

Mock Set B – Paper 3

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 3 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Write these numbers in order of size.
Start with the smallest number.

37.6% $0.\dot{3}\dot{7}$ $\frac{3}{8}$ $0.3\dot{7}$ 0.374

Write every number as a decimal, to at least 4 decimal places.

$$37.6\% = 0.376$$

$$0.\dot{3}\dot{7} = 0.373737 \dots \text{ (the 3 and the 7 both repeat)}$$

$$\frac{3}{8} = 3 \div 8 = 0.375$$

$$0.3\dot{7} = 0.37777 \dots \text{ (only the 7 repeats)}$$

$$0.374 = 0.3740$$

$$\text{Compare: } 0.3737 \dots < 0.3740 < 0.3750 < 0.3760 < 0.3777 \dots$$

Answer: $0.\dot{3}\dot{7}$, 0.374, $\frac{3}{8}$, 37.6%, $0.3\dot{7}$

Tip: Write the recurring decimals out to several places before comparing: $0.\dot{3}\dot{7} = 0.3737 \dots$ but $0.3\dot{7} = 0.3777 \dots$. The dots show which digits repeat.

Marks: 2 marks for all five in the correct order. 1 mark for four in the correct order, or for all five written as correct decimals.

2 (3 marks)

A park has two fountains.

- Fountain *A* starts a jet of water every 36 minutes.
- Fountain *B* starts a jet of water every 1 hour 30 minutes.

Both fountains start a jet of water at 10:15 am.

The park closes at 5 pm.

How many more times will both fountains start a jet of water at the same time before the park closes?

Change 1 hour 30 minutes into minutes: 90 minutes.

The fountains start together again after the lowest common multiple (LCM) of 36 and 90.

Multiples of 36: 36, 72, 108, 144, 180, ...

Multiples of 90: 90, 180, ...

LCM = 180 minutes = 3 hours

Together again at 1:15 pm, then 4:15 pm. The next time would be 7:15 pm, after the park closes.

So they start together 2 more times.

Answer: 2 (at 1:15 pm and 4:15 pm)

Tip: You need the lowest common multiple, not the highest common factor (18). Convert 1 hour 30 minutes to 90 minutes first: it is not 130 minutes or 1.3 hours.

Marks: 1 mark for using 90 minutes with multiples or prime factors of 36 and 90. 1 mark for 180 minutes (or 3 hours, or 1:15 pm). 1 mark for 2.

3 (2 marks)

Here are the first four terms of a linear sequence.

11 18 25 32

Abi says,

‘303 is one of the numbers in this sequence.’

Is Abi correct?

Tick a box.

You must show your working.

The terms go up by 7, so the n th term is $7n + 4$ (check: $7 \times 1 + 4 = 11$).

Set $7n + 4 = 303$

$7n = 299$

$n = 299 \div 7 = 42.71 \dots$

n must be a whole number, so 303 is not a term.

(The 42nd term is 298 and the 43rd term is 305.)

Answer: No (Abi is not correct). $7n + 4 = 303$ gives $n = 42.7 \dots$, which is not a whole number

Tip: A number is a term only if n comes out as a whole number. Saying ‘303 is odd’ is not a reason: the terms in this sequence alternate between odd and even.

Marks: 1 mark for the n th term $7n + 4$ (or $299 \div 7$). 1 mark for ‘No’ with working that shows why.

4 (2 marks)

Here is a list of ingredients for vegetable chilli for 8 people.

Vegetable chilli (serves 8)
2 onions
3 peppers
800 g chopped tomatoes
500 g kidney beans
560 ml vegetable stock
2 teaspoons chilli powder

Nour is making vegetable chilli for 75 people at a community lunch.

Vegetable stock is sold in 1 litre cartons.

How many cartons of vegetable stock does Nour need to buy?

Stock for 1 person: $560 \div 8 = 70$ ml

Stock for 75 people: $70 \times 75 = 5250$ ml

5250 ml = 5.25 litres

5 cartons is not enough, so she must buy 6 cartons.

Answer: 6

Tip: Round **up**: 5 cartons would only give 5 litres. Remember 1 litre = 1000 ml.

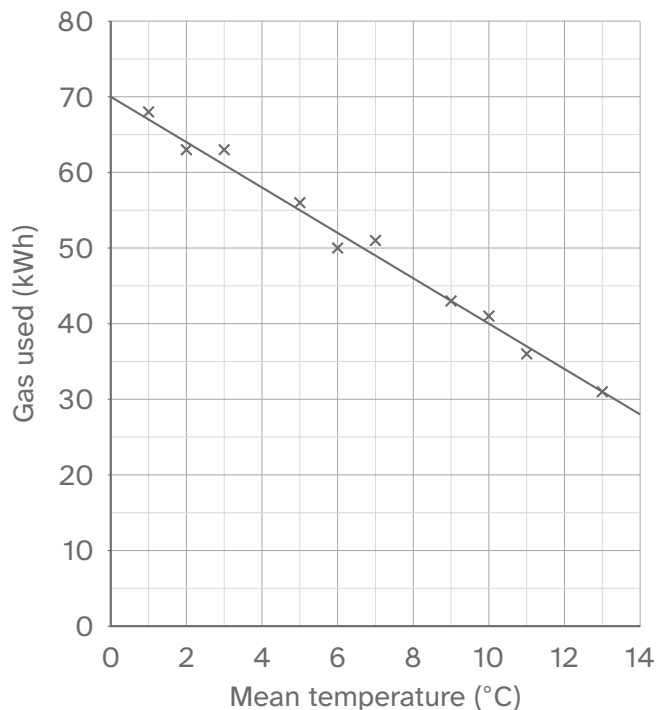
Marks: 1 mark for 5250 ml (or 5.25 litres). 1 mark for 6.

5 (2 marks)

Lara records the mean temperature and the amount of gas her house uses on 10 days in winter.

The scatter graph shows this information.

A line of best fit has been drawn.



- (a) Use the line of best fit to estimate the amount of gas used on a day when the mean temperature is 8 °C.

Go up from 8 °C on the horizontal axis to the line of best fit.

Go across to the vertical axis and read off the value: about 46 kWh.

Answer: 46 kWh (accept 44 to 48)

Tip: Each small square on the vertical axis is 5 kWh. Read from the line, not from the nearest cross.

- (b) Lara wants to use the line of best fit to estimate the amount of gas used on a day when the mean temperature is 25 °C.

Give a reason why this estimate may not be reliable.

The temperatures in the data only go from 1 °C to 13 °C.

25 °C is well outside this range, so we do not know that the pattern continues.

(Extending the line to 25 °C would give a negative amount of gas, which is impossible.)

Answer: 25 °C is outside the range of the data, so the trend may not continue

Tip: Using a line of best fit outside the range of the data is called extrapolation. Say what the range of the data is.

6 (3 marks)

Freddie and Jade each run a 5 km race in March and again in June.
The table shows their times.

	March	June
Freddie	34 minutes 0 seconds	31 minutes 15 seconds
Jade	27 minutes 30 seconds	25 minutes 15 seconds

Freddie says,

‘My time improved by more than Jade’s time, so I made the greater percentage improvement.’

Is Freddie correct?

Tick a box.

You must show your working.

Work in seconds. 1 minute = 60 seconds.

Freddie: 34 min = 2040 s and 31 min 15 s = 1875 s. Improvement = 165 s

Jade: 27 min 30 s = 1650 s and 25 min 15 s = 1515 s. Improvement = 135 s

Freddie did improve by more seconds.

$$\text{Freddie: } \frac{165}{2040} \times 100 = 8.08 \dots \%$$

$$\text{Jade: } \frac{135}{1650} \times 100 = 8.18 \dots \%$$

Jade's percentage improvement is greater, so Freddie is **not** correct.

Answer: No. Freddie improved by 8.09% but Jade improved by 8.18%

Tip: Percentage change = $\frac{\text{change}}{\text{original}} \times 100$. 31 minutes 15 seconds is 31.25 minutes, not 31.15 minutes.

Marks: 1 mark for a correct percentage method for one runner. 1 mark for both. 1 mark for 'No' with 8.08 ... % (or 8.09%) and 8.18%.

7 (3 marks)

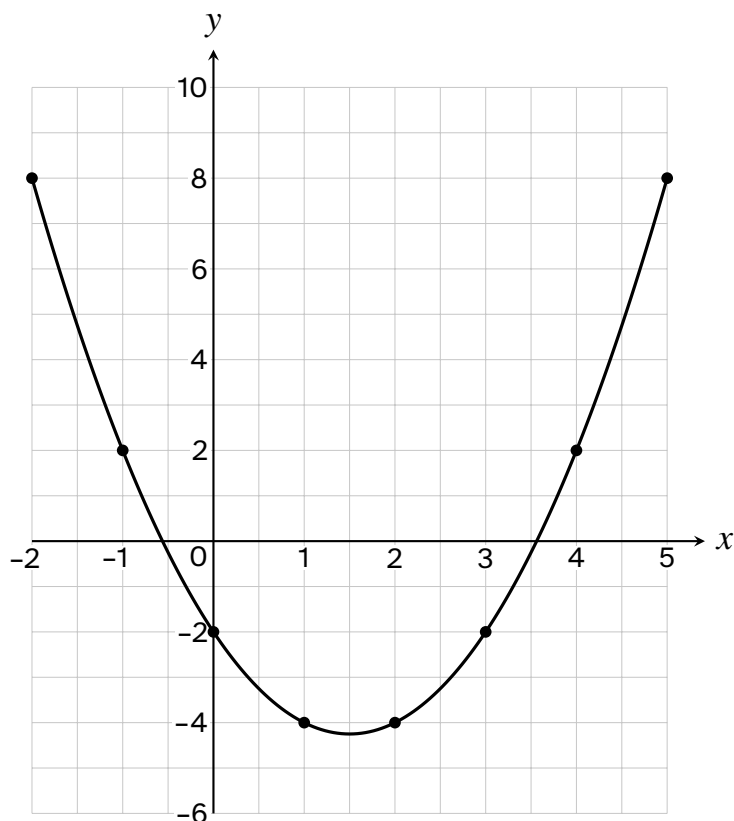
(a) Draw the graph of $y = x^2 - 3x - 2$ for values of x from -2 to 5

Complete the table by substituting each value of x .

$$x = -1: (-1)^2 - 3 \times (-1) - 2 = 1 + 3 - 2 = 2$$

$$x = 1: 1 - 3 - 2 = -4 \quad x = 2: 4 - 6 - 2 = -4 \quad x = 4: 16 - 12 - 2 = 2$$

Plot the eight points and join them with a smooth curve.



Answer: Smooth U-shaped curve through $(-2, 8)$, $(-1, 2)$, $(0, -2)$, $(1, -4)$, $(2, -4)$, $(3, -2)$, $(4, 2)$, $(5, 8)$

Tip: The lowest point of the curve is between $x = 1$ and $x = 2$, at $(1.5, -4.25)$. Draw a smooth curve through it, not a flat line from $(1, -4)$ to $(2, -4)$.

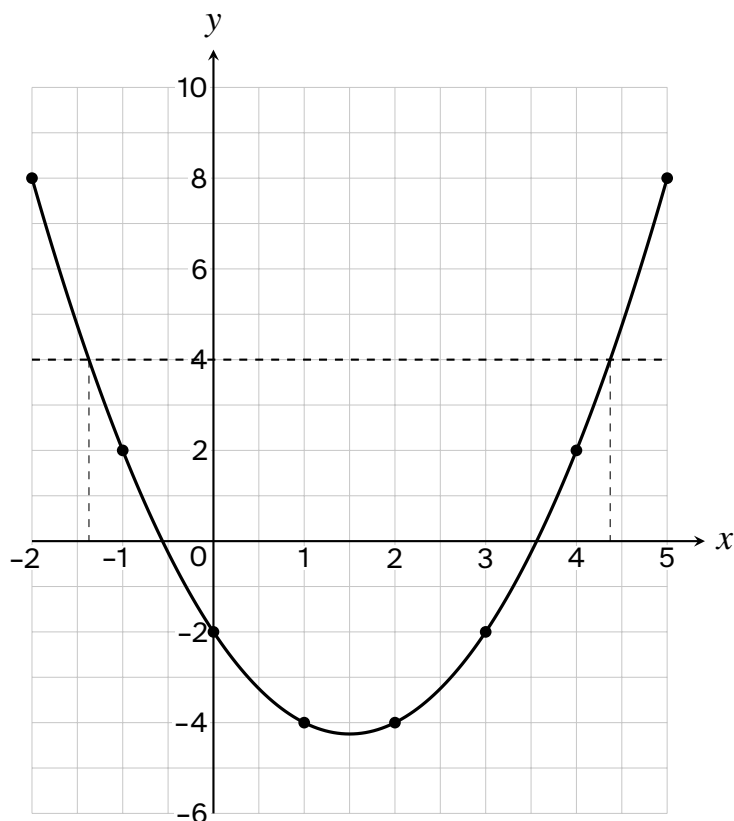
Marks: 2 marks for a fully correct smooth curve. 1 mark for at least six points plotted correctly.

- (b) Use your graph to estimate the solutions of $x^2 - 3x - 2 = 4$

$x^2 - 3x - 2 = 4$ means $y = 4$ on the graph.

Draw the line $y = 4$ and read off the x -coordinates where it crosses the curve.

$x \approx -1.4$ and $x \approx 4.4$



Answer: $x \approx -1.4$ and $x \approx 4.4$

Tip: Read the x -values where the line $y = 4$ crosses the curve. Do not read where the curve crosses the x -axis: that solves $x^2 - 3x - 2 = 0$.

8 (3 marks)

Ewa gives the same test to two classes.

- There are 24 students in class 10P. Their mean score is 15.5
- There are 28 students in class 10Q.
- The mean score of all 52 students is 16.2

Work out the mean score of the students in class 10Q.

Total score of all 52 students: $52 \times 16.2 = 842.4$

Total score of class 10P: $24 \times 15.5 = 372$

Total score of class 10Q: $842.4 - 372 = 470.4$

Mean for class 10Q: $470.4 \div 28 = 16.8$

Answer: 16.8

Tip: Work with totals, not means: the classes are different sizes. $2 \times 16.2 - 15.5 = 16.9$ is wrong.

Marks: 1 mark for 842.4 or 372. 1 mark for $(842.4 - 372) \div 28$. 1 mark for 16.8.

9 (3 marks)

$y = 650$ correct to 2 significant figures.

(a) Complete the error interval for y .

650 to 2 significant figures means y is rounded to the nearest 10.

Half of 10 is 5.

Lower limit: $650 - 5 = 645$. Upper limit: $650 + 5 = 655$

655 itself would round to 660, so the upper limit is not included.

Answer: $645 \leq y < 655$

Tip: The second significant figure of 650 is the tens digit, so y is rounded to the nearest 10, not the nearest 1 (649.5) or 100 (600).

Marks: 1 mark for 645. 1 mark for 655.

(b) Owain says,

‘When y is rounded to 1 significant figure, the answer must be 700’

Is Owain correct?

Tick a box.

Give a reason for your answer.

y can be anywhere from 645 up to (but not including) 655.

For example, $y = 647$ rounds to 600 to 1 significant figure.

So the answer could be 600, not always 700.

Answer: No. For example, $y = 647$ rounds to 600 to 1 significant figure

Tip: Values from 645 up to 650 round to 600; values from 650 up to 655 round to 700.

10 (3 marks)

The scale diagram shows a field with two gates, G and H , and a water tap, T .
Rory is going to put a water trough in the field.

Scale: 1 cm represents 10 m

The trough must be less than 40 m from the tap, T , and closer to gate G than to gate H . Show, by shading, the region where the trough can be placed.

Scale: 1 cm represents 10 m, so 40 m is 4 cm on the diagram.

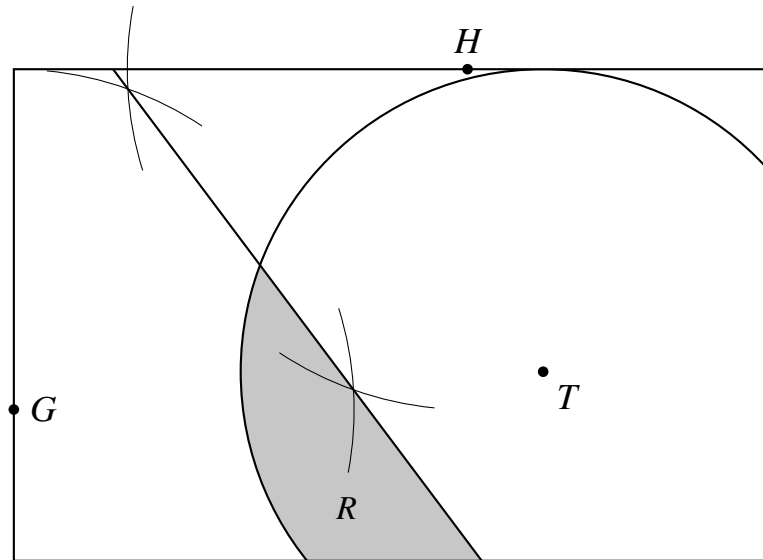
Draw a circle of radius 4 cm with centre T . Points less than 40 m from T are inside it.

Points the same distance from G and H lie on the perpendicular bisector of GH .

Construct it: with the compasses set to more than half of GH , draw arcs from G and from H on both sides of GH , then join the two points where the arcs cross.

Points closer to G are on G 's side of the bisector.

The region R is inside the circle **and** on G 's side of the bisector.



Answer: Region inside the circle of radius 4 cm, centre T , on the G side of the perpendicular bisector of GH

Tip: Convert the distance with the scale first (40 m is 4 cm). 'Closer to G than to H ' needs the perpendicular bisector of GH , not a circle round G . Leave your construction arcs showing.

Marks: 1 mark for the circle (or arc) of radius 4 cm about T . 1 mark for the perpendicular bisector of GH with construction arcs. 1 mark for the correct region shaded.

11 (3 marks)

A mobile crane stands on four legs.

- The total force of the crane on the ground is 96 000 newtons.
- The force is shared equally between the four legs.
- Each leg stands on a circular pad with diameter 60 cm.

The ground can safely take a pressure of up to 80 000 newtons/m²

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Is it safe to use the crane here?

You must show your working.

Force on one pad: $96\,000 \div 4 = 24\,000$ newtons

Radius of a pad: $60 \div 2 = 30$ cm = 0.3 m

Area of a pad: $\pi \times 0.3^2 = 0.2827 \dots \text{m}^2$

Pressure: $24\,000 \div 0.2827 \dots = 84\,883$ newtons/m² (to the nearest whole number)

84 883 is more than 80 000, so it is **not** safe.

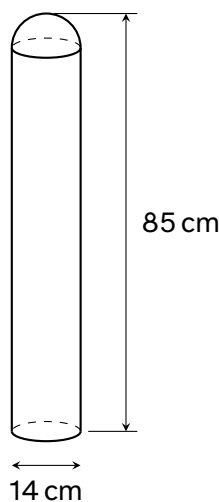
Answer: No. The pressure is about 84 900 newtons/m², which is more than 80 000 newtons/m²

Tip: The area must be in m² because the pressure is in newtons/m². Use the radius (0.3 m), not the diameter.

Marks: 1 mark for the area of a pad in m². 1 mark for dividing the force on one pad by that area. 1 mark for 'No' with 84 900 (or another correct comparison).

12 (3 marks)

A solid metal bollard is a cylinder with a hemisphere on top.



Not drawn accurately

The metal has a density of 7.2 g/cm³

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Work out the mass of the bollard.

Give your answer in kilograms to 3 significant figures.

Radius = $14 \div 2 = 7$ cm

The hemisphere is 7 cm tall, so the cylinder is $85 - 7 = 78$ cm tall.

Cylinder: $\pi \times 7^2 \times 78 = 12\,007.2 \dots$ cm³

Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3 = 718.3 \dots \text{cm}^3$

Total volume = $12\,725.5 \dots \text{cm}^3$

Mass = density $\times$ volume = $7.2 \times 12\,725.5 \dots = 91\,623.9 \dots \text{g}$

$91\,623.9 \dots \div 1000 = 91.6 \text{ kg (to 3 s.f.)}$

Answer: 91.6 kg

Tip: The 85 cm includes the hemisphere, so the cylinder is only 78 cm tall. Use half the volume of a sphere for the hemisphere, and the radius (7 cm), not the diameter.

Marks: 1 mark for the volume of the cylinder (height 78 cm) or of the hemisphere. 1 mark for the total volume $\times 7.2$. 1 mark for 91.6.

13 (3 marks)

(a) Solve $-3 \leq 5 - 2x < 11$

Subtract 5 from all three parts: $-8 \leq -2x < 6$

Divide all three parts by -2 . Dividing by a negative number reverses the inequality signs:

$$4 \geq x > -3$$

Write it the other way round: $-3 < x \leq 4$

Answer: $-3 < x \leq 4$

Tip: When you divide by -2 the signs flip: $-8 \leq -2x$ becomes $4 \geq x$. Check with a value: $x = 0$ gives 5, which is between -3 and 11.

Marks: 1 mark for $-8 \leq -2x < 6$ or one correct inequality. 1 mark for $-3 < x \leq 4$.

(b) n is an integer. Which set lists all the values of n for which $-3 \leq 5 - 2n < 11$ is true?

From part (a), $-3 < n \leq 4$.

-3 is **not** included ($<$), but 4 **is** included ($\leq$).

The integers are $-2, -1, 0, 1, 2, 3, 4$.

Answer: $\{-2, -1, 0, 1, 2, 3, 4\}$

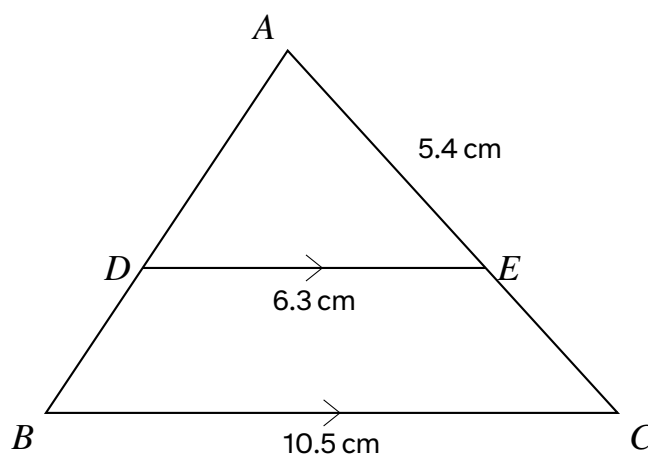
Tip: Check the end values in the original inequality: $n = -3$ gives $5 + 6 = 11$, and $11 < 11$ is false; $n = 4$ gives $5 - 8 = -3$, and $-3 \leq -3$ is true.

14 (3 marks)

ABC is a triangle.

D is a point on AB and E is a point on AC .

DE is parallel to BC .



Not drawn accurately

(a) Give a reason why triangle ADE is similar to triangle ABC .

DE is parallel to BC , so angle $ADE = \text{angle } ABC$ and angle $AED = \text{angle } ACB$ (corresponding angles).

Angle A is in both triangles.

So the three angles of triangle ADE are equal to the three angles of triangle ABC .

Answer: The angles are equal: corresponding angles are equal because DE is parallel to BC , and angle A is common

Tip: Similar means the same shape: all the angles match. Say why the angles are equal (corresponding angles, because the lines are parallel).

(b) Work out the length EC .

Scale factor from triangle ADE to triangle ABC : $10.5 \div 6.3 = \frac{5}{3} = 1.66 \dots$

$$AC = 5.4 \times \frac{5}{3} = 9 \text{ cm}$$

$$EC = AC - AE = 9 - 5.4 = 3.6 \text{ cm}$$

Answer: 3.6 cm

Tip: AC is the side of the big triangle that matches AE . Work out AC first, then subtract AE : 9 cm is AC , not EC .

Marks: 1 mark for the scale factor $\frac{5}{3}$ (or $AC = 9$). 1 mark for 3.6.

15 (3 marks)

Alina has a map of a nature reserve.

- The scale of the map is 1:50 000
- The area of the nature reserve on the map is 11.6 cm^2

Work out the real area of the nature reserve.

Give your answer in hectares.

$$1 \text{ hectare} = 10\,000 \text{ m}^2$$

1 cm on the map is 50 000 cm = 500 m in real life.

So 1 cm^2 on the map is $500 \times 500 = 250\,000 \text{ m}^2$ in real life.

$$\text{Real area} = 11.6 \times 250\,000 = 2\,900\,000 \text{ m}^2$$

$$\text{In hectares: } 2\,900\,000 \div 10\,000 = 290 \text{ hectares}$$

Answer: 290 hectares

Tip: Areas scale by the **square** of the scale factor. Multiplying 11.6 by 50 000 only once is the most common mistake.

Marks: 1 mark for 500 m (or $250\,000 \text{ m}^2$ for each cm^2). 1 mark for $2\,900\,000 \text{ m}^2$. 1 mark for 290.

16 (3 marks)

Make m the subject of $k = \sqrt{\frac{m+4}{m-1}}$

Square both sides: $k^2 = \frac{m+4}{m-1}$

Multiply both sides by $(m-1)$: $k^2(m-1) = m+4$

Expand: $k^2m - k^2 = m+4$

Collect the m terms on one side: $k^2m - m = k^2 + 4$

Factorise: $m(k^2 - 1) = k^2 + 4$

Divide by $(k^2 - 1)$: $m = \frac{k^2 + 4}{k^2 - 1}$

Answer: $m = \frac{k^2 + 4}{k^2 - 1}$

Tip: m appears twice, so collect both m terms on one side and then factorise:

$$k^2m - m = m(k^2 - 1).$$

Marks: 1 mark for squaring both sides. 1 mark for $k^2m - m = k^2 + 4$ (all m terms on one side). 1 mark for the final answer.

17 (3 marks)

Ruth is a forest ranger.

There are 3600 ash trees in a forest.

Ruth checks a random sample of 150 of these ash trees.

38 of the trees in her sample have a disease.

- (a) Work out an estimate for the number of ash trees in the forest that have the disease.

Fraction of the sample with the disease: $\frac{38}{150}$

Assume the same fraction of all the ash trees have the disease.

$$\frac{38}{150} \times 3600 = 912$$

Answer: 912

Tip: Find the fraction (or percentage) of the sample, then use the same fraction of the whole forest.

Marks: 1 mark for $\frac{38}{150} \times 3600$. 1 mark for 912.

- (b) Ruth says that her answer to part (a) is the exact number of ash trees in the forest that have the disease.

Give a reason why she may be wrong.

Ruth only checked 150 of the 3600 trees.

A different random sample would probably contain a different number of diseased trees.

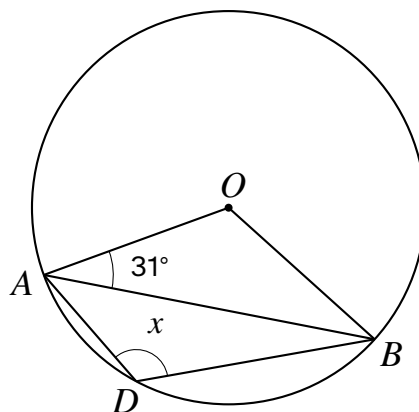
So her answer is an estimate, not an exact value.

Answer: It is only an estimate from a sample; a different sample would probably give a different answer

Tip: A sample gives an **estimate** for the whole population. Only checking every tree would give the exact number.

18 (3 marks)

A , B and D are points on a circle, centre O .
 D is on the minor arc AB .



Not drawn accurately

Work out the size of angle x .

You must show your working.

$OA = OB$ (radii), so triangle OAB is isosceles and angle $OBA = 31^\circ$.

Angle $AOB = 180^\circ - 31^\circ - 31^\circ = 118^\circ$ (angles in a triangle).

D is on the minor arc, so angle x is subtended by the **reflex** angle AOB .

Reflex angle $AOB = 360^\circ - 118^\circ = 242^\circ$ (angles at a point).

The angle at the centre is twice the angle at the circumference: $x = 242^\circ \div 2 = 121^\circ$

Answer: $x = 121^\circ$

Tip: 59° is the angle at the circumference on the **other** side of AB . Because D is on the minor arc, use the reflex angle 242° (or use opposite angles of a cyclic quadrilateral: $180 - 59 = 121$).

Marks: 1 mark for angle $AOB = 118^\circ$. 1 mark for 242° (or for an angle of 59° on the major arc). 1 mark for 121° .

19 (4 marks)

$$f(x) = x^2 - 5 \quad \text{and} \quad g(x) = 3x + 1$$

(a) Work out $fg(-3)$

$fg(-3)$ means work out $g(-3)$ first, then put the answer into f .

$$g(-3) = 3 \times (-3) + 1 = -8$$

$$f(-8) = (-8)^2 - 5 = 64 - 5 = 59$$

Answer: 59

Tip: Work from the inside out: $fg(x)$ means g first, then f . $gf(-3) = 13$ is the wrong order.

Marks: 1 mark for $g(-3) = -8$. 1 mark for 59.

(b) Solve $fg(x) = 4$

$$fg(x) = f(3x + 1) = (3x + 1)^2 - 5$$

$$(3x + 1)^2 - 5 = 4$$

$$(3x + 1)^2 = 9$$

$$3x + 1 = 3 \text{ or } 3x + 1 = -3$$

$$x = \frac{2}{3} \text{ or } x = -\frac{4}{3}$$

Answer: $x = \frac{2}{3}$ or $x = -\frac{4}{3}$

Tip: A square root has two values: $(3x + 1)^2 = 9$ gives $3x + 1 = 3$ **or** $3x + 1 = -3$. Many students lose the negative solution.

Marks: 1 mark for $(3x + 1)^2 - 5 = 4$. 1 mark for both solutions.

20 (3 marks)

Rahul's greyhound runs a 480 m race in 28.4 seconds. An ostrich can run at an average speed of 56 km/h. Suppose the ostrich ran the same 480 m race at this average speed. Which animal would finish first, and by how many seconds?

Change the ostrich's speed to metres per second.

$$56 \text{ km/h} = 56\,000 \text{ metres in } 3600 \text{ seconds}$$

$$56\,000 \div 3600 = 15.55 \dots \text{ m/s}$$

$$\text{Time for the ostrich to run 480 m: } 480 \div 15.55 \dots = 30.857 \dots \text{ seconds}$$

The greyhound takes 28.4 seconds, so the greyhound finishes first.

Difference: $30.857 \dots - 28.4 = 2.457 \dots$ seconds

Answer: The greyhound, by 2.46 seconds

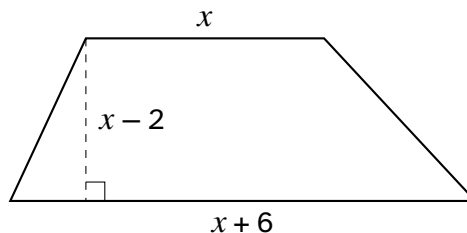
Tip: To change km/h to m/s, multiply by 1000 (metres) and divide by 3600 (seconds). Dividing by 60 only once gives a speed per minute.

Marks: 1 mark for changing 56 km/h to 15.6 m/s. 1 mark for the ostrich's time (30.9 seconds). 1 mark for the greyhound by 2.46 seconds.

21 (4 marks)

Here is a trapezium.

All measurements are in centimetres.



Not drawn accurately

The area of the trapezium is 40 cm^2

Work out the height of the trapezium.

Give your answer to 3 significant figures.

You must show your working.

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

$$\frac{1}{2}(x + x + 6)(x - 2) = 40$$

$$\frac{1}{2}(2x + 6) = x + 3, \text{ so } (x + 3)(x - 2) = 40$$

$$x^2 + x - 6 = 40$$

$$x^2 + x - 46 = 0$$

Quadratic formula with $a = 1$, $b = 1$, $c = -46$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-46)}}{2 \times 1} = \frac{-1 \pm \sqrt{185}}{2}$$

$$x = 6.3007 \dots \text{ or } x = -7.3007 \dots$$

A length cannot be negative, so $x = 6.3007 \dots$

Height = $x - 2 = 4.3007 \dots = 4.30$ cm (to 3 s.f.)

Answer: 4.30 cm

Tip: The question asks for the height, $x - 2$, not x . Keep the full value of x in your calculator until the end.

Marks: 1 mark for the area equation. 1 mark for $x^2 + x - 46 = 0$. 1 mark for a correct substitution into the quadratic formula. 1 mark for 4.30.

22 (4 marks)

Amara makes dog leads to sell at a craft fair.

- She has a rope of length 45 m, to the nearest metre.
- She cuts the rope into pieces.
- Each piece is 1.6 m long, to the nearest 10 cm.

Amara needs 27 pieces.

Can Amara be certain that she can cut 27 pieces from the rope?

Tick a box.

You must show your working.

To be certain, use the shortest possible rope and the longest possible pieces.

Lower bound of the rope: 44.5 m (rounded to the nearest metre)

Upper bound of each piece: 1.65 m (rounded to the nearest 10 cm = 0.1 m)

$44.5 \div 1.65 = 26.96 \dots$

In the worst case only 26 whole pieces can be cut.

(Or: $27 \times 1.65 = 44.55$ m, which is more than 44.5 m.)

So Amara **cannot** be certain.

Answer: No. In the worst case $44.5 \div 1.65 = 26.9 \dots$, so only 26 pieces are certain

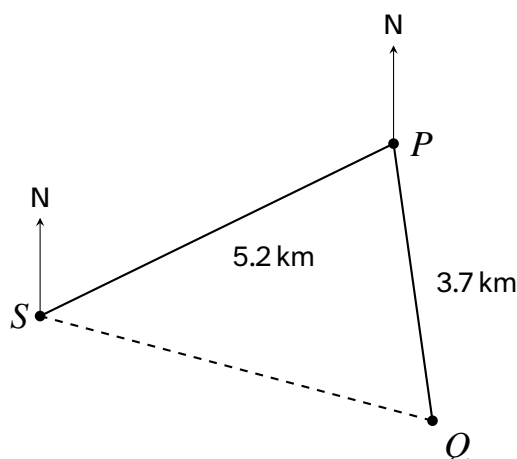
Tip: For the fewest possible pieces, divide the **smallest** rope by the **biggest** piece. Using $45 \div 1.6 = 28.1$ gives the wrong decision.

Marks: 1 mark for 44.5. 1 mark for 1.65. 1 mark for dividing the lower bound by the upper bound. 1 mark for 'No' with 26.9 ... (or $44.55 > 44.5$).

23 (5 marks)

A research ship sails from its base, S , to a point P , then to a point Q , and then straight back to S .

- P is 5.2 km from S on a bearing of 064°
- Q is 3.7 km from P on a bearing of 172°



Not drawn accurately

Work out the bearing of S from Q .

Give your answer to the nearest degree.

You must show your working.

Angle SPQ . The bearing of S from P is $064^\circ + 180^\circ = 244^\circ$.

Angle $SPQ = 244^\circ - 172^\circ = 72^\circ$

Distance SQ (cosine rule): $SQ^2 = 5.2^2 + 3.7^2 - 2 \times 5.2 \times 3.7 \times \cos 72^\circ$

$SQ^2 = 27.04 + 13.69 - 11.89 \dots = 28.839 \dots$, so $SQ = 5.370 \dots$ km

Angle PQS (sine rule): $\frac{\sin PQS}{5.2} = \frac{\sin 72^\circ}{5.370 \dots}$

$\sin PQS = 0.9209 \dots$, so angle $PQS = 67.06 \dots^\circ$

Bearing. The bearing of P from Q is $172^\circ + 180^\circ = 352^\circ$.

S is $67.06 \dots^\circ$ anticlockwise from the direction of P , so the bearing of S from Q is $352^\circ - 67.06 \dots^\circ = 284.9 \dots^\circ$

To the nearest degree: 285°

Answer: 285°

Tip: Draw north lines at every point and mark the angle inside the triangle at P first. A bearing is measured clockwise from north, so check your answer points the right way: S is to the west of Q , so the bearing must be between 180° and 360° .

Marks: 1 mark for angle $SPQ = 72^\circ$. 1 mark for using the cosine rule for SQ (5.37 km). 1 mark for using the sine rule (or cosine rule) for an angle of triangle SPQ . 1 mark for a correct method for the bearing. 1 mark for 285° .

24 (3 marks)

A tram leaves a tram stop.

The graph shows the distance, d metres, travelled by the tram in the first 20 seconds.

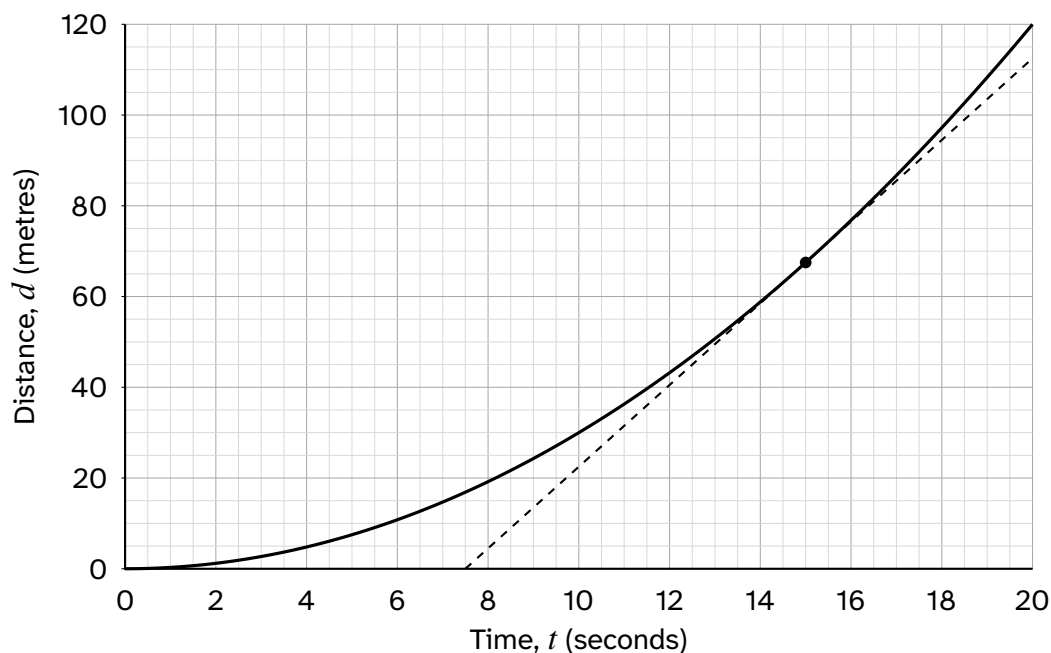
(a) By drawing a tangent, estimate the speed of the tram at 15 seconds.

Draw a tangent to the curve at $t = 15$: a straight line that just touches the curve there.

Pick two points on the tangent that are far apart, e.g. (7.5, 0) and (20, 112.5).

$$\text{Gradient} = \frac{\text{change in } d}{\text{change in } t} = \frac{112.5 - 0}{20 - 7.5} = \frac{112.5}{12.5} = 9$$

The gradient of a distance–time graph is the speed, so the speed is about 9 m/s.



Answer: About 9 m/s (accept 8 to 10)

Tip: Make the tangent long and pick two points on it that are far apart. The tangent must touch the curve at $t = 15$ without cutting across it.

Marks: 1 mark for a tangent drawn at $t = 15$ and a gradient method. 1 mark for an answer in the range 8 to 10.

- (b) Tariq divides the distance at 15 seconds by 15 to estimate the speed of the tram at 15 seconds.

Explain why his method does not give the speed at 15 seconds.

$67.5 \div 15 = 4.5$ is the gradient of the straight line from (0, 0) to (15, 67.5).

That is the **average** speed for the first 15 seconds.

The tram is speeding up (the curve gets steeper), so its speed at 15 seconds is more than its average speed.

The speed at 15 seconds is the gradient of the tangent at that point.

Answer: He has found the average speed over the first 15 seconds, not the speed at 15 seconds. The tram is accelerating, so its speed at 15 seconds is greater

Tip: Distance $\div$ time gives an average speed. For the speed at an instant, you need the gradient of the tangent.

25 (3 marks)

a and b are positive constants. $(x + a)^2 + b(x - 1) \equiv x^2 + 10x + 5$. Work out the values of a and b .

Expand the left-hand side: $x^2 + 2ax + a^2 + bx - b$

Compare the coefficients of x : $2a + b = 10$

Compare the constant terms: $a^2 - b = 5$

Add the two equations to eliminate b : $a^2 + 2a = 15$

$$a^2 + 2a - 15 = 0$$

$$(a + 5)(a - 3) = 0, \text{ so } a = -5 \text{ or } a = 3$$

a is positive, so $a = 3$

$$\text{Then } b = 10 - 2 \times 3 = 4$$

$$\text{Check: } (x + 3)^2 + 4(x - 1) = x^2 + 6x + 9 + 4x - 4 = x^2 + 10x + 5$$

Answer: $a = 3$, $b = 4$

Tip: In an identity the coefficients of each power of x must match. Write one equation for the x terms and one for the numbers, then solve them together.

Marks: 1 mark for both equations, $2a + b = 10$ and $a^2 - b = 5$. 1 mark for $a^2 + 2a - 15 = 0$ (or another correct method to eliminate b). 1 mark for $a = 3$ and $b = 4$.

26 (4 marks)

Vera checks the weather forecast every morning.

- The probability that the forecast says it will rain is 0.3
- If the forecast says it will rain, the probability that Vera takes an umbrella is 0.85
- If the forecast does not say it will rain, the probability that Vera takes an umbrella is 0.2

On one morning, Vera takes an umbrella.

Work out the probability that the forecast said it would rain on that morning.

There are two ways that Vera takes an umbrella.

Forecast rain **and** umbrella: $0.3 \times 0.85 = 0.255$

Forecast no rain **and** umbrella: $0.7 \times 0.2 = 0.14$

$P(\text{umbrella}) = 0.255 + 0.14 = 0.395$

We know she took an umbrella, so divide by $P(\text{umbrella})$:

$$P(\text{forecast rain given umbrella}) = \frac{0.255}{0.395} = 0.6455 \dots$$

$$= 0.646 \text{ (to 3 s.f.) or } \frac{51}{79}$$

Answer: $\frac{51}{79}$ (0.646 to 3 s.f.)

Tip: 'Given that she takes an umbrella' means the denominator is $P(\text{umbrella}) = 0.395$, not 1. A tree diagram helps you find both ways she can take an umbrella.

Marks: 1 mark for 0.3×0.85 . 1 mark for 0.7×0.2 . 1 mark for $0.255 \div (0.255 + 0.14)$. 1 mark for 0.646 or $\frac{51}{79}$.