

Mock Set B – Paper 1

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Work out $\sqrt{0.0036}$

Circle your answer.

Options: 0.0018, 0.006, 0.06, 0.6

$$0.0036 = \frac{36}{10\,000}$$

$$\sqrt{\frac{36}{10\,000}} = \frac{\sqrt{36}}{\sqrt{10\,000}} = \frac{6}{100} = 0.06$$

Check: $0.06 \times 0.06 = 0.0036$ ✓

Answer: 0.06

Tip: Check by squaring your answer. $0.006^2 = 0.000\,036$, not 0.0036. Halving (0.0018) is not the same as square rooting.

2 (2 marks)

By rounding each number to 1 significant figure, estimate the value of

$$\frac{8.17 \times 0.287}{0.0503}$$

You must show your working.

Round each number to 1 significant figure: $8.17 \approx 8$, $0.287 \approx 0.3$, $0.0503 \approx 0.05$

Top: $8 \times 0.3 = 2.4$

Divide: $2.4 \div 0.05 = 240 \div 5 = 48$

Answer: 48

Tip: To divide by 0.05, multiply the top and bottom by 100: $2.4 \div 0.05 = 240 \div 5$. Dividing by a number less than 1 makes the answer bigger.

Marks: 1 mark for rounding all three numbers correctly (8, 0.3 and 0.05). 1 mark for 48.

3 (3 marks)

The prize money for a quiz is shared between the teams that finish first, second and third.

- The team that finishes first gets $\frac{2}{5}$ of the prize money.
- The teams that finish second and third share the rest in the ratio 7:5
- The team that finishes second gets £40 more than the team that finishes third.

Work out the total prize money.

Second and third: $7 - 5 = 2$ parts, and 2 parts are worth £40

One part = $40 \div 2 = £20$

Second and third together: $7 + 5 = 12$ parts = $12 \times 20 = £240$

The first team gets $\frac{2}{5}$, so the other two teams share $\frac{3}{5}$ of the prize money.

$\frac{3}{5}$ of the total = £240, so $\frac{1}{5} = £80$ and the total = $5 \times 80 = £400$

Check: first £160, second £140, third £100; $160 + 140 + 100 = 400$ ✓

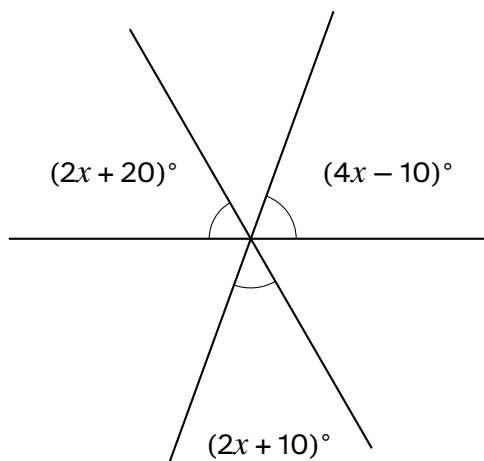
Answer: £400

Tip: The £40 is the **difference** between the second and third shares, so it is $7 - 5 = 2$ parts. The ratio 7:5 only shares the money left after the first team is paid.

Marks: 1 mark for one part = £20 (or second £140, third £100). 1 mark for £240 shown to be $\frac{3}{5}$ of the total. 1 mark for £400.

4 (3 marks)

Three straight lines meet at a point.



Not drawn accurately

Work out the size of the largest angle in the diagram.

The angle vertically opposite $(2x + 10)^\circ$ is between the $(4x - 10)^\circ$ and $(2x + 20)^\circ$ angles.

Vertically opposite angles are equal, so that angle is also $(2x + 10)^\circ$.

The three angles $(4x - 10)^\circ$, $(2x + 10)^\circ$ and $(2x + 20)^\circ$ lie on a straight line, so they add to 180° .

$$4x - 10 + 2x + 10 + 2x + 20 = 180$$

$$8x + 20 = 180$$

$$8x = 160, \text{ so } x = 20$$

The angles are $4 \times 20 - 10 = 70^\circ$, $2 \times 20 + 10 = 50^\circ$ and $2 \times 20 + 20 = 60^\circ$

Each angle appears twice (vertically opposite), so the largest angle is 70° .

Answer: 70°

Tip: The three marked angles do not add to 360° : there are six angles round the point. Use vertically opposite angles to get three angles on a straight line (180°), or double up to 360° .

Marks: 1 mark for an equation, e.g. $8x + 20 = 180$ or $16x + 40 = 360$. 1 mark for $x = 20$. 1 mark for 70° .

5 (3 marks)

Sana tries to solve the equation $6(2x - 3) = 4(x - 5) + 7$

Here is her working.

$$12x - 3 = 4x - 20 + 7$$

$$12x - 3 = 4x - 13$$

$$8x = -10$$

$$x = -1.25$$

(a) What mistake has Sana made?

In the first line Sana has worked out $6(2x - 3)$ as $12x - 3$.

She only multiplied the $2x$ by 6. She should also multiply the -3 by 6:

$$6(2x - 3) = 12x - 18$$

Answer: She expanded $6(2x - 3)$ as $12x - 3$ instead of $12x - 18$ (she did not multiply the 3 by 6)

Tip: When you expand a bracket, multiply **every** term inside by the number outside.

(b) Solve $6(2x - 3) = 4(x - 5) + 7$

Expand both brackets: $12x - 18 = 4x - 20 + 7$

Simplify the right-hand side: $12x - 18 = 4x - 13$

Subtract $4x$ from both sides: $8x - 18 = -13$

Add 18 to both sides: $8x = 5$

$$x = \frac{5}{8}$$

Answer: $x = \frac{5}{8}$ (or 0.625)

Tip: Your answer does not have to be a whole number. Leave it as a fraction, $\frac{5}{8}$, or write it as 0.625.

Marks: 1 mark for $12x - 18 = 4x - 13$ or $8x = 5$. 1 mark for $\frac{5}{8}$.

6 (4 marks)

$\xi = \{\text{integers from 1 to 20}\}$

$A = \{\text{multiples of 4}\}$

$B = \{\text{factors of 40}\}$

(a) List the members of $A \cap B$

$A = \{4, 8, 12, 16, 20\}$

$B = \{1, 2, 4, 5, 8, 10, 20\}$

$A \cap B$ means the numbers in A **and** in B : 4, 8 and 20

Answer: 4, 8, 20

Tip: $\cap$ means “and” (in both sets). 40 is a factor of 40, but it is not in ξ .

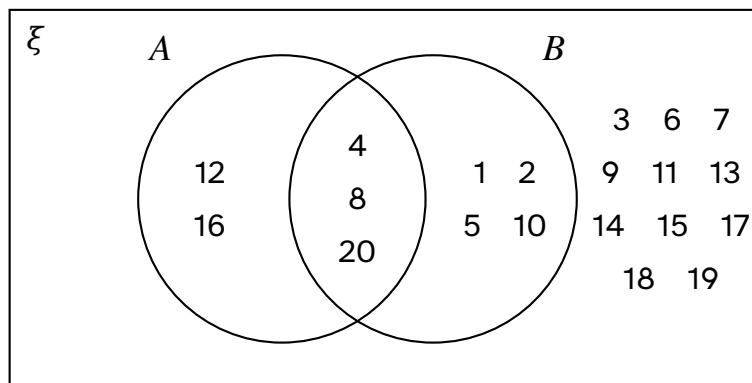
(b) Complete the Venn diagram.

Put 4, 8 and 20 in the overlap.

A only: 12, 16

B only: 1, 2, 5, 10

All the other numbers from 1 to 20 go outside both circles.



Answer: Overlap 4, 8, 20; A only 12, 16; B only 1, 2, 5, 10; outside 3, 6, 7, 9, 11, 13, 14, 15, 17, 18, 19

Tip: Fill in the overlap first. Every number in ξ must appear exactly once, including the ones outside both circles.

Marks: 1 mark for the two circles correct. 1 mark for the eleven numbers outside both circles.

- (c) A number is chosen at random from ξ .

Work out $P(A' \cap B)$

$A' \cap B$ means in B and **not** in A : 1, 2, 5, 10

That is 4 numbers out of 20.

$$P(A' \cap B) = \frac{4}{20} = \frac{1}{5}$$

Answer: $\frac{4}{20} = \frac{1}{5}$

Tip: A' means “not in A ”. Count the numbers in circle B that are outside circle A .

7 (3 marks)

On Saturday at a summer festival,

- $\frac{1}{4}$ of the visitors buy food from the pizza stall
- $\frac{2}{9}$ of the other visitors buy food from the noodle stall
- the remaining 1400 visitors do not buy food from either stall.

No visitor buys food from both stalls.

How many visitors were at the festival on Saturday?

After the pizza stall, $1 - \frac{1}{4} = \frac{3}{4}$ of the visitors are left.

Noodle stall: $\frac{2}{9}$ of $\frac{3}{4} = \frac{2}{9} \times \frac{3}{4} = \frac{6}{36} = \frac{1}{6}$ of all the visitors

Neither stall: $\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$ of the visitors

$\frac{7}{12}$ of the visitors = 1400, so $\frac{1}{12} = 1400 \div 7 = 200$

Total = $12 \times 200 = 2400$

Check: pizza 600, noodles $\frac{2}{9} \times 1800 = 400$, neither $2400 - 600 - 400 = 1400$ ✓

Answer: 2400

Tip: The $\frac{2}{9}$ is a fraction of the visitors who are **left**, not of all the visitors. Work out what fraction of the whole is left, then scale up.

Marks: 1 mark for $\frac{2}{9} \times \frac{3}{4}$ or $\frac{1}{6}$ (or $\frac{7}{9}$ of the rest). 1 mark for $\frac{7}{12}$ of the total = 1400. 1 mark for 2400.

8 (3 marks)

Deepa catches a train from Leeds to London.

- The train leaves Leeds at 10:30
- It travels 306 km at an average speed of 136 km/h.
- From the station in London, it takes Deepa 12 minutes to get to a meeting.
- The meeting starts at 13:00

Deepa says,

“I will get to my meeting on time.”

Is Deepa correct?

You must show your working.

$$\text{Time on the train} = \frac{\text{distance}}{\text{speed}} = \frac{306}{136} \text{ hours}$$

$$136 \times 2 = 272, \text{ and } 306 - 272 = 34, \text{ which is } \frac{1}{4} \text{ of } 136$$

$$\text{So the time is } 2\frac{1}{4} \text{ hours} = 2 \text{ hours } 15 \text{ minutes.}$$

$$\text{The train arrives at } 10:30 + 2 \text{ h } 15 \text{ min} = 12:45$$

$$\text{She gets to the meeting at } 12:45 + 12 \text{ minutes} = 12:57$$

12:57 is before 13:00, so Deepa is correct.

Answer: Yes: the train takes $2\frac{1}{4}$ hours (2 h 15 min), so she arrives in London at 12:45 and at the meeting at 12:57

Tip: 2.25 hours is 2 hours and 15 minutes, not 2 hours 25 minutes: 0.25 of an hour is $\frac{1}{4} \times 60 = 15$ minutes.

Marks: 1 mark for $306 \div 136$ or $2\frac{1}{4}$ hours. 1 mark for converting to 2 h 15 min (or 135 min) and finding the arrival time 12:45. 1 mark for Yes with 12:57 (or 3 minutes to spare).

9 (3 marks)

Write $0.\dot{8}\dot{4}$ as a fraction in its simplest form.

Let $x = 0.848484 \dots$

Two digits recur, so multiply by 100: $100x = 84.848484 \dots$

Subtract: $100x - x = 84.8484 \dots - 0.8484 \dots$

$$99x = 84$$

$$x = \frac{84}{99}$$

Divide the top and bottom by 3: $x = \frac{28}{33}$

Answer: $\frac{28}{33}$

Tip: Both digits recur, so the decimal is $0.848484 \dots$ (not $0.8444 \dots$). Multiply by 100 because two digits recur, and remember to simplify at the end.

Marks: 1 mark for $100x = 84.8484 \dots$ and $x = 0.8484 \dots$. 1 mark for $99x = 84$ or $\frac{84}{99}$. 1 mark for $\frac{28}{33}$.

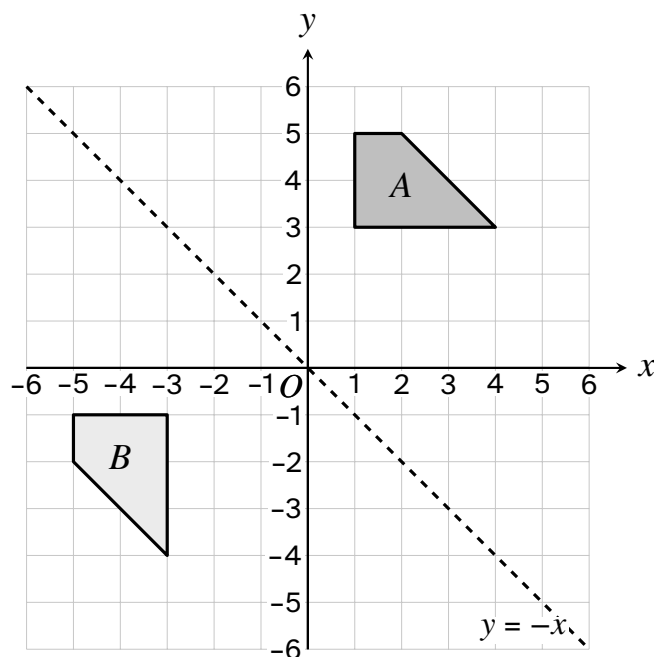
10 (4 marks)

Shape A is drawn on the grid.

- (a) Reflect shape A in the line $y = -x$. Label your image B .

Each point (x, y) reflects to $(-y, -x)$ in the line $y = -x$.

$(1, 3) \rightarrow (-3, -1)$, $(1, 5) \rightarrow (-5, -1)$, $(2, 5) \rightarrow (-5, -2)$, $(4, 3) \rightarrow (-3, -4)$



Answer: B has vertices $(-3, -1)$, $(-5, -1)$, $(-5, -2)$, $(-3, -4)$

Tip: Draw the line $y = -x$ first: it goes through $(0, 0)$, $(1, -1)$, $(2, -2)$, ... Each point of B must be the same distance from the line as the matching point of A , on the other side.

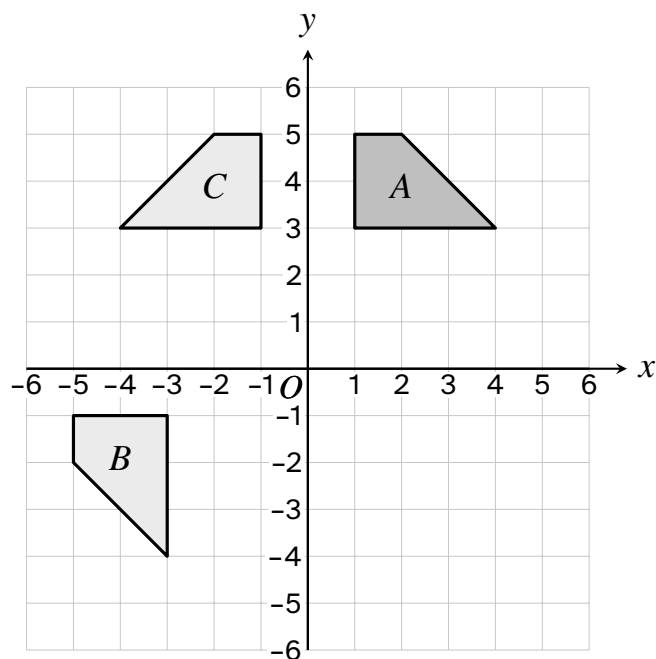
Marks: 2 marks for a correct reflection. 1 mark for a reflection in $y = x$ or for three vertices correct.

- (b) Shape B is rotated 90° clockwise about $(0, 0)$ to give shape C .

Describe fully the **single** transformation that maps shape A onto shape C .

Rotating 90° clockwise about $(0, 0)$ maps (x, y) to $(y, -x)$.

B : $(-3, -1) \rightarrow (-1, 3)$, $(-5, -1) \rightarrow (-1, 5)$, $(-5, -2) \rightarrow (-2, 5)$,
 $(-3, -4) \rightarrow (-4, 3)$



Compare A and C: $(1, 3) \rightarrow (-1, 3)$, $(4, 3) \rightarrow (-4, 3)$. The x -coordinates change sign and the y -coordinates stay the same.

So A maps onto C by a reflection in the y -axis.

Answer: Reflection in the y -axis (the line $x = 0$)

Tip: Draw shape C first, then look at A and C together. For a reflection you must give the equation of the mirror line.

Marks: 1 mark for reflection. 1 mark for the y -axis ($x = 0$).

11 (3 marks)

Solve the simultaneous equations $4x + 3y = 7$ and $6x - 5y = 39$

Make the y terms the same size: multiply the first equation by 5 and the second by 3.

$$20x + 15y = 35$$

$$18x - 15y = 117$$

Add to eliminate y : $38x = 152$, so $x = 4$

Substitute into $4x + 3y = 7$: $16 + 3y = 7$, so $3y = -9$ and $y = -3$

Check in the second equation: $6 \times 4 - 5 \times (-3) = 24 + 15 = 39 \checkmark$

Answer: $x = 4, y = -3$

Tip: When the signs of the matching terms are different ($+15y$ and $-15y$), **add** the equations. Check your answers in the equation you did not substitute into.

Marks: 1 mark for multiplying both equations so the x or y terms can be eliminated, and adding or subtracting correctly. 1 mark for one correct value. 1 mark for both.

12 (3 marks)

Here are the first three terms of a geometric progression.

$$2\sqrt{5} \qquad 10 \qquad 10\sqrt{5}$$

(a) Work out the next term of the sequence.

$$\text{Common ratio: } 10 \div 2\sqrt{5} = \frac{10}{2\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5} \quad (\text{check: } 10 \times \sqrt{5} = 10\sqrt{5} \checkmark)$$

$$\text{Next term: } 10\sqrt{5} \times \sqrt{5} = 10 \times 5 = 50$$

Answer: 50

Tip: In a geometric progression you multiply by the same number each time. $\sqrt{5} \times \sqrt{5} = 5$.

(b) Which term of the sequence is equal to 1250?
You must show your working.

Multiplying by $\sqrt{5}$ twice multiplies by 5, so every second term is 5 times the term two places before.

$$2\sqrt{5}, 10, 10\sqrt{5}, 50, 50\sqrt{5}, 250, 250\sqrt{5}, 1250$$

1250 is the 8th term.

Or: the n th term is $2\sqrt{5} \times (\sqrt{5})^{n-1} = 2 \times (\sqrt{5})^n$, and $2 \times (\sqrt{5})^n = 1250$ gives $(\sqrt{5})^n = 625 = 5^4 = (\sqrt{5})^8$, so $n = 8$

Answer: The 8th term

Tip: The even-numbered terms are whole numbers: 10, 50, 250, ..., multiplying by 5 each time. Count the terms carefully, including the ones with $\sqrt{5}$.

Marks: 1 mark for continuing the sequence correctly to 250 or beyond (or for $(\sqrt{5})^n = 625$). 1 mark for 8th.

13 (4 marks)

A gardening club grows 80 sunflowers in compost P and 80 sunflowers in compost Q . The cumulative frequency graph shows information about the heights of the sunflowers grown in compost P .

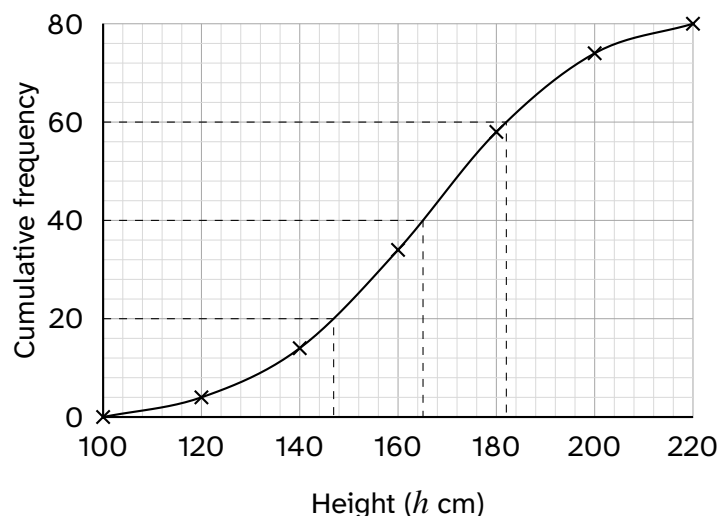
- (a) Use the graph to estimate the median and the inter-quartile range of the heights of the sunflowers grown in compost P .

There are 80 sunflowers, so the median is the $\frac{80}{2} = 40$ th value.

Read across from 40 on the cumulative frequency axis to the curve, then down:
median ≈ 165 cm

Lower quartile: read across from $\frac{1}{4} \times 80 = 20$: about 147 cm

Upper quartile: read across from $\frac{3}{4} \times 80 = 60$: about 182 cm



Inter-quartile range $\approx 182 - 147 = 35$ cm

Answer: Median 165 cm (accept 163 to 167); inter-quartile range 35 cm (accept 31 to 39)

Tip: Read across from 40 (half of 80) for the median, and from 20 and 60 for the quartiles. Subtract the quartiles to get the inter-quartile range.

Marks: 1 mark for the median. 1 mark for the inter-quartile range.

- (b) The box plot shows information about the heights of the 80 sunflowers grown in compost Q (minimum 118, LQ 150, median 158, UQ 172, maximum 204). Make two comparisons between the heights of the sunflowers grown in compost P and in compost Q .

From the box plot for compost Q : median = 158 cm, inter-quartile range = $172 - 150 = 22$ cm

Medians: P about 165 cm, Q 158 cm. On average, the sunflowers grown in compost P were taller.

Inter-quartile ranges: P about 35 cm, Q 22 cm. The heights of the sunflowers grown in compost Q were more consistent (less spread out).

Answer: On average the sunflowers in compost P were taller (median about 165 cm against 158 cm). The heights in compost Q were more consistent (IQR 22 cm against about 35 cm).

Tip: Compare an average (median) **and** a measure of spread (inter-quartile range), and say what each means for the sunflowers, e.g. “taller on average” or “more consistent heights”.

Marks: 1 mark for comparing the medians (or another average) in context. 1 mark for comparing the inter-quartile ranges (or spread) in context.

14 (3 marks)

Prove that $(2n + 5)^2 - (2n - 1)^2$ is a multiple of 24 for all positive integers n .

Expand each square:

$$(2n + 5)^2 = 4n^2 + 20n + 25$$

$$(2n - 1)^2 = 4n^2 - 4n + 1$$

$$\begin{aligned}\text{Subtract: } (4n^2 + 20n + 25) - (4n^2 - 4n + 1) &= 4n^2 + 20n + 25 - 4n^2 + 4n - 1 \\ &= 24n + 24 \\ &= 24(n + 1)\end{aligned}$$

$n + 1$ is an integer, so $24(n + 1)$ is a multiple of 24.

Answer: $(2n + 5)^2 - (2n - 1)^2 = 24n + 24 = 24(n + 1)$, which is $24 \times$ an integer, so it is a multiple of 24

Tip: Put the second expansion in a bracket before subtracting:

$-(4n^2 - 4n + 1) = -4n^2 + 4n - 1$. Trying a few values of n is not a proof.

Marks: 1 mark for expanding at least one square correctly. 1 mark for $24n + 24$. 1 mark for $24(n + 1)$ with a conclusion.

15 (3 marks)

A crate at a farm shop contains red apples and green apples only.

number of red apples : number of green apples = 3:2

40 green apples are added to the crate.

Now

number of red apples : number of green apples = 5:6

How many apples were in the crate at first?

At first: red = $3k$ and green = $2k$ for some number k .

After 40 green apples are added: red = $3k$ and green = $2k + 40$

$$\frac{3k}{2k + 40} = \frac{5}{6}$$

Cross-multiply: $18k = 5(2k + 40) = 10k + 200$

$8k = 200$, so $k = 25$

At first: red = 75, green = 50, total = 125

Check: $75:(50 + 40) = 75:90 = 5:6$ ✓

Answer: 125

Tip: The number of red apples does not change. Write both ratios with the same number of parts for red ($3:2 = 15:10$ and $5:6 = 15:18$): the green goes up by 8 parts, which is 40 apples.

Marks: 1 mark for an equation, e.g. $\frac{3k}{2k+40} = \frac{5}{6}$. 1 mark for $k = 25$ (or 75 red or 50 green). 1 mark for 125.

16 (4 marks)

- (a) Expand and simplify $(6 + \sqrt{2})(3 - \sqrt{2})$

Give your answer in the form $p + q\sqrt{2}$ where p and q are integers.

Multiply every term in the first bracket by every term in the second:

$$6 \times 3 = 18, \quad 6 \times (-\sqrt{2}) = -6\sqrt{2}, \quad \sqrt{2} \times 3 = 3\sqrt{2}, \quad \sqrt{2} \times (-\sqrt{2}) = -2$$

$$18 - 6\sqrt{2} + 3\sqrt{2} - 2 = 16 - 3\sqrt{2}$$

Answer: $16 - 3\sqrt{2}$

Tip: $\sqrt{2} \times \sqrt{2} = 2$, so $\sqrt{2} \times (-\sqrt{2}) = -2$. Collect the whole numbers and the $\sqrt{2}$ terms separately.

Marks: 1 mark for four correct terms (or three correct terms including -2). 1 mark for $16 - 3\sqrt{2}$.

- (b) $\sqrt{a} + \sqrt{50} = 9\sqrt{2}$ where a is an integer.

Work out the value of a .

$$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

$$\text{So } \sqrt{a} = 9\sqrt{2} - 5\sqrt{2} = 4\sqrt{2}$$

$$4\sqrt{2} = \sqrt{16} \times \sqrt{2} = \sqrt{32}, \text{ so } a = 32$$

Answer: $a = 32$

Tip: To turn $4\sqrt{2}$ back into a single square root, square the 4: $4\sqrt{2} = \sqrt{16 \times 2} = \sqrt{32}$.

Marks: 1 mark for $\sqrt{50} = 5\sqrt{2}$ (or $\sqrt{a} = 4\sqrt{2}$). 1 mark for 32.

17 (3 marks)

(a) Factorise $x^2 - 3x - 28$

Find two numbers that multiply to -28 and add to -3 : -7 and 4

$$x^2 - 3x - 28 = (x - 7)(x + 4)$$

Answer: $(x - 7)(x + 4)$

Tip: Check by expanding: $(x - 7)(x + 4) = x^2 + 4x - 7x - 28 = x^2 - 3x - 28$.

(b) Hence, solve $x^2 - 3x - 28 > 0$

$(x - 7)(x + 4) = 0$ when $x = 7$ or $x = -4$ (the critical values).

The graph of $y = x^2 - 3x - 28$ is a U-shaped curve crossing the x -axis at -4 and 7 .

$x^2 - 3x - 28 > 0$ where the curve is **above** the x -axis: outside the two roots.

So $x < -4$ or $x > 7$

Answer: $x < -4$ or $x > 7$

Tip: Sketch the curve. “ > 0 ” means above the x -axis, which for a U-shaped curve is the two outside parts, so the answer is two separate inequalities, not $-4 < x < 7$.

Marks: 1 mark for the critical values -4 and 7 . 1 mark for $x < -4$ or $x > 7$.

18 (3 marks)

- (a) Work out the value of $7^0 + 1000^{\frac{2}{3}}$

Any number to the power 0 is 1: $7^0 = 1$

$$1000^{\frac{2}{3}} = (\sqrt[3]{1000})^2 = 10^2 = 100$$

$$1 + 100 = 101$$

Answer: 101

Tip: $7^0 = 1$, not 0 or 7. For $1000^{\frac{2}{3}}$, take the cube root first, then square.

- (b) $\frac{\sqrt{5}}{25} = 5^n$

Work out the value of n .

$$\sqrt{5} = 5^{\frac{1}{2}} \text{ and } 25 = 5^2$$

$$\frac{5^{\frac{1}{2}}}{5^2} = 5^{\frac{1}{2}-2} = 5^{-\frac{3}{2}}$$

$$\text{So } n = -\frac{3}{2}$$

Answer: $n = -\frac{3}{2}$ (or -1.5)

Tip: Write everything as a power of 5. A square root is the power $\frac{1}{2}$, and dividing means subtracting the powers.

Marks: 1 mark for $5^{\frac{1}{2}}$ and 5^2 (or 5^{-2}). 1 mark for $-\frac{3}{2}$.

19 (3 marks)

t is inversely proportional to the square root of n .

When $n = 36$, $t = 10$

- (a) Work out the value of n when $t = 15$

$$t = \frac{k}{\sqrt{n}}$$

Substitute $n = 36$ and $t = 10$: $10 = \frac{k}{6}$, so $k = 60$ and $t = \frac{60}{\sqrt{n}}$

When $t = 15$: $15 = \frac{60}{\sqrt{n}}$, so $\sqrt{n} = 60 \div 15 = 4$

$$n = 4^2 = 16$$

Answer: $n = 16$

Tip: After finding $\sqrt{n} = 4$, remember to square to get n . $n = 4$ is a common wrong answer.

Marks: 1 mark for $k = 60$ or $t = \frac{60}{\sqrt{n}}$. 1 mark for 16.

(b) Mason says,

“When n is multiplied by 9, t is divided by 3”

Is Mason correct?

Tick a box.

Give a reason for your answer.

$t = \frac{60}{\sqrt{n}}$. If n is multiplied by 9, then $\sqrt{n}$ is multiplied by $\sqrt{9} = 3$.

Dividing 60 by a number 3 times as big gives an answer 3 times as small.

Example: $n = 4$ gives $t = 30$, and $n = 36$ gives $t = 10$, which is $30 \div 3$.

So Mason is correct.

Answer: Yes, because $\sqrt{9} = 3$, so $\sqrt{n}$ is multiplied by 3 and $t = \frac{60}{\sqrt{n}}$ is divided by 3

Tip: Inversely proportional to $\sqrt{n}$ does not mean “divide by the same number”. Find what happens to $\sqrt{n}$ first.

20 (3 marks)

Simplify fully $\frac{9x^2 - 49}{3x^2 - x - 14}$

Numerator: a difference of two squares: $9x^2 - 49 = (3x)^2 - 7^2 = (3x + 7)(3x - 7)$

Denominator: $3x^2 - x - 14 = (3x - 7)(x + 2)$ (check: $3x^2 + 6x - 7x - 14$ ✓)

$$\frac{(3x + 7)(3x - 7)}{(3x - 7)(x + 2)}$$

Cancel the common factor $(3x - 7)$: $\frac{3x + 7}{x + 2}$

Answer: $\frac{3x + 7}{x + 2}$

Tip: Factorise the top and bottom completely before you cancel. You can only cancel whole brackets that are factors, never single terms such as $9x^2$ and $3x^2$.

Marks: 1 mark for $(3x + 7)(3x - 7)$. 1 mark for $(3x - 7)(x + 2)$. 1 mark for $\frac{3x+7}{x+2}$.

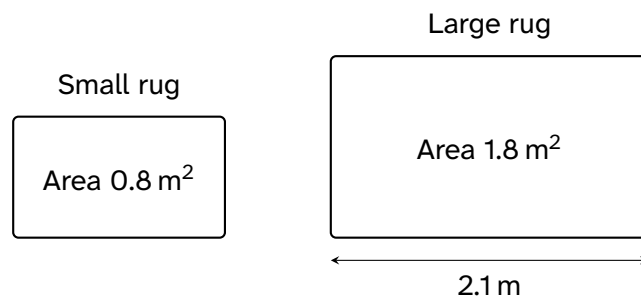
21 (3 marks)

A shop sells a small rug and a large rug.

The rugs are mathematically similar.

The small rug has area 0.8 m^2

The large rug has area 1.8 m^2 and length 2.1 m .



Not drawn accurately

Work out the length of the small rug.

Area scale factor from large to small: $\frac{0.8}{1.8} = \frac{8}{18} = \frac{4}{9}$

Length scale factor = $\sqrt{\frac{4}{9}} = \frac{2}{3}$

$$\text{Length of the small rug} = 2.1 \times \frac{2}{3} = 0.7 \times 2 = 1.4 \text{ m}$$

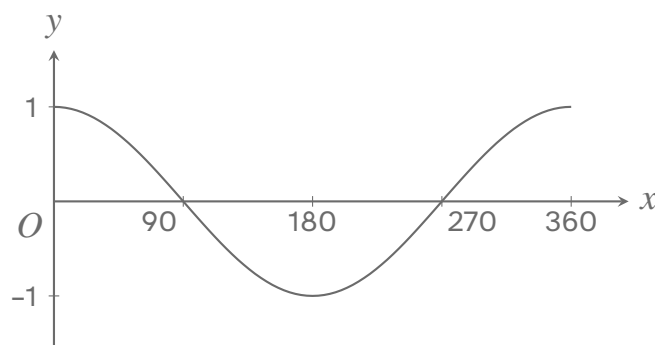
Answer: 1.4 m

Tip: For similar shapes, the area scale factor is the length scale factor **squared**. Square root the area ratio to get the length ratio.

Marks: 1 mark for the area ratio $\frac{4}{9}$ (or $\frac{9}{4}$). 1 mark for the length ratio $\frac{2}{3}$ (or $\frac{3}{2}$). 1 mark for 1.4 m.

22 (3 marks)

Here is a sketch of the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$



- (a) Write down the coordinates of the minimum point of the graph.

The lowest point of $y = \cos x$ for $0 \leq x \leq 360$ is at $x = 180$, where $\cos 180^\circ = -1$.

Answer: (180, -1)

Tip: Learn the shape of the cos graph: it starts at (0, 1), crosses the axis at 90 and 270, and is lowest at (180, -1).

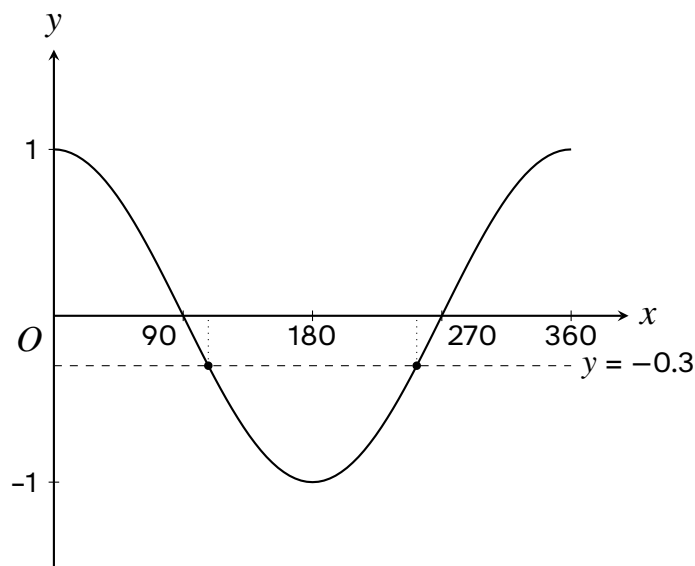
- (b) $\cos 72.54^\circ = 0.3$. Use this fact and the graph to solve $\cos x = -0.3$ for $0^\circ \leq x \leq 360^\circ$

The cos graph has rotational symmetry about (90, 0) and about (270, 0), and is symmetrical about $x = 180$.

$\cos x = -0.3$ is 0.3 **below** the axis, between 90° and 270° .

First value: $180 - 72.54 = 107.46^\circ$

Second value (by symmetry about $x = 180$): $180 + 72.54 = 252.54^\circ$



Answer: 107.46° and 252.54°

Tip: Sketch the lines $y = 0.3$ and $y = -0.3$ on the graph. $\cos x$ is negative between 90° and 270° , so both answers are in that range.

Marks: 1 mark for one correct value. 1 mark for both values.

23 (5 marks)

A box contains 8 green tea bags and 4 mint tea bags only.

Mei takes two tea bags at random from the box without replacement.

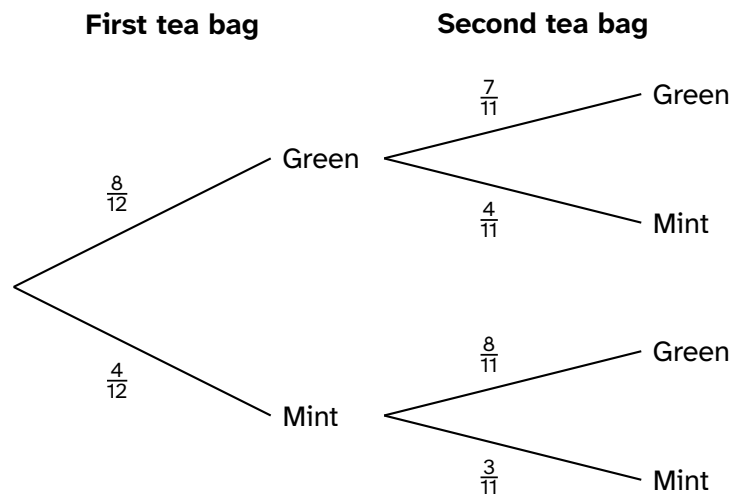
(a) Complete the tree diagram.

First tea bag: 12 tea bags, 8 green and 4 mint: $P(\text{green}) = \frac{8}{12}$, $P(\text{mint}) = \frac{4}{12}$

The first tea bag is not put back, so 11 are left for the second.

After a green: 7 green and 4 mint left: $\frac{7}{11}$ and $\frac{4}{11}$

After a mint: 8 green and 3 mint left: $\frac{8}{11}$ and $\frac{3}{11}$



Answer: First: green $\frac{8}{12}$, mint $\frac{4}{12}$; second after green: $\frac{7}{11}$, $\frac{4}{11}$; second after mint: $\frac{8}{11}$, $\frac{3}{11}$

Tip: Without replacement, the total goes down by 1 for the second tea bag, and so does the number of the type already taken.

Marks: 1 mark for $\frac{8}{12}$ and $\frac{4}{12}$. 1 mark for all four second branches correct.

(b) Mei says,

“I am more likely to take two tea bags of the same flavour than two tea bags of different flavours.”

Is Mei correct?

You must show your working.

$$P(\text{both green}) = \frac{8}{12} \times \frac{7}{11} = \frac{56}{132}$$

$$P(\text{both mint}) = \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$$

$$P(\text{same flavour}) = \frac{56}{132} + \frac{12}{132} = \frac{68}{132}$$

$$P(\text{different flavours}) = 1 - \frac{68}{132} = \frac{64}{132}$$

$\frac{68}{132} > \frac{64}{132}$, so Mei is correct.

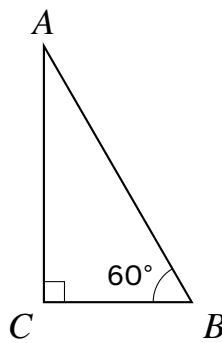
Answer: Yes: $P(\text{same}) = \frac{68}{132} = \frac{17}{33}$ and $P(\text{different}) = \frac{64}{132} = \frac{16}{33}$

Tip: “Same flavour” means green-green **or** mint-mint: multiply along each branch, then add. Compare with $P(\text{different})$ or with $\frac{1}{2}$.

Marks: 1 mark for one correct product (e.g. $\frac{8}{12} \times \frac{7}{11}$). 1 mark for $P(\text{same}) = \frac{68}{132}$. 1 mark for Yes, comparing with $P(\text{different}) = \frac{64}{132}$ or with $\frac{1}{2}$.

24 (3 marks)

ABC is a right-angled triangle.



Not drawn accurately

The area of triangle ABC is $24\sqrt{3} \text{ cm}^2$

Work out the length of BC .

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

You must show your working.

Let $BC = x \text{ cm}$.

$$\tan 60^\circ = \frac{AC}{BC}, \text{ and } \tan 60^\circ = \sqrt{3}, \text{ so } AC = \sqrt{3}x$$

$$\text{Area} = \frac{1}{2} \times BC \times AC = \frac{1}{2} \times x \times \sqrt{3}x = \frac{\sqrt{3}}{2}x^2$$

$$\frac{\sqrt{3}}{2}x^2 = 24\sqrt{3}, \text{ so } x^2 = 48$$

$$x = \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

Answer: $4\sqrt{3}$ cm

Tip: Learn the exact values: $\tan 60^\circ = \sqrt{3}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$. Call the unknown length x and write the other side in terms of x .

Marks: 1 mark for $AC = \sqrt{3} \times BC$ (using $\tan 60^\circ = \sqrt{3}$). 1 mark for $x^2 = 48$. 1 mark for $4\sqrt{3}$.

25 (5 marks)

The circle $x^2 + y^2 = 42$ and the line $x - y = 6$ intersect at two points. Work out the exact coordinates of the two points. Give your answers in their simplest form.

From the line: $y = x - 6$

Substitute into the circle: $x^2 + (x - 6)^2 = 42$

$$x^2 + x^2 - 12x + 36 = 42$$

$$2x^2 - 12x - 6 = 0$$

Divide by 2: $x^2 - 6x - 3 = 0$

Complete the square: $(x - 3)^2 - 9 - 3 = 0$, so $(x - 3)^2 = 12$

$$x - 3 = \pm\sqrt{12} = \pm 2\sqrt{3}, \text{ so } x = 3 + 2\sqrt{3} \text{ or } x = 3 - 2\sqrt{3}$$

$$y = x - 6: y = -3 + 2\sqrt{3} \text{ or } y = -3 - 2\sqrt{3}$$

The points are $(3 + 2\sqrt{3}, -3 + 2\sqrt{3})$ and $(3 - 2\sqrt{3}, -3 - 2\sqrt{3})$.

Answer: $(3 + 2\sqrt{3}, -3 + 2\sqrt{3})$ and $(3 - 2\sqrt{3}, -3 - 2\sqrt{3})$

Tip: Expand $(x - 6)^2$ as $x^2 - 12x + 36$, not $x^2 + 36$. Leave the answers exact: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$, and match each y with its own x .

Marks: 1 mark for substituting $y = x - 6$ (or $x = y + 6$). 1 mark for $x^2 - 6x - 3 = 0$ or $2x^2 - 12x - 6 = 0$. 1 mark for a correct method to solve it (completing the square or the formula). 1 mark for $x = 3 \pm 2\sqrt{3}$. 1 mark for both points with the y -coordinates.
