

Mock Set A – Paper 3

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 3 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Dafydd is planning a walk in the Peak District.

The route he wants to walk is 14.6 cm long on his map.

The scale of the map is 1:25 000

Work out the real length of the route.

Give your answer in kilometres.

The scale 1:25 000 means 1 cm on the map is 25 000 cm in real life.

$$14.6 \times 25\,000 = 365\,000 \text{ cm}$$

$$365\,000 \div 100 = 3650 \text{ m}$$

$$3650 \div 1000 = 3.65 \text{ km}$$

Answer: 3.65 km

Tip: There are 100 000 cm in a kilometre. Divide by 100 to get metres, then by 1000 to get kilometres.

Marks: 1 mark for $14.6 \times 25\,000$ (or 365 000). 1 mark for 3.65 km.

2 (1 mark)

Here are the first four terms of a linear sequence.

23 19 15 11

Which expression is the n th term of the sequence?

Circle your answer.

Options: $23 - 4n$, $4n + 27$, $27 - 4n$, $19 - 4n$

The terms go down by 4 each time, so the n th term includes $-4n$.

When $n = 1$, $-4n = -4$. To get the first term 23, add 27.

So the n th term is $27 - 4n$.

Check: $n = 2$ gives $27 - 8 = 19$, which is the second term.

Answer: $27 - 4n$

Tip: The number added is the term **before** the first term ($23 + 4 = 27$), not the first term itself.

3 (2 marks)

The length of a ribbon is l cm. l is **truncated** to 1 decimal place. The result is 46.3 cm. Complete the error interval for l .

Truncating to 1 decimal place means chopping off every digit after the first decimal place.

Any length from 46.3 up to (but not including) 46.4 truncates to 46.3.

For example, 46.39 truncates to 46.3, but 46.4 does not.

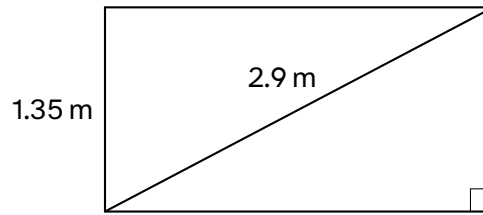
Answer: $46.3 \leq l < 46.4$

Tip: For truncation the lower limit is the value itself. $46.25 \leq l < 46.35$ is the error interval for **rounding**, not truncating.

Marks: 1 mark for 46.3 on the left. 1 mark for 46.4 on the right.

4 (3 marks)

The diagram shows a rectangular gate with a diagonal brace.



Not drawn accurately

Work out the perimeter of the gate.

The brace is the hypotenuse of a right-angled triangle.

$$\text{Width}^2 = 2.9^2 - 1.35^2$$

$$= 8.41 - 1.8225 = 6.5875$$

$$\text{Width} = \sqrt{6.5875} = 2.5666 \dots \text{ m}$$

$$\text{Perimeter} = 2 \times (2.5666 \dots + 1.35)$$

$$= 7.833 \dots \text{ m}$$

Answer: 7.83 m (to 3 s.f.)

Tip: You are finding a shorter side, so **subtract** the squares. Then remember the perimeter needs all four sides.

Marks: 1 mark for $2.9^2 - 1.35^2$. 1 mark for the width 2.566 ... (or 2.57). 1 mark for 7.83.

5 (3 marks)

Bethan buys dog biscuits for her dog.

Dog biscuits are sold in three sizes of bag.

Small bag	Medium bag	Large bag
2 kg	3 kg	5 kg
£5.80	£8.40	£11.90
	Now 20% extra free	

Which bag is the best value for money?

You must show your working.

Work out the price of 1 kg for each bag.

Small: $£5.80 \div 2 = £2.90$ per kg

Medium: 20% extra free gives $3 \times 1.2 = 3.6$ kg

$£8.40 \div 3.6 = £2.333 \dots$ per kg

Large: $£11.90 \div 5 = £2.38$ per kg

The medium bag has the lowest price per kg.

Answer: Medium bag (about £2.33 per kg)

Tip: Include the free biscuits: the medium bag holds 3.6 kg, not 3 kg. Without it, the medium bag works out at £2.80 per kg and the large bag looks best.

Marks: 1 mark for 3.6 kg. 1 mark for comparable values for all three bags. 1 mark for Medium with correct values.

6 (3 marks)

A cricket club orders new shirts.

- each of the first 12 shirts costs £16
- each shirt after the first 12 costs 25% less
- there is a delivery charge of £8.50 for the whole order.

The club pays £368.50 in total.

How many shirts did the club order?

The first 12 shirts cost $12 \times £16 = £192$

Each shirt after the first 12 costs $£16 \times 0.75 = £12$

Take off the delivery: $£368.50 - £8.50 = £360$

Left for the cheaper shirts: $£360 - £192 = £168$

Number of cheaper shirts: $168 \div 12 = 14$

Total: $12 + 14 = 26$ shirts

Answer: 26

Tip: Do not forget the first 12 shirts at the end: 14 is only the number of cheaper shirts.

Marks: 1 mark for 192 or 12 (the reduced price). 1 mark for $(368.50 - 8.50 - 192) \div 12$ or 14. 1 mark for 26.

7 (4 marks)

Sami catches a bus to school on Monday and on Tuesday.

- The probability that the bus is late on Monday is 0.2
- The probability that the bus is late on Tuesday is 0.35

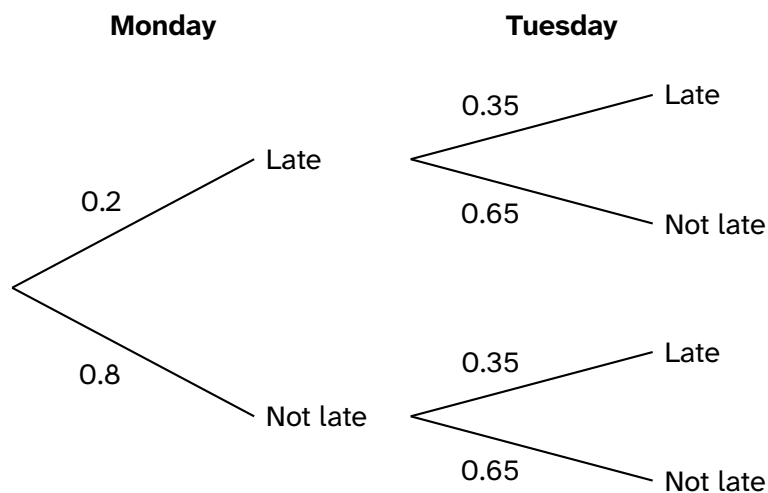
Whether the bus is late on Tuesday is independent of whether it is late on Monday.

(a) Complete the tree diagram.

The branches from each point add up to 1.

Monday not late: $1 - 0.2 = 0.8$

Tuesday is independent of Monday, so both pairs of Tuesday branches are 0.35 and 0.65.



Answer: Monday: 0.8; Tuesday: 0.65, 0.35, 0.65

Tip: Independent means the Tuesday probabilities are the same on both sets of branches.

Marks: 1 mark for 0.8. 1 mark for all three Tuesday values.

- (b) Work out the probability that the bus is late on at least one of the two mornings.

‘At least one late’ is everything except ‘not late on both mornings’.

$$P(\text{not late, not late}) = 0.8 \times 0.65 = 0.52$$

$$P(\text{at least one late}) = 1 - 0.52 = 0.48$$

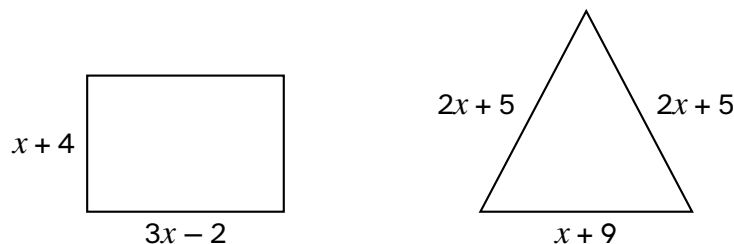
Answer: 0.48

Tip: Multiply along the branches and add the outcomes you want. Using $1 - P(\text{neither late})$ is quickest.

Marks: 1 mark for 0.8×0.65 (or for adding the three correct products). 1 mark for 0.48.

8 (3 marks)

Here are a rectangle and an isosceles triangle.
All measurements are in centimetres.



Not drawn accurately

The rectangle and the triangle have the same perimeter.
Work out the length of the side $x + 9$ of the triangle.

Perimeter of the rectangle: $2(3x - 2) + 2(x + 4) = 6x - 4 + 2x + 8 = 8x + 4$

Perimeter of the triangle: $(2x + 5) + (2x + 5) + (x + 9) = 5x + 19$

Set them equal: $8x + 4 = 5x + 19$

$3x = 15$, so $x = 5$

Side $x + 9 = 5 + 9 = 14$ cm

Answer: 14 cm

Tip: The rectangle has **two** sides of each length. Many students only add $(3x - 2) + (x + 4)$.

Marks: 1 mark for both perimeters ($8x + 4$ and $5x + 19$). 1 mark for $x = 5$. 1 mark for 14.

9 (4 marks)

Zara rides her horse from her stables to a village and back.

- On the way to the village she rides 16.8 km at an average speed of 12 km/h.
- On the way back she takes a shorter route of 12 km. This takes her 45 minutes.

Zara says,

‘My average speed for the whole ride was more than 13.5 km/h.’

Is Zara correct?

Tick a box.

You must show your working.

Time to the village: $16.8 \div 12 = 1.4$ hours

Time back: 45 minutes = $45 \div 60 = 0.75$ hours

Total distance: $16.8 + 12 = 28.8$ km

Total time: $1.4 + 0.75 = 2.15$ hours

Average speed = $28.8 \div 2.15 = 13.39 \dots$ km/h

$13.39 \dots$ km/h is **less** than 13.5 km/h, so Zara is **not** correct.

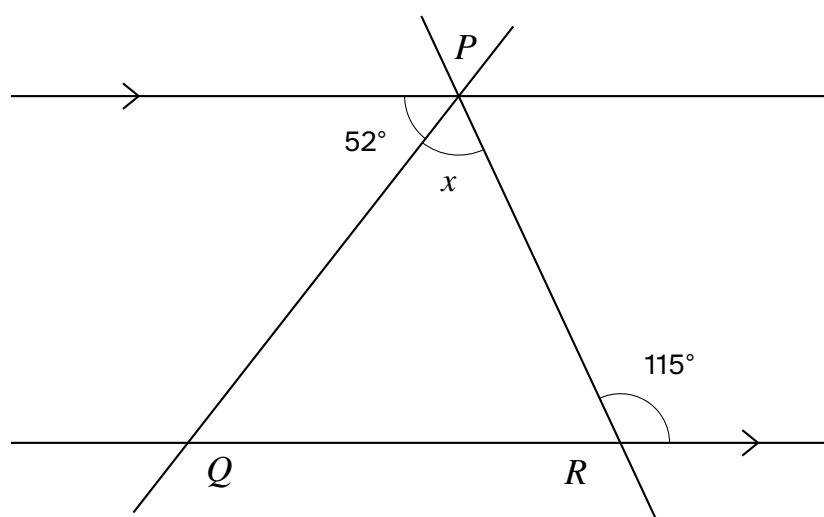
Answer: No. Her average speed was $28.8 \div 2.15 = 13.4$ km/h, which is less than 13.5 km/h

Tip: Average speed = total distance $\div$ total time. Do not find the mean of the two speeds: $(12 + 16) \div 2 = 14$ is wrong, because Zara rides for longer at the slower speed. Also, 45 minutes is 0.75 hours, not 0.45 hours.

Marks: 1 mark for 1.4 hours. 1 mark for 0.75 hours. 1 mark for $28.8 \div 2.15$. 1 mark for 13.4 and 'No'.

10 (3 marks)

The diagram shows two parallel lines and two straight lines that meet on the upper parallel line.



Not drawn accurately

Show that $x = 63^\circ$

The angle at Q inside the triangle is 52° (alternate angles are equal).

The angle at R inside the triangle is $180^\circ - 115^\circ = 65^\circ$ (angles on a straight line add up to 180°).

$$x = 180^\circ - 52^\circ - 65^\circ \text{ (angles in a triangle add up to } 180^\circ\text{)}$$

$$x = 63^\circ$$

Answer: Angle $Q = 52^\circ$ (alternate), angle $R = 180 - 115 = 65^\circ$ (straight line),
 $x = 180 - 52 - 65 = 63^\circ$ (triangle)

Tip: In a 'show that' question you must show every step. Writing the reasons, such as 'alternate angles', makes your method clear.

Marks: 1 mark for the 52° inside the triangle. 1 mark for the 65° . 1 mark for a full method that reaches 63° .

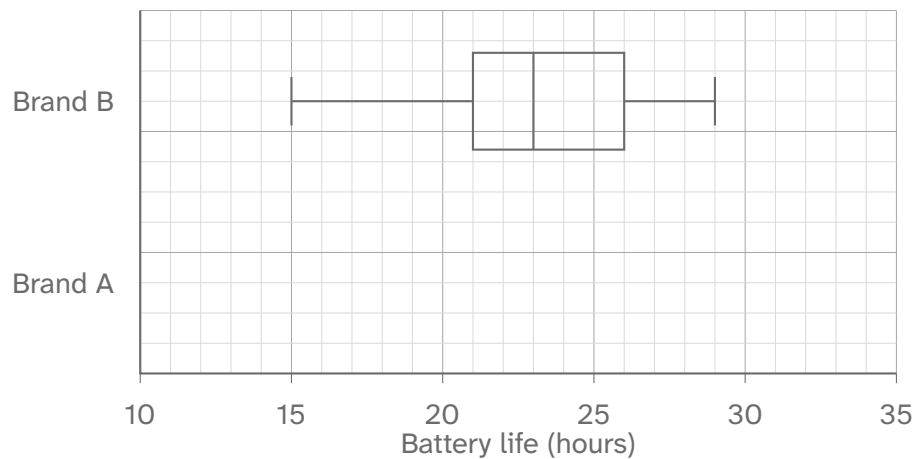
11 (4 marks)

Kemi tests the battery life of 60 pairs of Brand *A* headphones and 60 pairs of Brand *B* headphones.

Information about Brand *A* is shown in the table.

Shortest battery life	12 hours
Lower quartile	17 hours
Median	21 hours
Upper quartile	25 hours
Longest battery life	31 hours

The box plot for Brand *B* is drawn on the grid.



(a) On the grid, draw a box plot for Brand *A*.

Draw the box from the lower quartile (17) to the upper quartile (25).

Draw the median line at 21.

Draw the whiskers out to 12 and 31.



Answer: Box plot with whiskers to 12 and 31, box from 17 to 25, median at 21

Tip: Use a ruler and check the scale: each small square is 1 hour.

Marks: 2 marks for a fully correct box plot. 1 mark if at least three of the five values are plotted correctly.

- (b) Compare the battery lives of the two brands.
Give **two** comparisons.

Medians: Brand A 21 hours, Brand B 23 hours.

So on average, Brand B headphones have a longer battery life.

Interquartile ranges: Brand A $25 - 17 = 8$ hours, Brand B $26 - 21 = 5$ hours.

Brand B has the smaller IQR, so its battery lives are more consistent.

Answer: On average Brand B lasts longer (median $23 > 21$); Brand B is more consistent (IQR $5 < 8$)

Tip: Compare an average (the median) and a spread (the IQR), and say what they mean for battery life. 'Brand B has a bigger median' on its own is not enough.

Marks: 1 mark for comparing the medians in context. 1 mark for comparing the IQRs (or ranges) in context.

12 (3 marks)

Each flat in a block has a door entry code.

Each code is **two** letters followed by **four** digits.

- The letters are chosen from *A, B, C, D, E, F*
- The digits are chosen from 0 to 9

Letters and digits can be repeated.

- (a) Work out the number of different codes possible.

There are 6 choices for each letter and 10 choices for each digit.

$$6 \times 6 \times 10 \times 10 \times 10 \times 10 = 360\,000$$

Answer: 360 000

- (b) New rules are brought in.

- The two letters must be different.
- The last digit must be odd.

Jack says,

‘The new rules halve the number of different codes possible.’

Is Jack correct?

Tick a box.

You must show your working.

Letters: 6 choices, then 5 choices (the second letter must be different).

Digits: $10 \times 10 \times 10$, then 5 choices for the last digit (1, 3, 5, 7, 9).

$$6 \times 5 \times 10 \times 10 \times 10 \times 5 = 150\,000$$

Half of 360 000 is 180 000.

150 000 is not 180 000, so Jack is not correct.

Answer: No. There are 150 000 codes, but half of 360 000 is 180 000

Tip: Each rule changes one position. Work out the number of choices for every position, then multiply.

Marks: 1 mark for $6 \times 5 \times 10 \times 10 \times 10 \times 5$ or 150 000. 1 mark for ‘No’ with 150 000 compared with 180 000 (or 360 000).

13 (3 marks)

Owen has a water butt in his garden.

- When full, the water butt holds 0.35 m^3 of water.
- The water butt already contains 80 litres of water.
- Water flows into the water butt at a constant rate of 3.6 litres per minute.

$$1 \text{ m}^3 = 1000 \text{ litres}$$

Owen starts filling the water butt at 10:50 am.

Will the water butt be full by 12 noon?

You must show your working.

$$1 \text{ m}^3 = 1000 \text{ litres, so } 0.35 \text{ m}^3 = 350 \text{ litres.}$$

$$\text{Water still needed: } 350 - 80 = 270 \text{ litres}$$

$$\text{Time: } 270 \div 3.6 = 75 \text{ minutes}$$

$$75 \text{ minutes} = 1 \text{ hour } 15 \text{ minutes}$$

$$10:50 \text{ am} + 1 \text{ hour } 15 \text{ minutes} = 12:05 \text{ pm}$$

So the water butt will **not** be full by 12 noon (it is full at 12:05 pm).

Answer: No. It takes 75 minutes, so it is full at 12:05 pm

Tip: 1 m^3 is 1000 litres. From 10:50 am to 12 noon is only 70 minutes. Also, 75 minutes is 1 hour 15 minutes, not 1.15 hours.

Marks: 1 mark for 350 litres. 1 mark for $(350 - 80) \div 3.6$ or 75 minutes. 1 mark for 'No' with 12:05 pm (or 75 minutes compared with 70 minutes).

14 (3 marks)

Ali is solving $3x^2 - 7x - 20 = 0$

Here is his working.

$$3x^2 - 7x - 20 = 0$$

$$(3x - 5)(x + 4) = 0$$

$$x = \frac{5}{3} \text{ or } x = -4$$

- (a) Show that Ali's factorisation is wrong.

$$\begin{aligned}\text{Expand Ali's brackets: } (3x - 5)(x + 4) &= 3x^2 + 12x - 5x - 20 \\ &= 3x^2 + 7x - 20\end{aligned}$$

This has $+7x$, not $-7x$, so it is not equal to $3x^2 - 7x - 20$.

Answer: $(3x - 5)(x + 4) = 3x^2 + 7x - 20$, which is not $3x^2 - 7x - 20$

Tip: Expanding the brackets is the quickest check. You could also substitute: $x = -4$ gives $3 \times 16 + 28 - 20 = 56$, not 0.

- (b) Solve $3x^2 - 7x - 20 = 0$

$3 \times (-20) = -60$. Find two numbers that multiply to -60 and add to -7 : -12 and 5 .

$$3x^2 - 12x + 5x - 20 = 0$$

$$3x(x - 4) + 5(x - 4) = 0$$

$$(3x + 5)(x - 4) = 0$$

$$3x + 5 = 0 \text{ or } x - 4 = 0$$

$$x = -\frac{5}{3} \text{ or } x = 4$$

Answer: $x = -\frac{5}{3}$ or $x = 4$

Tip: Ali's signs were the wrong way round. Check by expanding:

$(3x + 5)(x - 4) = 3x^2 - 12x + 5x - 20 = 3x^2 - 7x - 20$. The bracket $(3x + 5)$ gives $x = -\frac{5}{3}$, not $x = -5$.

Marks: 1 mark for $(3x + 5)(x - 4)$ (or a correct substitution into the quadratic formula). 1 mark for both solutions.

15 (3 marks)

(a) Isla writes

$$5^4 \times 5^{-7} = 25^{-3}$$

What mistake has Isla made?

When multiplying powers of the same number, add the indices and keep the base the same.

Isla added the indices correctly ($4 + (-7) = -3$), but she also multiplied the bases: $5 \times 5 = 25$.

The base should stay as 5: $5^4 \times 5^{-7} = 5^{-3}$

Answer: She multiplied the bases ($5 \times 5 = 25$). The base should stay as 5, so the answer is 5^{-3}

Tip: Check with a calculator: $5^4 \times 5^{-7} = 0.008 = 5^{-3}$, but $25^{-3} = 0.000064$.

(b)
$$\frac{6^5 \times 10^3}{15^2} = 2^a \times 3^b \times 5^c$$

Work out the values of a , b and c .

Write each base as a product of primes.

$$6^5 = (2 \times 3)^5 = 2^5 \times 3^5$$

$$10^3 = (2 \times 5)^3 = 2^3 \times 5^3$$

$$15^2 = (3 \times 5)^2 = 3^2 \times 5^2$$

Powers of 2: $5 + 3 = 8$

Powers of 3: $5 - 2 = 3$

Powers of 5: $3 - 2 = 1$

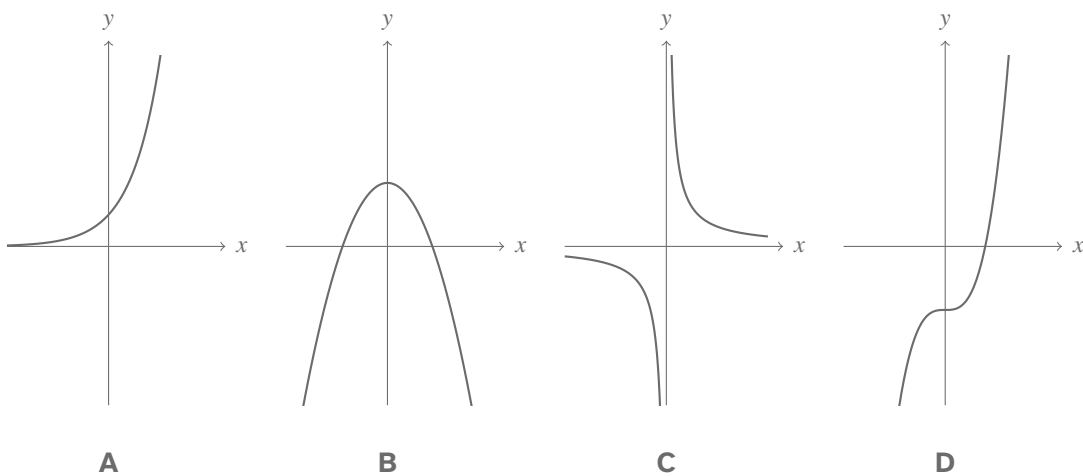
Answer: $a = 8, b = 3, c = 1$

Tip: On a calculator paper you can also work out the value (34 560) and write it as a product of prime factors to check.

Marks: 2 marks for all three correct. 1 mark for two correct.

16 (2 marks)

Here are four sketch graphs.



Write down the letter of the graph that could have each equation.

(a) $y = x^3 - 2$

$y = x^3 - 2$ is a cubic graph with a positive x^3 term, so it rises from bottom left to top right.

When $x = 0$, $y = -2$, so it crosses the y-axis below the origin.

Answer: D

(b) $y = 3^x$

$y = 3^x$ is an exponential graph.

It crosses the y -axis at $(0, 1)$, is always above the x -axis and gets steeper as x increases.

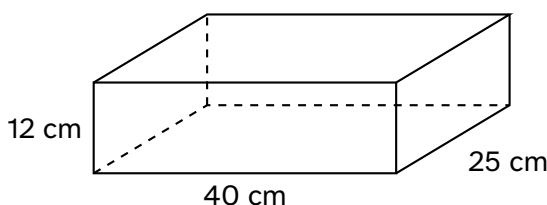
Answer: A

Tip: The other graphs are $y = 2 - x^2$ (B, a quadratic, symmetrical about the y -axis) and $y = \frac{1}{x}$ (C, a reciprocal graph in two separate parts). Only an exponential graph like $y = 3^x$ stays above the x -axis and passes through $(0, 1)$.

17 (3 marks)

A concrete block is a cuboid.

The block exerts a force of 150 newtons on the ground.



Not drawn accurately

The block can rest on any of its faces on horizontal ground.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the greatest pressure the block can exert on the ground.

Give your answer in newtons/m²

The greatest pressure comes from the smallest face.

Smallest face: 25 cm by 12 cm = 0.25 m by 0.12 m

Area = $0.25 \times 0.12 = 0.03 \text{ m}^2$

Pressure = $150 \div 0.03 = 5000 \text{ newtons/m}^2$

Answer: 5000 newtons/m²

Tip: Convert to metres **before** working out the area: 300 cm² is 0.03 m², not 3 m².

Marks: 1 mark for the area of the smallest face. 1 mark for dividing 150 by an area in m². 1 mark for 5000.

18 (4 marks)

A is the point $(-1, 8)$

B is the point $(7, 2)$

Line L is perpendicular to AB and passes through the midpoint of AB .

Work out the equation of line L .

Give your answer in the form $y = mx + c$

Midpoint of AB : $\left(\frac{-1+7}{2}, \frac{8+2}{2}\right) = (3, 5)$

Gradient of AB : $\frac{2-8}{7-(-1)} = \frac{-6}{8} = -\frac{3}{4}$

Perpendicular gradient: $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$)

$y = \frac{4}{3}x + c$ through $(3, 5)$: $5 = \frac{4}{3} \times 3 + c$

$5 = 4 + c$, so $c = 1$

$y = \frac{4}{3}x + 1$

Answer: $y = \frac{4}{3}x + 1$

Tip: Perpendicular gradients multiply to -1 : flip the fraction and change the sign. The perpendicular to $-\frac{3}{4}$ is $\frac{4}{3}$, not $\frac{3}{4}$ or $-\frac{4}{3}$.

Marks: 1 mark for the midpoint $(3, 5)$. 1 mark for the gradient of AB . 1 mark for the perpendicular gradient. 1 mark for the equation.

19 (3 marks)

Here are the first five terms of a quadratic sequence.

5 9 19 35 57

Work out an expression for the n th term.

First differences: 4, 10, 16, 22

Second differences: 6, 6, 6

Half of 6 is 3, so the n th term starts $3n^2$.

$3n^2$: 3, 12, 27, 48, 75

Sequence $-3n^2$: 2, -3, -8, -13, -18

This linear sequence has n th term $-5n + 7$.

So the n th term is $3n^2 - 5n + 7$.

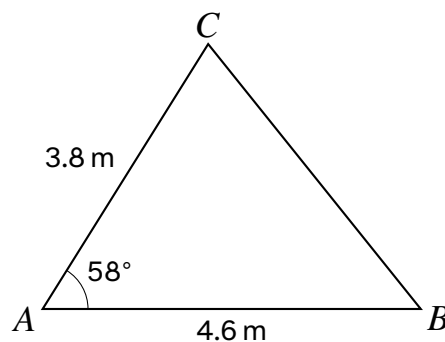
Answer: $3n^2 - 5n + 7$

Tip: Subtract $3n^2$ from the sequence (not the other way round), then find the n th term of what is left. Take care with the negatives: $9 - 12 = -3$.

Marks: 1 mark for $3n^2$. 1 mark for subtracting $3n^2$ to get 2, -3, -8, ... 1 mark for $3n^2 - 5n + 7$.

20 (5 marks)

The diagram shows a triangular patio ABC .



Not drawn accurately

(a) Work out the length BC .

Give your answer to 3 significant figures.

Two sides and the angle between them are known, so use the cosine rule.

$$BC^2 = 4.6^2 + 3.8^2 - 2 \times 4.6 \times 3.8 \times \cos 58^\circ$$

$$BC^2 = 21.16 + 14.44 - 18.525 \dots = 17.074 \dots$$

$$BC = \sqrt{17.074 \dots} = 4.1320 \dots$$

$$BC = 4.13 \text{ m (to 3 s.f.)}$$

Answer: 4.13 m

Tip: Work out $2 \times 4.6 \times 3.8 \times \cos 58^\circ$ first and subtract it. Do not do $(21.16 + 14.44 - 34.96) \times \cos 58^\circ$.

Marks: 1 mark for substituting into the cosine rule. 1 mark for 17.07 ... 1 mark for 4.13.

- (b) Megan is going to cover the patio with sealant.

One tin of sealant covers 2.4 m^2

Megan has 3 tins of sealant.

Does Megan have enough sealant?

You must show your working.

Area = $\frac{1}{2}ab \sin C$, using the two sides either side of the 58° angle.

$$\text{Area} = \frac{1}{2} \times 4.6 \times 3.8 \times \sin 58^\circ$$

$$= 7.411 \dots \text{ m}^2$$

$$3 \text{ tins cover } 3 \times 2.4 = 7.2 \text{ m}^2$$

7.2 m^2 is less than 7.41 m^2 , so Megan does **not** have enough sealant.

Answer: No. The area is 7.41 m^2 but 3 tins only cover 7.2 m^2

Tip: The angle in $\frac{1}{2}ab \sin C$ must be the angle **between** the two sides you use.

$\frac{1}{2} \times 4.6 \times 3.8 = 8.74$ is not the area: this is not a right-angled triangle.

Marks: 1 mark for $\frac{1}{2} \times 4.6 \times 3.8 \times \sin 58^\circ$. 1 mark for 'No' with 7.41 and 7.2 (or 3.08 tins).

21 (2 marks)

Enlarge shape A by scale factor -2 , centre $(1, 1)$

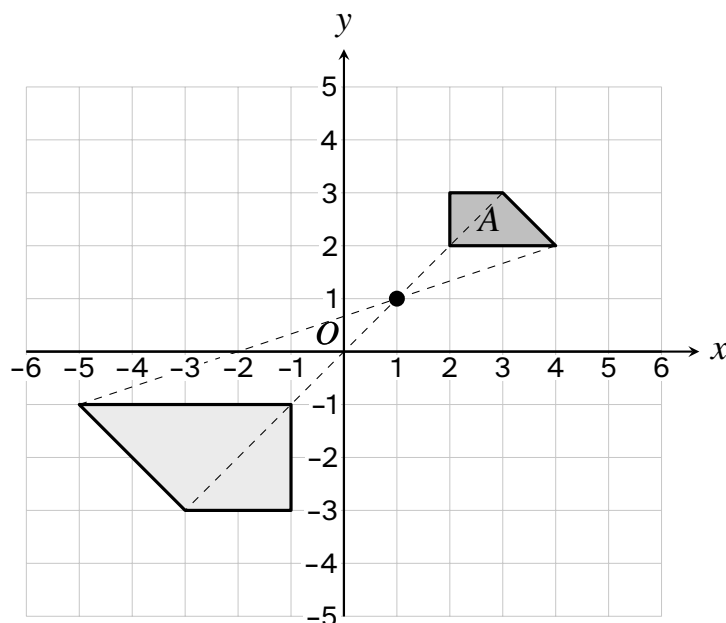
For each vertex, find how far it is from the centre $(1, 1)$, multiply by -2 , and count that from the centre.

$(2, 2)$ is 1 right, 1 up. $\times(-2)$: 2 left, 2 down $\rightarrow (-1, -1)$

$(4, 2)$ is 3 right, 1 up. $\times(-2)$: 6 left, 2 down $\rightarrow (-5, -1)$

(3, 3) is 2 right, 2 up. $\times(-2)$: 4 left, 4 down $\rightarrow (-3, -3)$

(2, 3) is 1 right, 2 up. $\times(-2)$: 2 left, 4 down $\rightarrow (-1, -3)$



Answer: Shape with vertices $(-1, -1)$, $(-5, -1)$, $(-3, -3)$, $(-1, -3)$

Tip: A negative scale factor puts the image on the **other side** of the centre, upside down.

Marks: 2 marks for a fully correct shape. 1 mark for the correct size and upside down in the wrong place, or three correct vertices.

22 (4 marks)

The ratio of fiction books to non-fiction books on a classroom shelf is 7:4

Mrs Hughes takes 8 fiction books off the shelf.

She puts 4 more non-fiction books on the shelf.

The ratio of fiction books to non-fiction books is now 4:3

How many fiction books were on the shelf at the start?

At the start: $7x$ fiction and $4x$ non-fiction.

Afterwards: $7x - 8$ fiction and $4x + 4$ non-fiction.

$$\frac{7x - 8}{4x + 4} = \frac{4}{3}$$

$$3(7x - 8) = 4(4x + 4)$$

$$21x - 24 = 16x + 16$$

$$5x = 40, \text{ so } x = 8$$

$$\text{Fiction books at the start: } 7 \times 8 = 56$$

$$\text{Check: } 48:36 = 4:3$$

Answer: 56

Tip: Use one letter for the size of one share: fiction = $7x$, non-fiction = $4x$. Then the new ratio gives one equation in x .

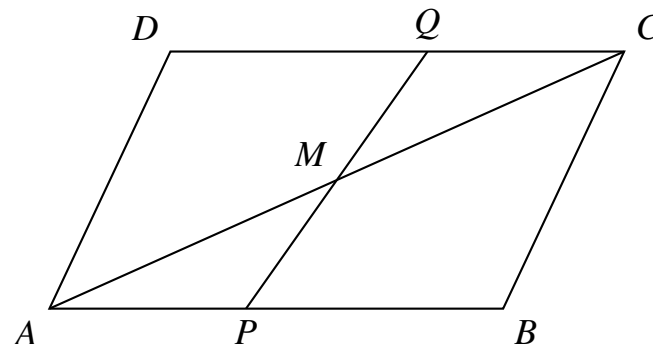
Marks: 1 mark for $7x - 8$ and $4x + 4$. 1 mark for the equation $3(7x - 8) = 4(4x + 4)$. 1 mark for $x = 8$. 1 mark for 56.

23 (4 marks)

$ABCD$ is a parallelogram.

M is the midpoint of the diagonal AC .

PMQ is a straight line with P on AB and Q on DC .



Not drawn accurately

Prove that triangle AMP is congruent to triangle CMQ .

$AM = CM$ (because M is the midpoint of AC).

Angle $PAM =$ angle QCM (alternate angles, because AB is parallel to DC).

Angle $AMP =$ angle CMQ (vertically opposite angles are equal).

So the triangles have two angles and the side between them equal.

Triangles AMP and CMQ are congruent (ASA).

Answer: $AM = CM$ (midpoint); angle $PAM =$ angle QCM (alternate angles, $AB \parallel DC$); angle $AMP =$ angle CMQ (vertically opposite); so congruent by ASA

Tip: Every statement needs a reason. Name the condition at the end: two angles and the included side is ASA. AAA does not prove congruence.

Marks: 1 mark for each correct pair of equal sides or angles with a reason (3 marks). 1 mark for a fully correct proof ending with ASA (or AAS).

24 (5 marks)

The straight line $x + y = 4$ meets the curve $y = 2x^2 + x - 8$ at the points A and B .

Work out the length AB .

Give your answer to 3 significant figures.

Do **not** use trial and improvement.

You must show your working.

At A and B both equations are true, so solve them simultaneously.

From the line, $y = 4 - x$.

Substitute: $4 - x = 2x^2 + x - 8$

$$0 = 2x^2 + 2x - 12$$

Divide by 2: $x^2 + x - 6 = 0$

$$(x + 3)(x - 2) = 0, \text{ so } x = -3 \text{ or } x = 2$$

When $x = -3$: $y = 4 - (-3) = 7$. When $x = 2$: $y = 4 - 2 = 2$.

So the points are $(-3, 7)$ and $(2, 2)$.

Horizontal distance: $2 - (-3) = 5$. Vertical distance: $7 - 2 = 5$.

$$AB^2 = 5^2 + 5^2 = 50$$

$$AB = \sqrt{50} = 7.071... = 7.07 \text{ (to 3 s.f.)}$$

Answer: 7.07 (the points are $(-3, 7)$ and $(2, 2)$; $AB = \sqrt{50} = 5\sqrt{2}$)

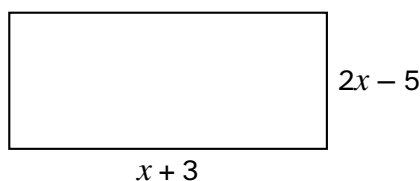
Tip: Draw a sketch of the two points and make a right-angled triangle between them. The sides of the triangle are the differences in the x and y coordinates.

Marks: 1 mark for substituting $y = 4 - x$ into the curve. 1 mark for $x^2 + x - 6 = 0$ (or $2x^2 + 2x - 12 = 0$). 1 mark for both points. 1 mark for using Pythagoras with the differences between the points. 1 mark for 7.07.

25 (4 marks)

Here is a rectangle.

All measurements are in centimetres.



Not drawn accurately

The area of the rectangle is less than 30 cm^2

Work out the range of possible values of x .

You must show your working.

$$\text{Area: } (x + 3)(2x - 5) < 30$$

$$2x^2 - 5x + 6x - 15 < 30$$

$$2x^2 + x - 45 < 0$$

$$\text{Solve } 2x^2 + x - 45 = 0: (2x - 9)(x + 5) = 0, \text{ so } x = 4.5 \text{ or } x = -5$$

The graph of $y = 2x^2 + x - 45$ is a U shape, so it is below 0 between the roots:
 $-5 < x < 4.5$

The sides must have positive length: $2x - 5 > 0$, so $x > 2.5$ (then $x + 3 > 0$ too).

Combine: $2.5 < x < 4.5$

Answer: $2.5 < x < 4.5$

Tip: Two conditions are needed: the area condition gives $x < 4.5$, and the side $2x - 5$ must be positive, which gives $x > 2.5$.

Marks: 1 mark for $2x^2 + x - 45 < 0$ (or $= 0$). 1 mark for the critical values 4.5 and -5 . 1 mark for $x < 4.5$ (or $-5 < x < 4.5$). 1 mark for $2.5 < x < 4.5$.
