: BC is a side of the octagon and BP is a side of the square, and both are the same length as AB . Say this in your reasons.

Marks: 1 mark for the interior angle 135° . 1 mark for angle $PBC = 135^\circ$. 1 mark for 22.5° . 1 mark for the reasons, including why triangle BCP is isosceles.

6 (6 marks)

(a) Complete the table of values for $y = x^2 - 2x - 1$

x	-2	-1	0	1	2	3	4
y	7	2	-1	-2	-1	2	7

Substitute each missing x -value into $y = x^2 - 2x - 1$.

$$x = 0: 0 - 0 - 1 = -1$$

$$x = 1: 1 - 2 - 1 = -2$$

$$x = 3: 9 - 6 - 1 = 2$$

Check: the y -values are symmetrical about $x = 1$.

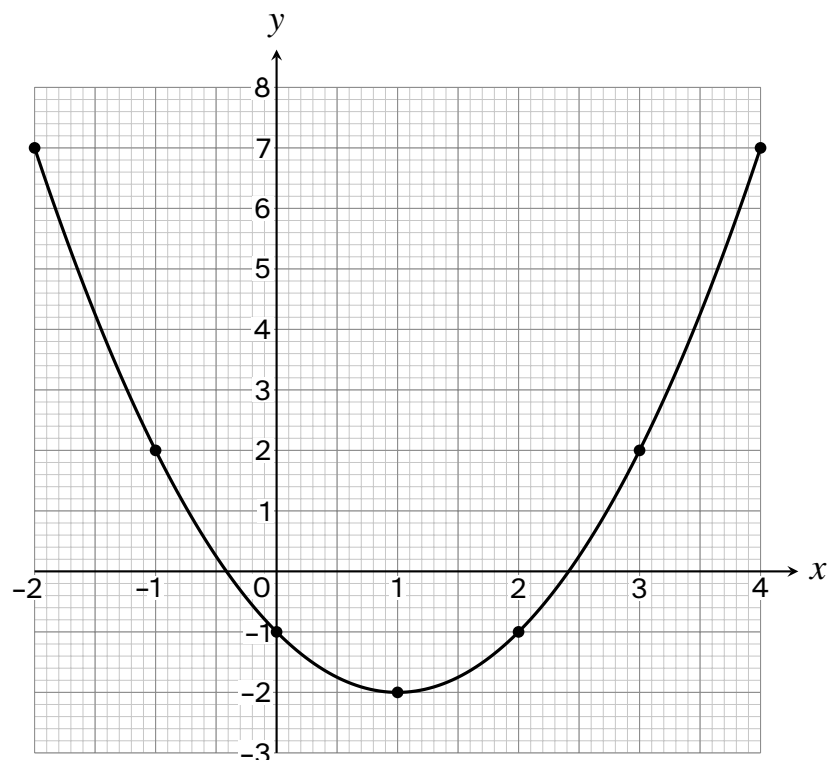
Answer: -1, -2, 2

Marks: 2 marks for all three values. 1 mark for one or two correct.

(b) On the grid, draw the graph of $y = x^2 - 2x - 1$ for all values of x from -2 to 4

Plot the seven points from the table.

Join them with one smooth U-shaped curve.



Answer: Smooth curve through $(-2, 7)$, $(-1, 2)$, $(0, -1)$, $(1, -2)$, $(2, -1)$, $(3, 2)$, $(4, 7)$

Tip: Use one smooth curve, not straight lines between the points. The bottom of the curve should be rounded, with its lowest point at $(1, -2)$.

Marks: 1 mark for plotting at least 5 points correctly. 1 mark for a smooth curve through all 7 points.

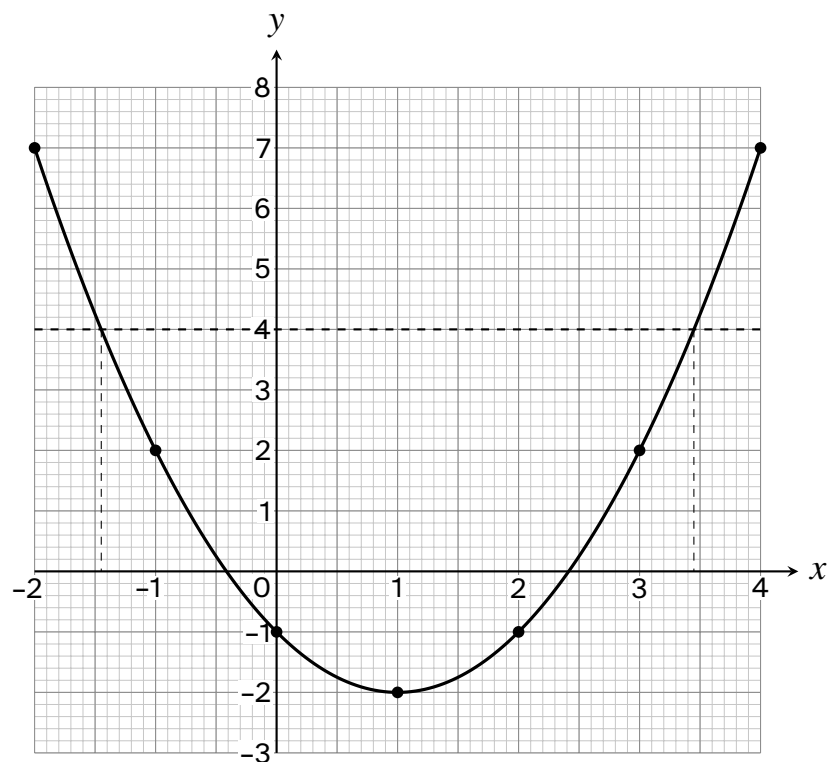
- (c) Use your graph to find estimates of the solutions of $x^2 - 2x - 5 = 0$

Rearrange so that the left-hand side matches the graph: add 4 to both sides.

$$x^2 - 2x - 5 = 0 \text{ becomes } x^2 - 2x - 1 = 4$$

So the solutions are where the curve $y = x^2 - 2x - 1$ meets the line $y = 4$.

Draw the horizontal line $y = 4$ and read off the x -values where it crosses the curve.



$x \approx -1.45$ and $x \approx 3.45$

Any answers from -1.4 to -1.5 and from 3.4 to 3.5 are fine.

Answer: $x \approx -1.45$ and $x \approx 3.45$

Tip: The graph is $y = x^2 - 2x - 1$, so you need $x^2 - 2x - 1 = \text{something}$. Adding 4 to both sides gives $x^2 - 2x - 1 = 4$: draw $y = 4$, not $y = 0$ or $y = -5$.

Marks: 1 mark for drawing the line $y = 4$ (or one correct value). 1 mark for both values.

7 (3 marks)

The table shows information about the times, t minutes, that 50 customers spent queuing at a bakery on a Saturday morning.

Time (t minutes)	Frequency
$0 < t \leq 2$	7
$2 < t \leq 4$	12
$4 < t \leq 6$	15
$6 < t \leq 8$	10
$8 < t \leq 10$	6

Work out an estimate for the mean time.

Use the midpoint of each class as an estimate of each time.

Time (t minutes)	Frequency f	Midpoint x	$f \times x$
$0 < t \leq 2$	7	1	7
$2 < t \leq 4$	12	3	36
$4 < t \leq 6$	15	5	75
$6 < t \leq 8$	10	7	70
$8 < t \leq 10$	6	9	54
Total	50		242

Total of $f \times x$: $7 + 36 + 75 + 70 + 54 = 242$

Estimate of mean = $242 \div 50$

= 4.84 minutes

Answer: 4.84 minutes

Tip: Divide by the total frequency (50), not by the number of classes (5).

Marks: 1 mark for products of frequency and values in each class. 1 mark for using midpoints and dividing the total by 50. 1 mark for 4.84.

8 (2 marks)

Find the exact value of $\left(\frac{8}{343}\right)^{-\frac{2}{3}}$

The negative power means turn the fraction upside down: $\left(\frac{343}{8}\right)^{\frac{2}{3}}$

The power $\frac{1}{3}$ means the cube root: $\sqrt[3]{343} = 7$, $\sqrt[3]{8} = 2$

$$\left(\frac{343}{8}\right)^{\frac{1}{3}} = \frac{7}{2}$$

Square it: $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

Answer: $\frac{49}{4}$

Tip: A negative power does not make the answer negative. It means the reciprocal: turn the fraction upside down.

Marks: 1 mark for using the reciprocal or the cube root correctly. 1 mark for $\frac{49}{4}$.

9 (4 marks)

A school buys some calculators and some geometry sets.

5 calculators and 2 geometry sets cost a total of £53

3 calculators cost £7 more than 5 geometry sets.

Work out the cost of one calculator and the cost of one geometry set.

You must show all your working.

Let one calculator cost £ c and one geometry set cost £ g .

$$5c + 2g = 53$$

“3 calculators cost £7 more than 5 geometry sets”: $3c = 5g + 7$, so $3c - 5g = 7$

Make the g -coefficients the same size: multiply the first equation by 5 and the second by 2.

$$25c + 10g = 265$$

$$6c - 10g = 14$$

The signs of $10g$ are different, so add: $31c = 279$

$$c = 279 \div 31 = 9$$

Substitute $c = 9$ into $5c + 2g = 53$: $45 + 2g = 53$

$$2g = 8, \text{ so } g = 4$$

Check: $3 \times 9 = 27$ and $5 \times 4 + 7 = 27$ ✓

Answer: calculator £9, geometry set £4

Tip: “3 calculators cost £7 more than 5 geometry sets” means $3c = 5g + 7$. Writing $3c + 7 = 5g$ puts the £7 on the wrong side.

Marks: 1 mark for writing two correct equations. 1 mark for making the coefficients of c or g the same and adding or subtracting correctly. 1 mark for substituting to find the other value. 1 mark for both answers.

10 (4 marks)

$\xi = \{\text{whole numbers from 1 to 18}\}$

$A = \{\text{factors of 48}\}$

$B = \{\text{multiples of 3}\}$

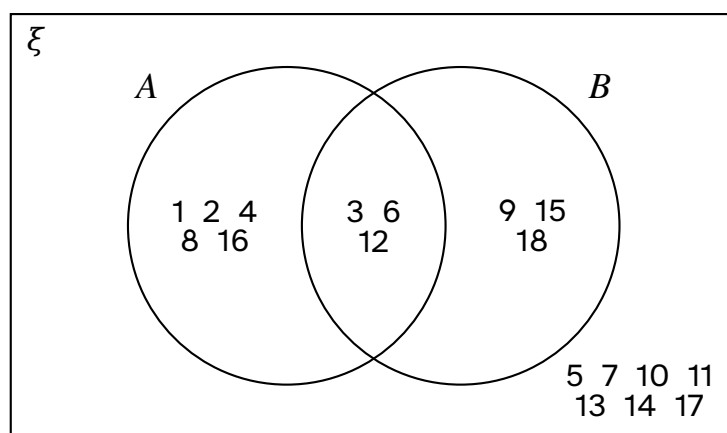
(a) Complete the Venn diagram.

Factors of 48 from 1 to 18: 1, 2, 3, 4, 6, 8, 12, 16

Multiples of 3 from 1 to 18: 3, 6, 9, 12, 15, 18

In both (the overlap): 3, 6, 12

All the other numbers go outside the circles: 5, 7, 10, 11, 13, 14, 17



Answer: See the completed Venn diagram

Tip: Use every number from 1 to 18 exactly once. Numbers in neither set go inside the rectangle but outside both circles.

Marks: 2 marks for a fully correct diagram. 1 mark if three of the four regions are correct.

(b) Nia picks a number at random from ξ .

She says,

“The probability that my number is in $A \cup B$ is more than $\frac{2}{3}$ ”

Is Nia correct?

You must show how you get your answer.

$A \cup B$ means in A **or** in B (or in both): every number inside the circles.

A only: 1, 2, 4, 8, 16 (5 numbers)

Both: 3, 6, 12 (3 numbers)

B only: 9, 15, 18 (3 numbers)

$$5 + 3 + 3 = 11, \text{ so } P(A \cup B) = \frac{11}{18}$$

$$\frac{2}{3} = \frac{12}{18}, \text{ and } \frac{11}{18} \text{ is less than } \frac{12}{18}$$

So Nia is **not** correct.

Answer: No. $P(A \cup B) = \frac{11}{18}$, which is less than $\frac{2}{3} = \frac{12}{18}$

Tip: $\cup$ means “or”: count every number in A , in B or in both, but count the overlap only once. Then compare using a common denominator.

Marks: 1 mark for $\frac{11}{18}$. 1 mark for “No” with a correct comparison, such as $\frac{11}{18} < \frac{12}{18}$.

11 (3 marks)

Prove algebraically that the recurring decimal $0.2\dot{4}\dot{5}$ can be written as $\frac{27}{110}$

$$\text{Let } x = 0.2454545 \dots$$

$$1000x = 245.454545 \dots$$

$$10x = 2.454545 \dots$$

$$\text{Subtract: } 1000x - 10x = 245.4545 \dots - 2.4545 \dots$$

$$990x = 243$$

$$x = \frac{243}{990}$$

Divide top and bottom by 9: $x = \frac{27}{110}$

Answer: $990x = 243$, so $x = \frac{243}{990} = \frac{27}{110}$

Tip: Choose the two multiples so that the recurring parts line up: here $1000x$ and $10x$ both end in ...4545...

Marks: 1 mark for two recurring decimals that subtract to give a whole number (or terminating decimal). 1 mark for $990x = 243$ or $\frac{243}{990}$. 1 mark for completing the proof to $\frac{27}{110}$.

12 (3 marks)

The diagram shows a logo for a café sign.

The logo is made from a sector OAB of a circle, centre O , and a right-angled triangle OBC .

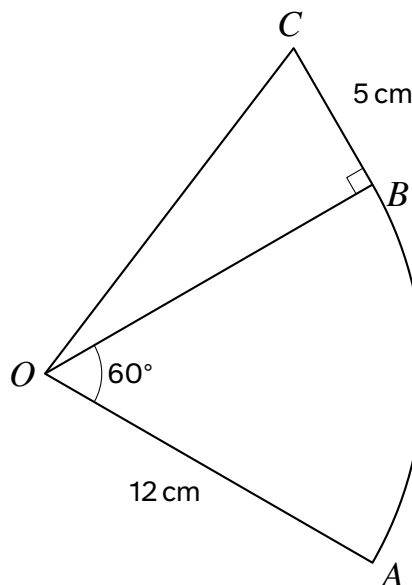


Diagram **NOT** accurately drawn

Work out the perimeter of the logo.

Give your answer in the form $a + b\pi$, where a and b are integers.

$OB = OA = 12$ cm (radii of the circle).

$$\text{Arc } AB = \frac{60}{360} \times 2 \times \pi \times 12 = \frac{1}{6} \times 24\pi = 4\pi \text{ cm}$$

Pythagoras in triangle OBC : $OC^2 = 12^2 + 5^2 = 144 + 25 = 169$

$$OC = \sqrt{169} = 13 \text{ cm}$$

The perimeter goes round the outside: OA , arc AB , BC and CO (OB is inside the shape).

$$\text{Perimeter} = 12 + 4\pi + 5 + 13 = 30 + 4\pi$$

Answer: $(30 + 4\pi)$ cm

Tip: The perimeter is only the outside edge. Do not include OB , which is inside the logo.

Marks: 1 mark for the arc length 4π . 1 mark for $OC = 13$. 1 mark for $30 + 4\pi$.

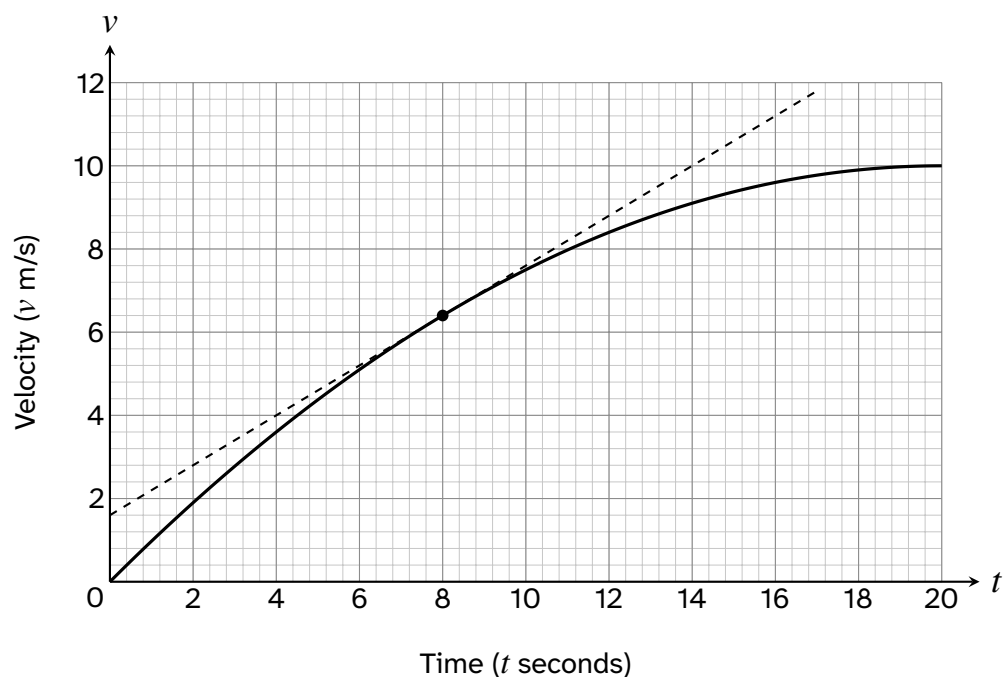
13 (4 marks)

Zara cycles away from a set of traffic lights. The graph shows her velocity, v m/s, t seconds after she starts.

(a) Work out an estimate for the gradient of the graph at $t = 8$

At $t = 8$ the velocity is 6.4 m/s.

Draw a tangent to the curve at $(8, 6.4)$: a straight line that just touches the curve there (the dashed line).



Two points on the tangent, far apart: (0, 1.6) and (17, 11.8)

$$\begin{aligned}\text{Gradient} &= \frac{\text{change in } v}{\text{change in } t} = \frac{11.8 - 1.6}{17 - 0} = \frac{10.2}{17} \\ &= 0.6\end{aligned}$$

Answer: about 0.6 (accept 0.5 to 0.7)

Tip: You must draw a tangent. Dividing the coordinates of the point ($6.4 \div 8 = 0.8$) does not give the gradient of the curve.

Marks: 1 mark for drawing a tangent at $t = 8$. 1 mark for change in $v \div$ change in t for your tangent. 1 mark for an answer from 0.5 to 0.7.

- (b) Zara says, “My acceleration is greater at $t = 16$ than at $t = 8$ ”

Is Zara correct?

Give a reason for your answer.

The gradient of a velocity–time graph is the acceleration.

The curve is less steep at $t = 16$ than at $t = 8$: a tangent at $t = 16$ has a gradient of only about 0.2.

So Zara’s acceleration at 16 seconds is **less** than at 8 seconds.

Zara is **not** correct.

Answer: No. The graph is less steep at $t = 16$ than at $t = 8$, and the gradient is the acceleration, so her acceleration is less at 16 seconds.

Tip: Her velocity is greater at $t = 16$, but acceleration is the **gradient**, not the height of the graph. Compare how steep the curve is.

14 (3 marks)

m is directly proportional to the cube of r .

When $r = 3$, $m = 108$

Find the value of m when $r = 5$

Write the proportion as an equation: $m = kr^3$

Substitute $r = 3$, $m = 108$: $108 = k \times 27$

$$k = 108 \div 27 = 4$$

So $m = 4r^3$

When $r = 5$: $m = 4 \times 125 = 500$

Answer: $m = 500$

Tip: “The cube of r ” means r^3 . Using $m = kr$ gives the wrong answer.

Marks: 1 mark for $m = kr^3$ (or $108 = 27k$). 1 mark for $k = 4$. 1 mark for 500.

15 (4 marks)

Make x the subject of $y = \frac{4x + 3}{x - 2}$

Multiply both sides by $(x - 2)$: $y(x - 2) = 4x + 3$

Expand the bracket: $xy - 2y = 4x + 3$

Get the x terms on one side: $xy - 4x = 2y + 3$

Factorise: $x(y - 4) = 2y + 3$

Divide by $(y - 4)$: $x = \frac{2y + 3}{y - 4}$

Answer: $x = \frac{2y + 3}{y - 4}$

Tip: x appears twice. Collect both x terms on the same side, then take x out as a factor.

Marks: 1 mark for $y(x - 2) = 4x + 3$. 1 mark for getting both x terms on one side and the other terms on the other. 1 mark for factorising. 1 mark for the answer.

16 (4 marks)

At a party, a bowl contains n milk chocolates and 3 dark chocolates.

Ellis takes a chocolate at random and eats it.

He then takes a second chocolate at random and eats it.

The probability that Ellis eats two milk chocolates is $\frac{5}{14}$

Work out the number of chocolates in the bowl before Ellis took any.

You must show all your working.

There are $n + 3$ chocolates at the start.

$$P(\text{first is milk}) = \frac{n}{n+3}$$

After eating one milk chocolate there are $n - 1$ milk out of $n + 2$: $P(\text{second is milk})$
 $= \frac{n-1}{n+2}$

$$\frac{n}{n+3} \times \frac{n-1}{n+2} = \frac{5}{14}$$

Cross-multiply: $14n(n-1) = 5(n+3)(n+2)$

$$14n^2 - 14n = 5n^2 + 25n + 30$$

$$9n^2 - 39n - 30 = 0$$

Divide by 3: $3n^2 - 13n - 10 = 0$

Factorise: $(3n+2)(n-5) = 0$

$n = 5$ (the number of chocolates cannot be $-\frac{2}{3}$)

Total at the start = $5 + 3 = 8$

$$\text{Check: } \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14} \quad \checkmark$$

Answer: 8

Tip: Without replacement: the second fraction has 1 less on top **and** 1 less on the bottom.

Read the question: it asks for the total, $n + 3$, not n .

Marks: 1 mark for $\frac{n}{n+3} \times \frac{n-1}{n+2}$. 1 mark for an equation without fractions. 1 mark for the quadratic $3n^2 - 13n - 10 = 0$ and solving it. 1 mark for 8.

17 (3 marks)

- (a) Simplify $\sqrt{245}$

Find a square factor of 245: $245 = 49 \times 5$

$$\sqrt{245} = \sqrt{49} \times \sqrt{5} = 7\sqrt{5}$$

Answer: $7\sqrt{5}$

- (b) Rationalise the denominator of $\frac{30}{\sqrt{12}}$

Give your answer in its simplest form.

Multiply the top and bottom by $\sqrt{12}$: $\frac{30}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{30\sqrt{12}}{12}$

$$= \frac{5\sqrt{12}}{2}$$

Simplify the surd: $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$$\frac{5\sqrt{12}}{2} = \frac{5 \times 2\sqrt{3}}{2} = 5\sqrt{3}$$

Answer: $5\sqrt{3}$

Tip: Check the surd at the end: $\frac{5\sqrt{12}}{2}$ is not in its simplest form, because 12 has the square factor 4.

Marks: 1 mark for multiplying the top and bottom by $\sqrt{12}$ (or $\sqrt{3}$). 1 mark for $5\sqrt{3}$.

18 (4 marks)

A , B and C are points on a circle, centre O .
 TAS is the tangent to the circle at A .

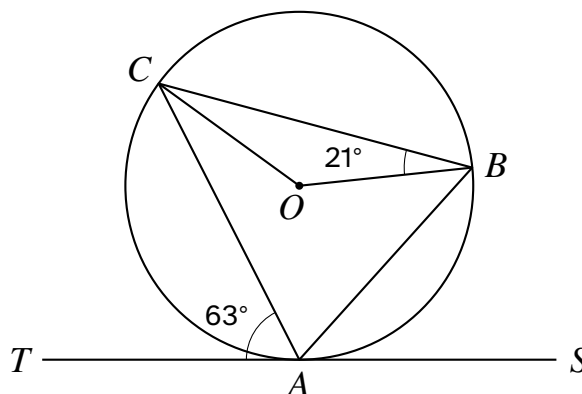


Diagram **NOT** accurately drawn

Work out the size of angle ACB .

You must give a reason for each stage of your working.

Angle $ABC = 63^\circ$. Reason: the angle between a tangent and a chord is equal to the angle in the alternate segment.

$OB = OC$ (radii), so triangle OBC is isosceles and angle $OCB = 21^\circ$. Reason: base angles of an isosceles triangle are equal.

Angle $BOC = 180 - 21 - 21 = 138^\circ$. Reason: angles in a triangle add up to 180° .

Angle $BAC = 138 \div 2 = 69^\circ$. Reason: the angle at the centre is twice the angle at the circumference.

Angle $ACB = 180 - 63 - 69 = 48^\circ$. Reason: angles in a triangle add up to 180° .

Another way: angle $OAT = 90^\circ$ (a tangent is perpendicular to the radius), so angle $OAC = 90 - 63 = 27^\circ$; $OA = OC$, so angle $OCA = 27^\circ$; then angle $ACB = 27 + 21 = 48^\circ$.

Answer: 48°

Tip: Write each reason in full, using the key words: “alternate segment”, “angle at the centre is twice the angle at the circumference”.

Marks: 1 mark for angle $ABC = 63^\circ$ or angle $BOC = 138^\circ$. 1 mark for angle $BAC = 69^\circ$. 1 mark for 48° . 1 mark for correct reasons, including the circle theorems.

19 (4 marks)

The functions f and g are such that

$$f(x) = \frac{2x+7}{3} \qquad g(x) = 3x - 2$$

(a) Find $fg(x)$

Give your answer in its simplest form.

$fg(x) = f(g(x))$: put $g(x) = 3x - 2$ into f .

$$fg(x) = \frac{2(3x-2)+7}{3}$$

$$= \frac{6x-4+7}{3} = \frac{6x+3}{3}$$

$$= 2x + 1$$

Answer: $fg(x) = 2x + 1$

Tip: $fg(x)$ means do g first, then f . It does not mean $f(x) \times g(x)$.

Marks: 1 mark for $\frac{2(3x-2)+7}{3}$. 1 mark for $2x + 1$.

(b) Find $f^{-1}(x)$

Write $y = \frac{2x+7}{3}$

Multiply by 3: $3y = 2x + 7$

Subtract 7: $3y - 7 = 2x$

Divide by 2: $x = \frac{3y-7}{2}$

Swap the letters: $f^{-1}(x) = \frac{3x-7}{2}$

Answer: $f^{-1}(x) = \frac{3x-7}{2}$

Tip: Undo the operations in reverse order: the last thing f does is divide by 3, so multiply by 3 first.

Marks: 1 mark for a correct first step, such as $3y = 2x + 7$. 1 mark for $\frac{3x-7}{2}$.

20 (3 marks)

Prove algebraically that, for any two consecutive multiples of 3, the difference between their squares is always an odd multiple of 9

Let the two consecutive multiples of 3 be $3n$ and $3n + 3$, where n is an integer.

$$(3n + 3)^2 - (3n)^2 = 9n^2 + 18n + 9 - 9n^2$$

$$= 18n + 9$$

$$= 9(2n + 1)$$

$2n$ is even, so $2n + 1$ is odd.

So the difference is $9 \times$ an odd number: an odd multiple of 9.

Answer: $(3n + 3)^2 - (3n)^2 = 18n + 9 = 9(2n + 1)$, and $2n + 1$ is odd, so it is an odd multiple of 9.

Tip: Trying a few numbers is not a proof. Use algebra: $3n$ and $3n + 3$ work for every integer n .

Marks: 1 mark for $3n$ and $3n + 3$. 1 mark for expanding and simplifying to $18n + 9$. 1 mark for $9(2n + 1)$ with a statement that $2n + 1$ is odd.

21 (5 marks)

C is the circle with equation $x^2 + y^2 = 52$

The line L is the tangent to C at the point P .

The x -coordinate of P is positive.

L is parallel to the line with equation $4x + 6y = 15$

Find an equation of L .

You must show all your working.

Gradient of $4x + 6y = 15$: $6y = -4x + 15$, so $y = -\frac{2}{3}x + \frac{5}{2}$. The gradient is $-\frac{2}{3}$

L is parallel to this line, so the gradient of L is $-\frac{2}{3}$

The radius OP is perpendicular to the tangent L , so the gradient of OP is $\frac{3}{2}$ (negative reciprocal of $-\frac{2}{3}$).

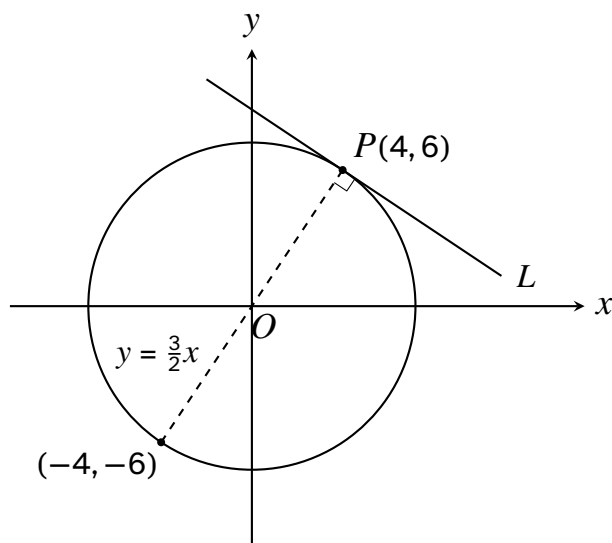
OP goes through the centre $O(0, 0)$, so P lies on the line $y = \frac{3}{2}x$

Substitute into the circle: $x^2 + \left(\frac{3}{2}x\right)^2 = 52$

$$x^2 + \frac{9}{4}x^2 = 52, \text{ so } \frac{13}{4}x^2 = 52, x^2 = 16$$

$x = 4$ (the x -coordinate of P is positive), so $y = \frac{3}{2} \times 4 = 6$: P is $(4, 6)$

Check: $4^2 + 6^2 = 16 + 36 = 52$ ✓



Equation of L : $y - 6 = -\frac{2}{3}(x - 4)$

$$y = -\frac{2}{3}x + \frac{8}{3} + 6$$

$$y = -\frac{2}{3}x + \frac{26}{3} \quad (\text{or } 2x + 3y = 26)$$

Answer: $y = -\frac{2}{3}x + \frac{26}{3}$ (or $2x + 3y = 26$)

Tip: You are not told P . Use the two facts you know: the radius to P is perpendicular to the tangent, and P is on the circle. Then use P to find the equation of L .

Marks: 1 mark for using the gradient $-\frac{2}{3}$ to get the gradient of the radius, $\frac{3}{2}$. 1 mark for substituting $y = \frac{3}{2}x$ into the circle equation. 1 mark for $P(4, 6)$. 1 mark for a method to find the equation of L through P . 1 mark for a correct equation.

22 (5 marks)

$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

D is the point on OA such that $OD:DA = 2:1$

CBF is a straight line such that B is the midpoint of CF .

E is the point on AB such that D, E and F lie on a straight line.

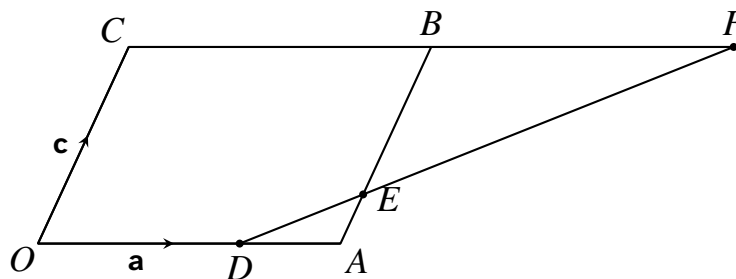


Diagram **NOT** accurately drawn

Work out the ratio $AE:EB$

You must show all your working.

$$OD:DA = 2:1, \text{ so } \overrightarrow{DA} = \frac{1}{3}\mathbf{a}$$

$$OABC \text{ is a parallelogram, so } \overrightarrow{AB} = \overrightarrow{OC} = \mathbf{c} \text{ and } \overrightarrow{CB} = \overrightarrow{OA} = \mathbf{a}$$

$$B \text{ is the midpoint of } CF, \text{ so } \overrightarrow{CF} = 2\mathbf{a}$$

$$\overrightarrow{DF} = \overrightarrow{DO} + \overrightarrow{OC} + \overrightarrow{CF} = -\frac{2}{3}\mathbf{a} + \mathbf{c} + 2\mathbf{a} = \frac{4}{3}\mathbf{a} + \mathbf{c}$$

E is on AB , so $\overrightarrow{AE} = \lambda \mathbf{c}$ for some number λ .

$$\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE} = \frac{1}{3}\mathbf{a} + \lambda \mathbf{c}$$

D , E and F are on a straight line, so $\overrightarrow{DE} = k\overrightarrow{DF}$ for some number k :

$$\frac{1}{3}\mathbf{a} + \lambda \mathbf{c} = k\left(\frac{4}{3}\mathbf{a} + \mathbf{c}\right) = \frac{4}{3}k\mathbf{a} + k\mathbf{c}$$

Compare the $\mathbf{a}$ parts: $\frac{1}{3} = \frac{4}{3}k$, so $k = \frac{1}{4}$

Compare the $\mathbf{c}$ parts: $\lambda = k = \frac{1}{4}$

$$\text{So } \overrightarrow{AE} = \frac{1}{4}\mathbf{c} \text{ and } \overrightarrow{EB} = \frac{3}{4}\mathbf{c}$$

$$AE:EB = \frac{1}{4}:\frac{3}{4} = 1:3$$

Answer: 1:3

Tip: You do not know where E is, so call $\overrightarrow{AE} = \lambda \mathbf{c}$. Points on a straight line give vectors that are multiples of each other: compare the $\mathbf{a}$ parts and the $\mathbf{c}$ parts.

Marks: 1 mark for $\overrightarrow{DF}$ (or $\overrightarrow{OF}$). 1 mark for $\overrightarrow{DE}$ written with an unknown, such as $\frac{1}{3}\mathbf{a} + \lambda \mathbf{c}$. 1 mark for using $\overrightarrow{DE} = k\overrightarrow{DF}$. 1 mark for finding k or λ . 1 mark for 1:3.
