

Mock Set B – Paper 2

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

- (a) Write 905 000 000 in standard form.

Put the decimal point after the first non-zero digit: 9.05

To get from 9.05 to 905 000 000 the digits move 8 places to the left.

$$905\,000\,000 = 9.05 \times 10^8$$

Answer: 9.05×10^8

Tip: The first number must be at least 1 and less than 10. 90.5×10^7 has the right value but is not in standard form.

- (b) Work out $\frac{8.1 \times 10^7 + 5.6 \times 10^6}{2.4 \times 10^{-3}}$. Give your answer in standard form to 3 significant figures.

Use the $\times 10^x$ key and put brackets round the top and the bottom, or use the fraction key.

$$\text{Top: } 8.1 \times 10^7 + 5.6 \times 10^6 = 81\,000\,000 + 5\,600\,000 = 86\,600\,000$$

$$86\,600\,000 \div 0.0024 = 36\,083\,333.333\ldots$$

$$= 3.6083 \ldots \times 10^{10}$$

To 3 significant figures: 3.61×10^{10}

Answer: 3.61×10^{10}

Tip: You cannot add numbers in standard form by adding the front numbers and the powers: $8.1 \times 10^7 + 5.6 \times 10^6$ is 8.66×10^7 , not 13.7×10^{13} .

Marks: 1 mark for $3.608 \dots \times 10^{10}$ (or 36 083 333 333). 1 mark for 3.61×10^{10} .

2 (3 marks)

2850 people visited a castle. 36% were members of a heritage trust and did not pay. $\frac{5}{8}$ of the other visitors bought tickets online. The rest paid at the gate. How many paid at the gate?

Visitors who were not members: $100\% - 36\% = 64\%$

$$0.64 \times 2850 = 1824$$

$\frac{5}{8}$ of these bought online, so $\frac{3}{8}$ of them paid at the gate.

$$\frac{3}{8} \times 1824 = 684$$

Answer: 684

Tip: The $\frac{5}{8}$ is a fraction of the **other** visitors (the 1824 non-members), not of all 2850 visitors.

Marks: 1 mark for 1824 (the visitors who were not members). 1 mark for $\frac{3}{8}$ of 1824 (or $1824 - 1140$). 1 mark for 684.

3 (1 mark)

Which expression is equal to $x^2 - 169$?

Circle your answer.

Options: $(x - 13)^2$, $(x + 13)(x - 13)$, $(x + 13)^2$, $x(x - 169)$

$x^2 - 169$ is a difference of two squares: $x^2 - 13^2$

$$a^2 - b^2 = (a + b)(a - b)$$

$$\text{So } x^2 - 169 = (x + 13)(x - 13)$$

Check: $(x + 13)(x - 13) = x^2 - 13x + 13x - 169 = x^2 - 169$

Answer: $(x + 13)(x - 13)$

Tip: $(x - 13)^2 = x^2 - 26x + 169$, which has an x term. A difference of two squares has no x term because $+13x$ and $-13x$ cancel.

4 (3 marks)

Asha rolls a biased six-sided dice 250 times.

The table shows her results.

The frequency for 6 is not shown.

Number on dice	1	2	3	4	5	6
Frequency	31	44	38	29	40	

- (a) Work out the relative frequency of rolling a 6

Frequency of 6: $250 - (31 + 44 + 38 + 29 + 40) = 250 - 182 = 68$

$$\text{Relative frequency} = \frac{\text{frequency}}{\text{total number of rolls}} = \frac{68}{250}$$

$$= 0.272$$

Answer: 0.272 (or $\frac{68}{250} = \frac{34}{125}$)

Tip: Relative frequency is a fraction (or decimal) of the total number of trials. Give $\frac{68}{250}$ or 0.272, not 68.

Marks: 1 mark for 68 (the frequency of 6). 1 mark for 0.272 or $\frac{68}{250}$.

- (b) Pip is going to roll the same dice 500 times. She says, "I will roll a 6 exactly 136 times." Is Pip correct?

$500 \times 0.272 = 136$, so 136 is a sensible **estimate** of the number of 6s.

But each roll is random, so the actual number of 6s will vary. It may not be exactly 136.

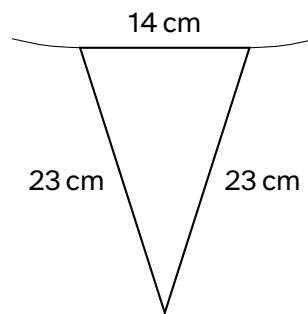
No, Pip is not correct.

Answer: No: 136 is only an estimate; the actual number of 6s could be more or less than 136.

Tip: Relative frequency gives an estimate of what to expect, not what will definitely happen.

5 (3 marks)

The diagram shows one flag from a string of bunting.
The flag is an isosceles triangle.



Not drawn accurately

Work out the area of the flag.
Give your answer to 3 significant figures.

Split the flag down the middle into two right-angled triangles.

Each one has hypotenuse 23 cm and base $14 \div 2 = 7$ cm.

$$\text{Height}^2 = 23^2 - 7^2 = 529 - 49 = 480$$

$$\text{Height} = \sqrt{480} = 21.908 \dots \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times 14 \times 21.908 \dots = 153.36 \dots \text{ cm}^2$$

$$= 153 \text{ cm}^2 \text{ (3 s.f.)}$$

Answer: 153 cm^2

Tip: Use half the base (7 cm), not 14 cm, in Pythagoras. The longest side is 23 cm, so subtract the squares.

Marks: 1 mark for $23^2 - 7^2$ (or 480). 1 mark for $\frac{1}{2} \times 14 \times \sqrt{480}$. 1 mark for 153.

6 (3 marks)

Lily wants a phone plan for 24 months and uses 9 GB of data each month. Plan A: £95 at the start, then £17.50 per month. Plan B: £15 per month including 6 GB, and £1.40 for each extra GB. Which plan is cheaper?

Plan A: $95 + 24 \times 17.50 = 95 + 420 = £515$

Plan B: each month Lily uses $9 - 6 = 3$ GB of extra data.

Cost each month: $15 + 3 \times 1.40 = 15 + 4.20 = £19.20$

For 24 months: $24 \times 19.20 = £460.80$

$460.80 < 515$, so Plan B is cheaper (by £54.20).

Answer: Plan B (£460.80 compared with £515)

Tip: Only the data over 6 GB is charged extra: 3 GB a month, not 9 GB.

Marks: 1 mark for the Plan A total (£515). 1 mark for the Plan B total (£460.80). 1 mark for Plan B with both totals.

7 (4 marks)

Poppy is 3 times as old as Luke. Georgia is 5 years younger than Poppy. Six years ago, Georgia was 4 times as old as Luke. Work out Georgia's age now.

Let Luke's age now be x years.

Poppy is $3x$. Georgia is $3x - 5$.

Six years ago **each** of them was 6 years younger: Luke $x - 6$ and Georgia

$$3x - 5 - 6 = 3x - 11$$

Georgia was 4 times as old as Luke: $3x - 11 = 4(x - 6)$

$$3x - 11 = 4x - 24$$

$$x = 13$$

Georgia is $3 \times 13 - 5 = 34$ years old now.

Check: six years ago Luke was 7 and Georgia was $28 = 4 \times 7$ ✓

Answer: 34 years

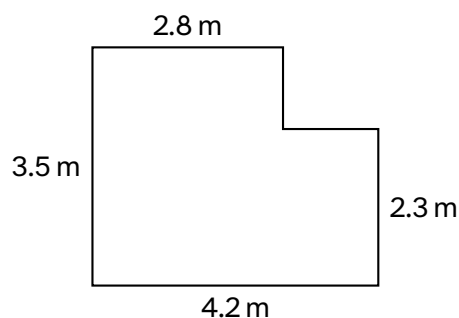
Tip: Six years ago **both** people were 6 years younger, so take 6 off both ages before using “4 times as old”. Then read the question again: it asks for Georgia’s age now.

Marks: 1 mark for $3x$ and $3x - 5$. 1 mark for the equation $3x - 11 = 4(x - 6)$. 1 mark for $x = 13$. 1 mark for 34.

8 (3 marks)

Sophie is going to tile her kitchen floor.

The diagram shows the shape of the floor.



Not drawn accurately

Sophie tiles her L-shaped kitchen floor with square tiles of side 40 cm, sold in packs of 9. Assume no tiles are wasted. How many packs does she need?

Split the floor into two rectangles, or take the missing corner away from a 4.2 m by 3.5 m rectangle.

Missing corner: $4.2 - 2.8 = 1.4$ m wide and $3.5 - 2.3 = 1.2$ m deep.

$$\text{Area} = 4.2 \times 3.5 - 1.4 \times 1.2 = 14.7 - 1.68 = 13.02 \text{ m}^2$$

$$\text{Area of one tile: } 0.4 \times 0.4 = 0.16 \text{ m}^2 \text{ (40 cm = 0.4 m)}$$

$$\text{Number of tiles: } 13.02 \div 0.16 = 81.375, \text{ so she needs 82 tiles.}$$

Packs: $82 \div 9 = 9.1\dots$, so she needs 10 packs.

Answer: 10 packs

Tip: Work in one unit. $1\text{ m}^2 = 10\,000\text{ cm}^2$, not 100 cm^2 , so it is easiest to change 40 cm to 0.4 m first. Round the number of packs **up**.

Marks: 1 mark for the floor area (13.02 m^2 or $130\,200\text{ cm}^2$). 1 mark for dividing by the area of a tile in the same units. 1 mark for 10.

9 (3 marks)

Van *A* costs £46 500 and its value decreases by 14% each year. Van *B* costs £37 500 and its value decreases by 8% each year. After how many complete years will van *A* first be worth less than van *B*?

Multipliers: van *A* $1 - 0.14 = 0.86$; van *B* $1 - 0.08 = 0.92$

After 1 year: *A* = £39 990, *B* = £34 500

After 2 years: *A* = £34 391.40, *B* = £31 740

After 3 years: *A* = £29 576.60, *B* = £29 200.80 (*A* is still worth more)

After 4 years: *A* = £25 435.88, *B* = £26 864.74 (*A* is now worth less)

So van *A* is first worth less than van *B* after 4 years.

Answer: 4 years

Tip: Van *A* loses more each year, so it catches up with van *B*. Work out both values year by year (or use 0.86^n and 0.92^n) and compare them each year.

Marks: 1 mark for using 0.86 for van *A* for at least two years. 1 mark for using 0.92 for van *B* for at least two years. 1 mark for 4 with the values after 3 and 4 years.

10 (3 marks)

The number of members of a cycling club increased by 15% during 2024.

(a) Write down the decimal multiplier for an increase of 15%

An increase of 15% means the new amount is $100\% + 15\% = 115\%$ of the old amount.

$$115\% = 1.15$$

Answer: 1.15

Tip: 0.15 is the multiplier to find 15% **of** an amount. To **increase** by 15% use 1.15.

- (b) Over the two years 2024 and 2025, the number of members increased by 38%. Work out the percentage increase in the number of members during 2025.

Overall multiplier for the two years: 1.38

$$1.15 \times (\text{multiplier for 2025}) = 1.38$$

$$\text{Multiplier for 2025} = 1.38 \div 1.15 = 1.2$$

1.2 means an increase of 20%.

Check with 100 members: $100 \rightarrow 115 \rightarrow 138$, and $\frac{23}{115} = 0.2$ ✓

Answer: 20%

Tip: Percentage increases do not add or subtract: $38 - 15 = 23\%$ is wrong, because the 2025 increase is a percentage of the larger 2024 total. Divide the multipliers.

Marks: 1 mark for $1.38 \div 1.15$ (or $100 \rightarrow 115 \rightarrow 138$ and $\frac{23}{115}$). 1 mark for 20.

11 (3 marks)

- (a) Write $x^2 + 16x - 7$ in the form $(x + a)^2 + b$

Halve the coefficient of x : $16 \div 2 = 8$

$(x + 8)^2 = x^2 + 16x + 64$, which is 64 too many (compared with the $x^2 + 16x$ part).

$$\text{So } x^2 + 16x = (x + 8)^2 - 64$$

$$x^2 + 16x - 7 = (x + 8)^2 - 64 - 7 = (x + 8)^2 - 71$$

Answer: $(x + 8)^2 - 71$

Tip: Subtract $8^2 = 64$ **and** the 7: $-64 - 7 = -71$. A common error is $-64 + 7 = -57$.

Marks: 1 mark for $(x + 8)^2$. 1 mark for -71 .

- (b) Use your answer to part (a) to explain why the equation $x^2 + 16x - 7 = -75$ has no solutions.

$$x^2 + 16x - 7 = (x + 8)^2 - 71$$

$(x + 8)^2$ is a square, so it can never be negative: $(x + 8)^2 \geq 0$

So $(x + 8)^2 - 71 \geq -71$ for every value of x .

The expression can never be as small as -75 , so the equation has no solutions.

(Or: $(x + 8)^2 = -4$, and a square cannot be negative.)

Answer: $(x + 8)^2 \geq 0$, so $(x + 8)^2 - 71$ is never less than -71 . It cannot equal -75 .

Tip: The completed-square form shows the smallest value an expression can take: here it is -71 , when $x = -8$.

12 (1 mark)

A circle has equation $x^2 + y^2 = 38$

What is the radius of the circle?

Circle your answer.

Options: 19, 38, 1444, $\sqrt{38}$

The circle $x^2 + y^2 = r^2$ has centre $(0, 0)$ and radius r .

Here $r^2 = 38$, so $r = \sqrt{38}$.

Answer: $\sqrt{38}$

Tip: The number on the right is the radius **squared**. Square root it to find the radius.

13 (3 marks)

Solve by factorising $6x^2 = 7x + 5$

Rearrange so that one side is 0: $6x^2 - 7x - 5 = 0$

Find two numbers that multiply to $6 \times (-5) = -30$ and add to -7 : -10 and 3

$$6x^2 - 10x + 3x - 5 = 0$$

$$2x(3x - 5) + 1(3x - 5) = 0$$

$$(3x - 5)(2x + 1) = 0$$

$$3x - 5 = 0 \text{ or } 2x + 1 = 0$$

$$x = \frac{5}{3} \text{ or } x = -\frac{1}{2}$$

Answer: $x = \frac{5}{3}$ and $x = -\frac{1}{2}$

Tip: Rearrange to ... = 0 before factorising. Check your brackets by expanding:

$$(3x - 5)(2x + 1) = 6x^2 + 3x - 10x - 5.$$

Marks: 1 mark for $6x^2 - 7x - 5 = 0$. 1 mark for $(3x - 5)(2x + 1)$. 1 mark for both solutions.

14 (3 marks)

360 people went to an ice rink on Saturday.

Each person either hired skates or brought their own skates.

- $\frac{5}{9}$ of the people were children.
- 45 of the adults brought their own skates.
- Altogether, 290 people hired skates.

(a) Complete the two-way table.

$$\text{Children: } \frac{5}{9} \times 360 = 200. \text{ Adults: } 360 - 200 = 160$$

$$\text{Adults who hired skates: } 160 - 45 = 115$$

$$\text{Children who hired skates: } 290 - 115 = 175. \text{ Children with their own skates: } 200 - 175 = 25$$

$$\text{Own skates total: } 45 + 25 = 70 \text{ (check: } 360 - 290 = 70)$$

	Hired skates	Own skates	Total
Adults	115	45	160
Children	175	25	200
Total	290	70	360

Answer: Adults: 115, 45, 160; Children: 175, 25, 200; Total: 290, 70, 360

Tip: Fill in the totals first (200 children and 160 adults), then use subtraction to complete each row and column.

Marks: 2 marks for a fully correct table. 1 mark for at least four correct values.

- (b) One of the people who hired skates is chosen at random.
Work out the probability that this person is an adult.

Only the people who hired skates can be chosen: there are 290 of them.

115 of them are adults.

$$\text{Probability} = \frac{115}{290} = \frac{23}{58}$$

Answer: $\frac{115}{290} = \frac{23}{58}$

Tip: The person is chosen from those who hired skates, so the denominator is 290, not 360 or 160.

15 (3 marks)

Members of a leisure centre use a four-digit PIN to open a locker.
Each digit is chosen from 0 to 9

- (a) For the gym lockers, the four digits of a PIN must all be different.
Work out the number of different PINs that are possible for the gym lockers.

There are 10 choices for the first digit, then 9 for the second (it must be different), then 8, then 7.

$$10 \times 9 \times 8 \times 7 = 5040$$

Answer: 5040

Tip: Multiply the number of choices for each position. Each digit that is used cannot be used again, so the number of choices goes down by 1 each time.

- (b) For the pool lockers, the first digit must be 5 or more and digits can be repeated. Kamal says there are more possible PINs for the pool lockers than for the gym lockers. Is Kamal correct?

First digit: 5, 6, 7, 8 or 9, so 5 choices.

Second, third and last digits: 10 choices each (repeats are allowed).

Pool lockers: $5 \times 10 \times 10 \times 10 = 5000$

$5000 < 5040$, so there are fewer PINs for the pool lockers.

No, Kamal is not correct.

Answer: No: $5 \times 10 \times 10 \times 10 = 5000$, which is less than 5040

Tip: Allowing repeats does not always give more choices. Work out the number of choices for each position (the first digit has only 5), multiply, then compare the two totals.

Marks: 1 mark for $5 \times 10 \times 10 \times 10$ (or 5000). 1 mark for No with 5000 compared with 5040.

16 (3 marks)

Machine *A* fills 96 bottles every minute and starts at 8:00 am. Machine *B* fills 2 bottles every second and starts 20 minutes later. Both then work until a total of 30 000 bottles have been filled. At what time have 30 000 bottles been filled?

Machine *B*: $2 \times 60 = 120$ bottles per minute.

For the first 20 minutes only machine *A* works: $20 \times 96 = 1920$ bottles.

Bottles still to fill: $30\,000 - 1920 = 28\,080$

Together: $96 + 120 = 216$ bottles per minute.

Time with both machines: $28\,080 \div 216 = 130$ minutes

Total time: $20 + 130 = 150$ minutes = 2 hours 30 minutes

8:00 am + 2 hours 30 minutes = 10:30 am

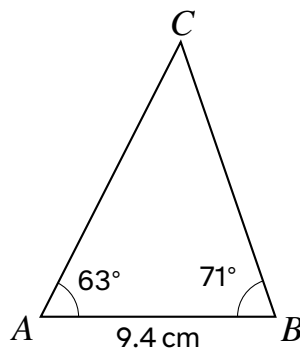
Answer: 10:30 am

Tip: Machine *A* works on its own for the first 20 minutes. Take those bottles off first, then use the combined rate for the rest. Remember to add the first 20 minutes back on.

Marks: 1 mark for 120 bottles per minute for machine *B* (or 1920 bottles from machine *A* in the first 20 minutes). 1 mark for $28\,080 \div 216$ ($= 130$). 1 mark for 10:30 am.

17 (3 marks)

ABC is a triangle.



Not drawn accurately

Work out the length of *BC*.

Give your answer to 3 significant figures.

BC is opposite the angle at *A*. The side we know, *AB*, is opposite the angle at *C*.

$$\text{Angle } C = 180^\circ - 63^\circ - 71^\circ = 46^\circ$$

$$\text{Sine rule: } \frac{BC}{\sin 63^\circ} = \frac{9.4}{\sin 46^\circ}$$

$$BC = \frac{9.4 \times \sin 63^\circ}{\sin 46^\circ} = 11.643 \dots$$

$$BC = 11.6 \text{ cm (3 s.f.)}$$

Answer: 11.6 cm

Tip: In the sine rule, each side goes with the angle **opposite** it. 9.4 cm is opposite angle C , so you need angle C first.

Marks: 1 mark for angle $C = 46^\circ$. 1 mark for the sine rule with BC , $\sin 63^\circ$, 9.4 and $\sin 46^\circ$. 1 mark for 11.6.

18 (5 marks)

The table shows information about the ages of 121 people at a cinema screening.

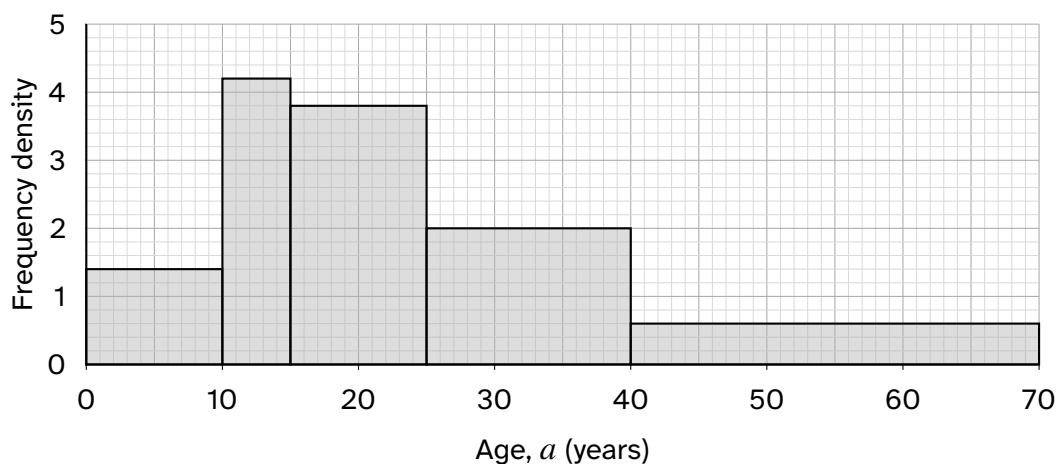
Age, a (years)	Frequency
$0 < a \leq 10$	14
$10 < a \leq 15$	21
$15 < a \leq 25$	38
$25 < a \leq 40$	30
$40 < a \leq 70$	18

- (a) Draw a histogram to represent this information.

Frequency density = frequency $\div$ class width

Age, a (years)	Frequency	Frequency density
$0 < a \leq 10$	14	1.4
$10 < a \leq 15$	21	4.2
$15 < a \leq 25$	38	3.8
$25 < a \leq 40$	30	2
$40 < a \leq 70$	18	0.6

Each bar goes across its whole class (e.g. from 10 to 15) with height equal to its frequency density.



Answer: Bars of height 1.4, 4.2, 3.8, 2 and 0.6 over 0–10, 10–15, 15–25, 25–40 and 40–70

Tip: The heights are frequency densities, not frequencies. Each small square up the axis is 0.2.

Marks: 3 marks for all five bars correct. 2 marks for four correct bars. 1 mark for two correct bars, or three correct frequency densities.

- (b) Estimate the number of people at the screening who were aged between 20 and 30

The ages from 20 to 30 are in two classes: part of $15 < a \leq 25$ and part of $25 < a \leq 40$.

20 to 25 is 5 years of the 10-year class $15 < a \leq 25$: $\frac{5}{10} \times 38 = 19$ (or $5 \times 3.8 = 19$)

25 to 30 is 5 years of the 15-year class $25 < a \leq 40$: $\frac{5}{15} \times 30 = 10$ (or $5 \times 2 = 10$)

Total: $19 + 10 = 29$

Answer: 29

Tip: Assume the people are spread evenly through each class. The range 20 to 30 crosses two classes, so take a fraction of each one: $\frac{5}{10}$ of the first and $\frac{5}{15}$ of the second.

Marks: 1 mark for 19 or 10 (one of the two parts). 1 mark for 29.

19 (4 marks)

A rugby match ticket's original price was increased by 16%, then reduced by 5%, then a £7.25 booking fee was added. Ryan paid £260.71. Work out the original price.

Take off the booking fee first: $260.71 - 7.25 = £253.46$

Increase by 16%: multiplier 1.16. Decrease by 5%: multiplier 0.95

Overall: original $\times 1.16 \times 0.95 = 253.46$

$1.16 \times 0.95 = 1.102$

Original price = $253.46 \div 1.102 = £230$

Check: $230 \times 1.16 = 266.80$, $266.80 \times 0.95 = 253.46$, $253.46 + 7.25 = 260.71$ ✓

Answer: £230

Tip: Undo the changes in reverse order: take off the fee, then divide by the multipliers. Do not add 5% and take off 16%: percentages of different amounts cannot be undone like that.

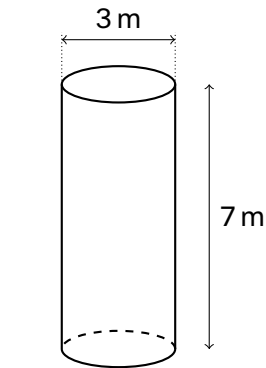
Marks: 1 mark for $260.71 - 7.25$. 1 mark for using 1.16 and 0.95 (or 1.102). 1 mark for $253.46 \div 1.102$ (or dividing by 0.95 then 1.16). 1 mark for £230.

20 (4 marks)

Gabriel has a grain silo on his farm.

The silo is a closed cylinder.

It stands on its circular base on the ground.



Not drawn accurately

Gabriel paints the outside of the closed cylindrical silo (diameter 3 m, height 7 m), but not the base. 1 litre of paint covers 5 m^2 . Paint is sold in 5-litre tins costing £32.50 each. Work out the cost of the paint he needs.

$$\text{Radius} = 3 \div 2 = 1.5 \text{ m}$$

$$\text{Curved surface: } \pi \times 3 \times 7 = 21\pi = 65.97 \dots \text{ m}^2 \text{ (circumference} \times \text{height)}$$

$$\text{Top: } \pi \times 1.5^2 = 2.25\pi = 7.068 \dots \text{ m}^2 \text{ (the base is not painted)}$$

$$\text{Total area} = 23.25\pi = 73.04 \dots \text{ m}^2$$

$$\text{Paint needed: } 73.04 \dots \div 5 = 14.6 \dots \text{ litres}$$

$$14.6 \dots \div 5 = 2.92 \dots, \text{ so he needs 3 tins.}$$

$$\text{Cost: } 3 \times 32.50 = \text{£}97.50$$

Answer: £97.50

Tip: The diagram gives the diameter, so use radius 1.5 m. Paint the top but not the base: one circle, not two. You can only buy whole tins, so round up.

Marks: 1 mark for the curved surface area. 1 mark for the area of one circle. 1 mark for dividing the total area by 5 and working out the number of tins. 1 mark for £97.50.

21 (5 marks)

- (a) Show that the equation $x^3 + 5x^2 - 9 = 0$ can be rearranged to give $x = \frac{3}{\sqrt{x+5}}$

$$x^3 + 5x^2 - 9 = 0$$

Add 9 to both sides: $x^3 + 5x^2 = 9$

Factorise: $x^2(x + 5) = 9$

Divide by $(x + 5)$: $x^2 = \frac{9}{x+5}$

Square root both sides: $x = \sqrt{\frac{9}{x+5}} = \frac{\sqrt{9}}{\sqrt{x+5}} = \frac{3}{\sqrt{x+5}}$

Answer: $x^2(x + 5) = 9$, so $x^2 = \frac{9}{x+5}$ and $x = \frac{3}{\sqrt{x+5}}$

Tip: In a “show that” question every step must be written down. Factorise x^2 out of $x^3 + 5x^2$ first.

Marks: 1 mark for $x^2(x + 5) = 9$ (or $x^2 = \frac{9}{x+5}$). 1 mark for the complete rearrangement with the square root step shown.

- (b) Starting with $x_1 = 1$, use the iterative formula $x_{n+1} = \frac{3}{\sqrt{x_n + 5}}$ to find a solution to $x^3 + 5x^2 - 9 = 0$ to 3 decimal places.

$$x_2 = \frac{3}{\sqrt{1+5}} = 1.224744 \dots$$

$$x_3 = \frac{3}{\sqrt{1.224744 \dots + 5}} = 1.202431 \dots$$

$$x_4 = 1.204592 \dots$$

$$x_5 = 1.204382 \dots$$

$$x_6 = 1.204403 \dots$$

x_5 and x_6 both round to 1.204 (3 d.p.), so the solution is 1.204 to 3 decimal places.

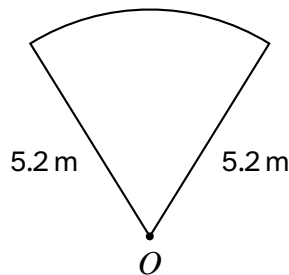
Answer: $x = 1.204$

Tip: Use the ANS key so that each step uses the full value of the one before. Keep going until two values in a row round to the same answer.

Marks: 1 mark for $x_2 = 1.2247 \dots$ 1 mark for continuing correctly to at least x_5 . 1 mark for 1.204 with two consecutive values that round to it.

22 (3 marks)

A lawn sprinkler waters a region of grass in the shape of a sector of a circle, centre O . The radius of the sector is 5.2 m. The area of the region is 15 m^2



Not drawn accurately

Work out the perimeter of the region.
Give your answer to 1 decimal place.

Let the angle of the sector be θ .

$$\text{Area: } \frac{\theta}{360} \times \pi \times 5.2^2 = 15$$

$$\theta = \frac{15 \times 360}{\pi \times 5.2^2} = 63.567 \dots^\circ$$

$$\text{Arc length} = \frac{63.567 \dots}{360} \times 2 \times \pi \times 5.2 = 5.769 \dots \text{ m}$$

$$\text{Perimeter} = 5.2 + 5.2 + 5.769 \dots = 16.169 \dots \text{ m}$$

= 16.2 m (1 d.p.)

Answer: 16.2 m

Tip: The perimeter is the arc **plus** the two radii. A quick route: arc length = $\frac{2 \times \text{area}}{\text{radius}} = \frac{30}{5.2}$.

Marks: 1 mark for the angle (63.6°). 1 mark for the arc length (5.77 m). 1 mark for 16.2.

23 (3 marks)

Shade the region that satisfies all three inequalities $x - y \geq -2$, $3y \geq 12 - 2x$, $x < 5$. Label it R .

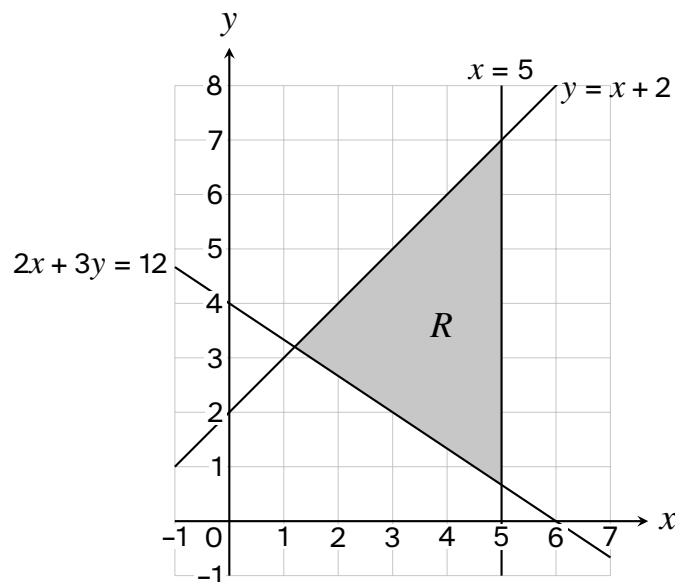
Rearrange each inequality to match a line on the grid:

$x - y \geq -2$ means $x + 2 \geq y$, so $y \leq x + 2$: on or **below** the line $y = x + 2$.

$3y \geq 12 - 2x$ means $2x + 3y \geq 12$: on or **above** the line $2x + 3y = 12$.

$x < 5$: to the **left** of the line $x = 5$.

R is the triangle where all three overlap, with vertices $(1.2, 3.2)$, $(5, 7)$ and $(5, \frac{2}{3})$.



Check with a point inside, e.g. (4, 3): $4 - 3 = 1 \geq -2$ ✓ $9 \geq 12 - 8 = 4$ ✓ $4 < 5$ ✓

Answer: Triangle with vertices (1.2, 3.2), (5, 7) and $(5, \frac{2}{3})$ shaded and labelled R

Tip: Rearrange each inequality so it matches the equation of a line (for $x - y \geq -2$, add y and 2 to both sides to get $x + 2 \geq y$, which avoids dividing by a negative), then test a point such as (0, 0) to see which side you need. (0, 0) gives $0 \geq 12$, which is false, so R is on the other side of $2x + 3y = 12$ from the origin.

Marks: 3 marks for the correct region labelled R . 2 marks for a region with two correct boundaries. 1 mark for one correct side of one line.

24 (4 marks)

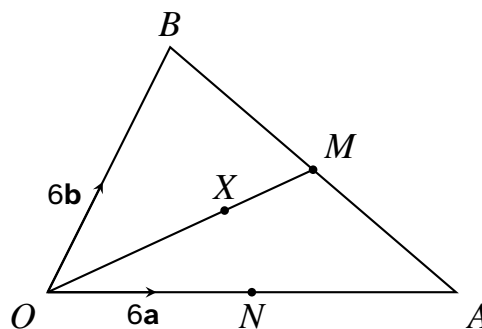
OAB is a triangle.

$$\overrightarrow{OA} = 6\mathbf{a} \quad \text{and} \quad \overrightarrow{OB} = 6\mathbf{b}$$

M is the midpoint of AB .

X is the point on OM such that $OX:XM = 2:1$

N is the midpoint of OA .



Not drawn accurately

Prove that B , X and N lie on a straight line.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -6\mathbf{a} + 6\mathbf{b}$$

$$M \text{ is the midpoint of } AB: \overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = 6\mathbf{a} - 3\mathbf{a} + 3\mathbf{b} = 3\mathbf{a} + 3\mathbf{b}$$

$$OX:XM = 2:1, \text{ so } \overrightarrow{OX} = \frac{2}{3}\overrightarrow{OM} = 2\mathbf{a} + 2\mathbf{b}$$

$$\overrightarrow{BX} = \overrightarrow{BO} + \overrightarrow{OX} = -6\mathbf{b} + 2\mathbf{a} + 2\mathbf{b} = 2\mathbf{a} - 4\mathbf{b}$$

$$N \text{ is the midpoint of } OA, \text{ so } \overrightarrow{ON} = 3\mathbf{a}$$

$$\overrightarrow{BN} = \overrightarrow{BO} + \overrightarrow{ON} = 3\mathbf{a} - 6\mathbf{b}$$

$$3\mathbf{a} - 6\mathbf{b} = \frac{3}{2}(2\mathbf{a} - 4\mathbf{b}), \text{ so } \overrightarrow{BN} = \frac{3}{2}\overrightarrow{BX}$$

So BX and BN are parallel, and they both pass through B .

Therefore B , X and N lie on a straight line.

Answer: $\overrightarrow{BX} = 2\mathbf{a} - 4\mathbf{b}$ and $\overrightarrow{BN} = 3\mathbf{a} - 6\mathbf{b} = \frac{3}{2}\overrightarrow{BX}$; parallel with the common point B , so B , X , N are on a straight line.

Tip: Showing $\overrightarrow{BX} + \overrightarrow{XN} = \overrightarrow{BN}$ proves nothing: that is true for any three points. You must show one vector is a multiple of the other **and** that they share a point. $OX:XM = 2:1$ means OX is $\frac{2}{3}$ of OM .

Marks: 1 mark for $\overrightarrow{OM} = 3\mathbf{a} + 3\mathbf{b}$ (or $\overrightarrow{AB}$ and a correct route). 1 mark for $\overrightarrow{OX} = 2\mathbf{a} + 2\mathbf{b}$ (or $\overrightarrow{BX}$). 1 mark for $\overrightarrow{BN} = 3\mathbf{a} - 6\mathbf{b}$. 1 mark for showing one is a multiple of the other and stating that they are parallel with a common point.

25 (4 marks)

Show that $\frac{9}{2x^2 + 7x - 4} - \frac{5}{2x^2 + 3x - 2}$ simplifies to $\frac{2}{x^2 + 6x + 8}$

Factorise each denominator: $2x^2 + 7x - 4 = (2x - 1)(x + 4)$ and $2x^2 + 3x - 2 = (2x - 1)(x + 2)$

Common denominator: $(2x - 1)(x + 4)(x + 2)$

$$\frac{9(x + 2)}{(2x - 1)(x + 4)(x + 2)} - \frac{5(x + 4)}{(2x - 1)(x + 4)(x + 2)}$$

Numerator: $9x + 18 - 5x - 20 = 4x - 2 = 2(2x - 1)$

$$\frac{2(2x - 1)}{(2x - 1)(x + 4)(x + 2)} = \frac{2}{(x + 4)(x + 2)} \text{ (cancel } 2x - 1)$$

$(x + 4)(x + 2) = x^2 + 6x + 8$, so the expression simplifies to $\frac{2}{x^2 + 6x + 8}$

Answer:
$$\frac{9(x + 2) - 5(x + 4)}{(2x - 1)(x + 4)(x + 2)} = \frac{2(2x - 1)}{(2x - 1)(x + 4)(x + 2)} = \frac{2}{(x + 4)(x + 2)} = \frac{2}{x^2 + 6x + 8}$$

Tip: Both denominators have the factor $2x - 1$. Factorise first, so you can use the simplest common denominator, $(2x - 1)(x + 4)(x + 2)$. Put the $5(x + 4)$ in a bracket so that both terms are subtracted, then look for $2x - 1$ in the numerator.

Marks: 1 mark for factorising a denominator. 1 mark for writing both fractions over a common denominator. 1 mark for the numerator $4x - 2$ written as $2(2x - 1)$. 1 mark for cancelling to reach $\frac{2}{x^2 + 6x + 8}$ with $(x + 4)(x + 2)$ seen.
