

Mock Set A – Paper 2

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 2 (Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

- (a) Write 756 as a product of its prime factors.
Give your answer in index form.

Divide by primes: $756 \div 2 = 378$, $378 \div 2 = 189$

$189 \div 3 = 63$, $63 \div 3 = 21$, $21 \div 3 = 7$

7 is prime, so stop.

$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$

In index form: $2^2 \times 3^3 \times 7$

Answer: $2^2 \times 3^3 \times 7$

Tip: Index form means using powers: write 3^3 , not $3 \times 3 \times 3$.

Marks: 1 mark for a correct method, such as a factor tree or repeated division with at most one error. 1 mark for the answer in index form.

- (b) $N = 2^3 \times 3^2 \times 5$
Work out the lowest common multiple (LCM) of 756 and N .

Take the highest power of each prime that appears in either number.

2: 2^2 and 2^3 , so use 2^3 . 3: 3^3 and 3^2 , so use 3^3 .

Also include 5 and 7.

$$\text{LCM} = 2^3 \times 3^3 \times 5 \times 7 = 7560$$

Answer: 7560 (or $2^3 \times 3^3 \times 5 \times 7$)

Tip: For the LCM use the highest power of every prime. For the HCF use the lowest power of the primes that appear in both.

2 (1 mark)

Which line is parallel to $y = 3x - 5$?

Circle your answer.

Options: $y = 5x - 3$, $y = -3x - 5$, $y = 2 + 3x$, $y = -\frac{1}{3}x - 5$

Parallel lines have the same gradient.

$y = 3x - 5$ has gradient 3.

$y = 2 + 3x$ is the same as $y = 3x + 2$, which also has gradient 3.

Answer: $y = 2 + 3x$

Tip: Look at the number in front of x , not the constant. $y = -\frac{1}{3}x - 5$ is perpendicular to $y = 3x - 5$, not parallel.

3 (2 marks)

(a) Use your calculator to work out

$$\frac{\sqrt{48.7} + 2.3^3}{6.1 - 0.84^2}$$

Write down all the figures on your calculator display.

Put brackets round the whole top and the whole bottom, or use the fraction key.

Top: $\sqrt{48.7} + 2.3^3 = 6.978538 \dots + 12.167 = 19.145538 \dots$

Bottom: $6.1 - 0.84^2 = 6.1 - 0.7056 = 5.3944$

$$19.145538 \dots \div 5.3944 = 3.549150699 \dots$$

Answer: 3.549150699

Tip: Use the fraction key, or put brackets round the numerator and the denominator. Typing it without brackets divides only part of the top by part of the bottom.

- (b) Give your answer to part (a) to 3 significant figures.

The first three significant figures are 3.54

The next digit is 9, which is 5 or more, so round up.

3.55

Answer: 3.55

4 (2 marks)

Harriet buys a bicycle for £145 and sells it for £203. She says, “My percentage profit is 28.6%”. Is Harriet correct?

$$\text{Profit} = 203 - 145 = \text{£}58$$

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100$$

$$= \frac{58}{145} \times 100 = 40\%$$

Harriet worked out $\frac{58}{203} \times 100 = 28.6\%$: she divided by the selling price instead of the price she paid.

No, Harriet is not correct. Her percentage profit is 40%.

Answer: No: her percentage profit is 40%

Tip: Percentage profit is always worked out as a percentage of the cost price (what she paid), not the selling price.

Marks: 1 mark for $\frac{58}{145}$ (or $203 \div 145 = 1.4$). 1 mark for “No” with 40%.

5 (1 mark)

The council in Brantwell wants to know what the people of the town think about the opening hours of the town library.

Four ways of choosing a sample of 100 people are described below.

Which sample is most likely to be biased?

Tick **one** box.

People leaving the library already use it, and only at the times it is open now.

People who cannot get there at the current times (for example, people who work on Saturdays) are left out.

So the views of this sample are unlikely to represent the whole town.

Answer: 100 people leaving the library on a Saturday morning

Tip: A sample is biased if some groups of the population have no chance (or less chance) of being chosen.

6 (3 marks)

In a sale, the price of an electric scooter is reduced by 12%

The sale price is £338.80

Work out the price of the scooter before the sale.

After a 12% reduction, the sale price is $100\% - 12\% = 88\%$ of the original price.

88% of the original price = £338.80

Original price = $338.80 \div 0.88$

= £385

Answer: £385

Tip: Do not add 12% of £338.80 back on. The 12% was taken off the original price, so divide by 0.88.

Marks: 1 mark for 88% (or 0.88). 1 mark for $338.80 \div 0.88$. 1 mark for £385.

7 (4 marks)

The table shows information about the lengths, t minutes, of 40 episodes of a podcast.

Length (t minutes)	Frequency
$0 < t \leq 20$	7
$20 < t \leq 40$	12
$40 < t \leq 60$	11
$60 < t \leq 90$	6
$90 < t \leq 120$	4

- (a) Work out an estimate of the mean length of an episode.

Use the midpoint of each class as an estimate of each length.

Length (t minutes)	Frequency	Midpoint	Midpoint $\times$ frequency
$0 < t \leq 20$	7	10	70
$20 < t \leq 40$	12	30	360
$40 < t \leq 60$	11	50	550
$60 < t \leq 90$	6	75	450
$90 < t \leq 120$	4	105	420
Total	40		1850

Total of midpoint $\times$ frequency = 1850

Total frequency = 40

Mean $\approx 1850 \div 40 = 46.25$ minutes

Answer: 46.25 minutes

Tip: Divide by the total frequency (40), not by the number of classes (5). Check that your mean lies between 0 and 120.

Marks: 1 mark for using the midpoints with the frequencies. 1 mark for dividing their total by 40. 1 mark for 46.25.

- (b) Each of the 4 longest episodes lasted more than 105 minutes. Niamh says, “So the actual mean must be more than the estimate in part (a).” Is Niamh correct?

In part (a) the 4 longest episodes were each counted as 105 minutes (the midpoint), so for those 4 the estimate is too low.

But we do not know the lengths of the other 36 episodes. They could be shorter than the midpoints of their classes.

So the actual mean could be more than 46.25, or less. It does not **have** to be more.

No, Niamh is not correct.

Answer: No: the other 36 episodes could be shorter than their class midpoints, so the actual mean could still be less than the estimate.

Tip: An estimate of the mean uses a midpoint for every class. Knowing about one class is not enough: the errors in the other classes could go either way.

8 (4 marks)

Imani has £6000 to save for 5 years. Account A pays simple interest at 3.6% per year. Account B pays compound interest at 3.4% per year. Which account will give Imani more money at the end of 5 years, and how much more?

Account A: simple interest is the same every year.

Interest each year = $6000 \times 0.036 = \text{£}216$

After 5 years: $6000 + 5 \times 216 = \text{£}7080$

Account B: compound interest is added on each year, so use a multiplier.

Multiplier = 1.034

After 5 years: $6000 \times 1.034^5 = \text{£}7091.758 \dots = \text{£}7091.76$

Account B gives more: $7091.76 - 7080 = \text{£}11.76$ more.

Answer: Account B, by $\text{£}11.76$

Tip: Compound interest: multiply by 1.034 five times (1.034^5). Do not work out 3.4% of $\text{£}6000$ and add it on five times: that is simple interest.

Marks: 1 mark for the Account A total ($\text{£}7080$) or interest ($\text{£}1080$). 1 mark for 6000×1.034^5 . 1 mark for $\text{£}7091.76$. 1 mark for Account B and $\text{£}11.76$.

9 (3 marks)

Expand and simplify $(2x - 3)(x + 4)(3 - x)$

Multiply two of the brackets first: $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12 = 2x^2 + 5x - 12$

Now multiply by the third bracket: $(2x^2 + 5x - 12)(3 - x)$

$$= 6x^2 - 2x^3 + 15x - 5x^2 - 36 + 12x$$

Collect like terms: $-2x^3 + x^2 + 27x - 36$

Answer: $-2x^3 + x^2 + 27x - 36$

Tip: Take care with the bracket $(3 - x)$: the x term is **negative**, so every term multiplied by $-x$ changes sign. You should get six terms before simplifying.

Marks: 1 mark for expanding two of the brackets correctly. 1 mark for multiplying by the third bracket (six terms, at least four correct). 1 mark for the simplified answer.

10 (4 marks)

The Okafor family drive 225 miles on a road trip.

- Their car travels 16 kilometres on 1 litre of petrol.
- Petrol costs 148.9p per litre.

- (a) Use 5 miles = 8 kilometres. Work out the cost of the petrol for the road trip. Give your answer in pounds.

Change miles to kilometres: $225 \div 5 \times 8 = 360$ km

Petrol used: $360 \div 16 = 22.5$ litres

Cost: $22.5 \times 148.9 = 3350.25\text{p}$

In pounds: $3350.25 \div 100 = \text{£}33.5025$, so $\text{£}33.50$

Answer: $\text{£}33.50$

Tip: Write the units at each step (miles, km, litres, pence) so you know what each number means. Money in pounds needs two decimal places: $\text{£}33.50$, not $\text{£}33.5025$.

Marks: 1 mark for 360 km. 1 mark for 22.5 litres multiplied by 148.9. 1 mark for $\text{£}33.50$.

- (b) In fact, 5 miles is slightly more than 8 kilometres. What does this mean about the actual cost of the petrol for the road trip?

If 5 miles is more than 8 km, then 225 miles is more than 360 km.

A longer distance in km means more litres of petrol are used.

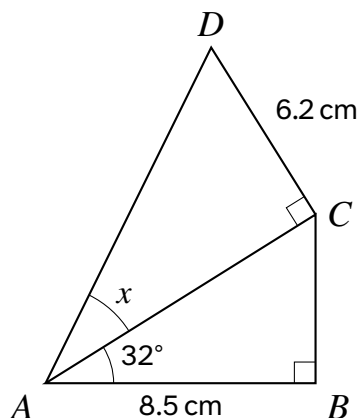
So the actual cost is more than the answer to part (a).

Answer: It is more than the answer to part (a)

Tip: Follow the change through each step: more kilometres $\rightarrow$ more litres $\rightarrow$ higher cost.

11 (4 marks)

The diagram shows two right-angled triangles, ABC and ACD .



Not drawn accurately

Work out the size of angle x .

Give your answer to 1 decimal place.

In triangle ABC : AB is adjacent to 32° and AC is the hypotenuse, so use $\cos$.

$$\cos 32^\circ = \frac{8.5}{AC}$$

$$AC = \frac{8.5}{\cos 32^\circ} = 10.0230 \dots \text{ cm}$$

In triangle ACD : CD is opposite x and AC is adjacent, so use $\tan$.

$$\tan x = \frac{6.2}{10.0230 \dots}$$

$$x = \tan^{-1}(0.6185 \dots) = 31.739 \dots^\circ$$

$$x = 31.7^\circ \text{ (1 d.p.)}$$

Answer: 31.7°

Tip: Keep the full value of AC in your calculator (use the answer key). Rounding AC too early can change the final answer.

Marks: 1 mark for a correct trig statement for AC . 1 mark for $AC = 10.02 \dots$. 1 mark for $\tan x = 6.2 \div \text{their } AC$. 1 mark for 31.7 .

12 (3 marks)

A metalworker melts together

- 210 g of copper, which has a density of 8.96 g/cm^3
- 90 g of zinc, which has a density of 7.13 g/cm^3

to make brass.

- (a) Work out the density of the brass.
Give your answer to 3 significant figures.

Volume = mass $\div$ density

Copper: $210 \div 8.96 = 23.4375 \text{ cm}^3$

Zinc: $90 \div 7.13 = 12.622 \dots \text{ cm}^3$

Total mass = $210 + 90 = 300 \text{ g}$. Total volume = $36.060 \dots \text{ cm}^3$

Density = $300 \div 36.060 \dots = 8.319 \dots$

= 8.32 g/cm^3 (3 s.f.)

Answer: 8.32 g/cm^3

Tip: Density of the mixture = total mass $\div$ total volume. Find each volume first, using
volume = mass $\div$ density.

Marks: 1 mark for working out both volumes (or the total volume $36.06 \dots$). 1 mark for 8.32.

- (b) Yusuf says,

“The density of the brass is the mean of 8.96 g/cm^3 and 7.13 g/cm^3 ”

Explain why Yusuf is wrong.

The mean of 8.96 and 7.13 would only be right if the two metals had the same volume.

There is much more copper than zinc, so the density is closer to 8.96 than to 7.13.

Answer: The volumes of copper and zinc are not equal: there is more copper, so the density is closer to 8.96.

13 (3 marks)

Solve $x(3x - 4) = 3x + 7$

Give your solutions to 2 decimal places.

Expand the bracket: $3x^2 - 4x = 3x + 7$

Rearrange to = 0: $3x^2 - 7x - 7 = 0$

$a = 3$, $b = -7$, $c = -7$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 3 \times (-7)}}{2 \times 3} = \frac{7 \pm \sqrt{133}}{6}$$

$$x = \frac{7 + 11.532...}{6} = 3.088... \text{ or } x = \frac{7 - 11.532...}{6} = -0.755...$$

$$x = 3.09 \text{ or } x = -0.76$$

Answer: $x = 3.09$ and $x = -0.76$

Tip: Rearrange so that one side is 0 before using the formula, and take care with the signs: here $b = -7$ and $c = -7$. "2 decimal places" tells you to use the formula.

Marks: 1 mark for rearranging to $3x^2 - 7x - 7 = 0$. 1 mark for substituting correctly into the quadratic formula. 1 mark for both solutions.

14 (3 marks)

Ruby tests a remote-controlled car on a straight track.

- The car travels 150 metres, to the nearest 10 metres.
- It takes 9.6 seconds, to the nearest 0.1 second.

Ruby says, "The average speed of the car could have been more than 16.1 m/s". Is Ruby correct?

Average speed = distance $\div$ time

The greatest possible speed uses the **largest** distance and the **smallest** time.

Upper bound of the distance = $150 + 5 = 155$ m (half of 10 m above 150 m)

Lower bound of the time = $9.6 - 0.05 = 9.55$ s

Upper bound of the speed = $155 \div 9.55 = 16.23... \text{ m/s}$

$16.23... > 16.1$, so the speed could have been more than 16.1 m/s.

Yes, Ruby is correct.

Answer: Yes: $155 \div 9.55 = 16.23 \dots$, which is more than 16.1

Tip: To make a division as big as possible, divide the upper bound by the lower bound.
 $155 \div 9.65$ gives the wrong answer. Using the measured values ($150 \div 9.6 = 15.6$) is not enough: the question asks what the speed **could** have been.

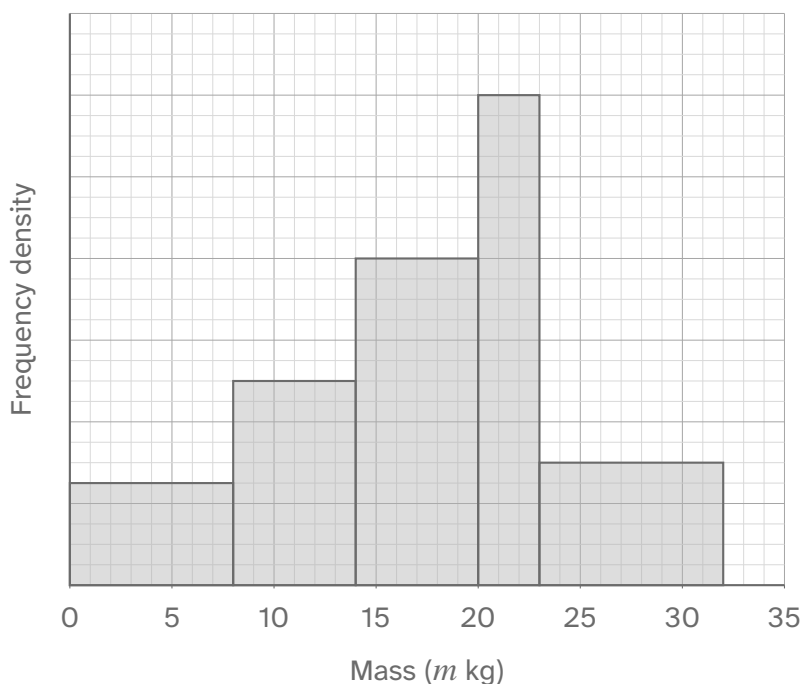
Marks: 1 mark for 155 or 9.55. 1 mark for $155 \div 9.55$. 1 mark for 16.23 ... and Yes.

15 (4 marks)

The histogram shows information about the masses of the suitcases checked in for a flight.

20 of the suitcases had a mass of 8 kg or less.

The scale on the frequency density axis is not shown.

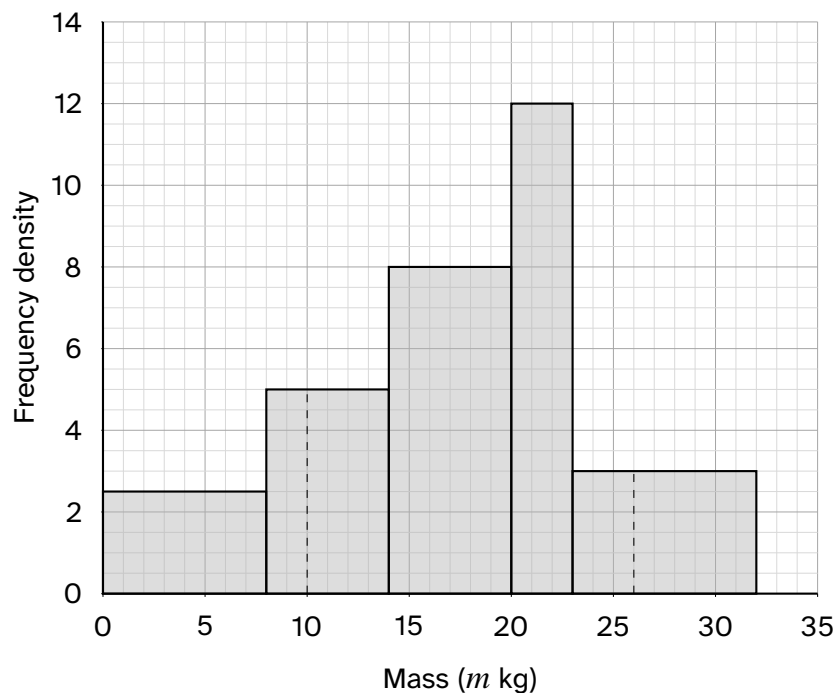


Estimate the number of suitcases with a mass between 10 kg and 26 kg.

Frequency density = frequency $\div$ class width

First bar: $20 \div 8 = 2.5$. This bar is 5 small squares high, so each small square up the axis is 0.5

With the scale filled in, the bar heights are 2.5, 5, 8, 12 and 3:



Frequency = frequency density $\times$ class width. Split 10 to 26 kg at the class boundaries.

10 to 14 kg: width 4, height 5, $4 \times 5 = 20$

14 to 20 kg: $6 \times 8 = 48$

20 to 23 kg: $3 \times 12 = 36$

23 to 26 kg: width 3, height 3, $3 \times 3 = 9$

Total: $20 + 48 + 36 + 9 = 113$

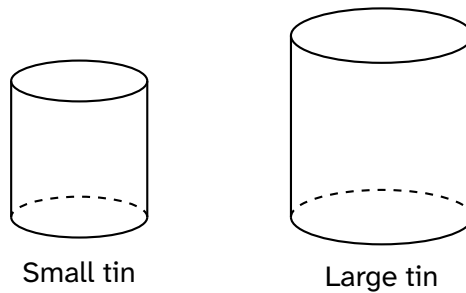
Answer: 113

Tip: When the axis has no scale, use the class you know: frequency $\div$ width gives its frequency density, and that fixes the scale. For part of a class use only the width you need: 10 to 14 is 4 kg wide.

Marks: 1 mark for the scale (frequency density 2.5 for the first bar). 1 mark for one correct frequency between 10 kg and 26 kg. 1 mark for adding all four parts. 1 mark for 113.

16 (3 marks)

Two tins of paint are mathematically similar.



Not drawn accurately

The surface area of the small tin is 900 cm^2

The surface area of the large tin is 1600 cm^2

The small tin holds 1.35 litres of paint.

How many litres of paint does the large tin hold?

$$\text{Area scale factor} = 1600 \div 900 = \frac{16}{9}$$

$$\text{Length scale factor} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$$\text{Volume scale factor} = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

$$\text{Large tin: } 1.35 \times \frac{64}{27} = 3.2 \text{ litres}$$

Answer: 3.2 litres

Tip: Do not multiply the volume by the area scale factor. Go from area to length (square root), then from length to volume (cube).

Marks: 1 mark for the length scale factor $\frac{4}{3}$ (the square root of $\frac{1600}{900}$). 1 mark for cubing it and multiplying by 1.35. 1 mark for 3.2.

17 (4 marks)

At a museum open day

- 24% of the adults went on a guided tour
- 40% of the children went on a guided tour
- the same number of adults and children went on a guided tour.

There were 1440 people at the open day altogether.

How many people went on a guided tour?

Let there be a adults and c children. The numbers on tours are equal, so $0.24a = 0.4c$

So $a:c = 0.4:0.24 = 40:24 = 5:3$

$5 + 3 = 8$ parts. One part $= 1440 \div 8 = 180$

Adults $= 5 \times 180 = 900$. Children $= 3 \times 180 = 540$

Adults on a tour: $0.24 \times 900 = 216$. Children on a tour: $0.4 \times 540 = 216$ (equal, as it should be)

Total on a guided tour $= 216 + 216 = 432$

Answer: 432

Tip: Do not split 1440 in the ratio 24:40. The **numbers** on tours are equal, so the group with the smaller percentage must be the bigger group.

Marks: 1 mark for $0.24a = 0.4c$ (or equivalent). 1 mark for the ratio 5:3. 1 mark for 900 adults or 540 children. 1 mark for 432.

18 (3 marks)

- (a) Show that the equation $x^3 + 4x - 11 = 0$ has a solution between $x = 1$ and $x = 2$

Substitute $x = 1$: $1^3 + 4 \times 1 - 11 = -6$

Substitute $x = 2$: $2^3 + 4 \times 2 - 11 = 5$

The value changes from negative to positive, so $x^3 + 4x - 11 = 0$ somewhere between $x = 1$ and $x = 2$.

Answer: At $x = 1$ the value is -6 and at $x = 2$ it is 5 ; there is a change of sign, so there is a solution between 1 and 2.

- (b) Starting with $x_1 = 1.5$, use the iterative formula $x_{n+1} = \sqrt[3]{11 - 4x_n}$ to work out the value of x_4 . Give your answer to 4 decimal places.

$$x_2 = \sqrt[3]{11 - 4 \times 1.5} = \sqrt[3]{5} = 1.709975 \dots$$

$$x_3 = \sqrt[3]{11 - 4 \times 1.709975 \dots} = 1.608302 \dots$$

$$x_4 = \sqrt[3]{11 - 4 \times 1.608302 \dots} = 1.659091 \dots$$

$$x_4 = 1.6591 \text{ (4 d.p.)}$$

Answer: $x_4 = 1.6591$

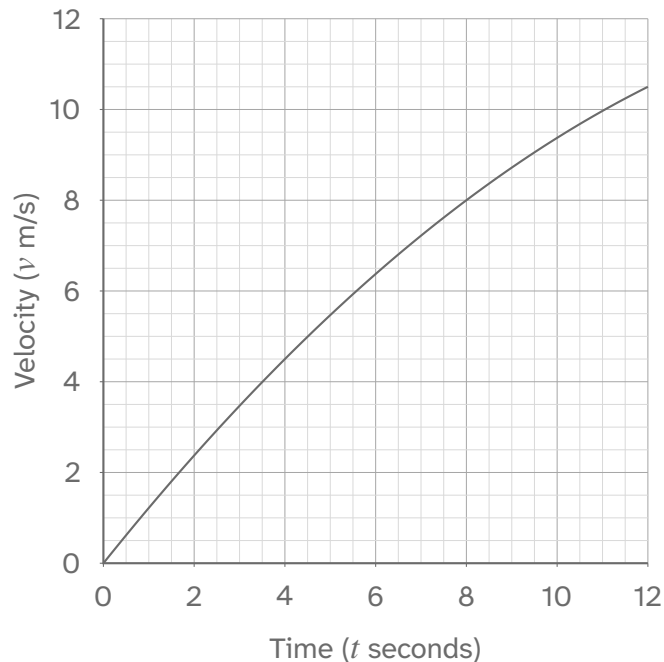
Tip: Use the ANS key so that each step uses the full value of the one before. x_1 is the starting value, so three more steps are needed to reach x_4 .

Marks: 1 mark for x_2 and x_3 correct. 1 mark for $x_4 = 1.6591$.

19 (4 marks)

A cyclist starts from rest at a set of traffic lights.

The graph shows her velocity, v metres per second, for the first 12 seconds.



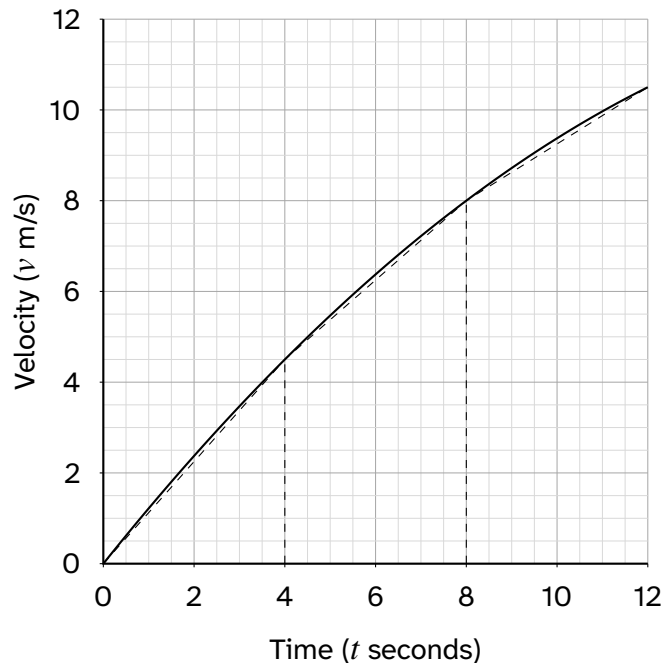
- (a) Use three strips of equal width to estimate the distance the cyclist travels in the first 12 seconds.

The distance travelled is the area under the velocity-time graph.

Three strips from $t = 0$ to $t = 12$, so each strip is 4 seconds wide.

Read the velocities at $t = 0, 4, 8, 12$: 0, 4.5, 8, 10.5

The strips are a triangle and two trapezia (dashed on the graph).



$$\frac{1}{2} \times 4 \times 4.5 + \frac{1}{2} \times (4.5 + 8) \times 4 + \frac{1}{2} \times (8 + 10.5) \times 4$$

$$= 9 + 25 + 37 = 71 \text{ metres}$$

Answer: 71 metres

Tip: Use the number of strips you are told to use, and use trapezia (straight lines joining points on the curve), not rectangles.

Marks: 1 mark for reading the velocities at 4, 8 and 12 seconds. 1 mark for adding the areas of the three strips. 1 mark for 71.

(b) Is your answer to part (a) an overestimate or an underestimate?

Tick a box.

Give a reason for your answer.

The graph gets less steep as time goes on (it bends over), so the curve is **above** the straight tops of the strips.

So the strips miss some of the area under the curve.

Underestimate.

Answer: Underestimate: the curve is above the tops of the strips, so some area is missed.

20 (4 marks)

$$f(x) = 5 - 2x \quad \text{and} \quad g(x) = x^2 + 1$$

- (a) Work out an expression for $gf(x)$
Give your answer in the form $ax^2 + bx + c$

$gf(x)$ means do f first, then g .

$$gf(x) = g(5 - 2x) = (5 - 2x)^2 + 1$$

$$(5 - 2x)^2 = 25 - 20x + 4x^2$$

$$gf(x) = 4x^2 - 20x + 26$$

Answer: $4x^2 - 20x + 26$

Tip: $gf(x)$ means put $f(x)$ into g . Square the whole of $(5 - 2x)$: $(5 - 2x)^2$ is not $25 + 4x^2$.

Marks: 1 mark for $(5 - 2x)^2 + 1$. 1 mark for $4x^2 - 20x + 26$.

- (b) Work out $f^{-1}(x)$

Write $y = 5 - 2x$ and make x the subject.

$$2x = 5 - y$$

$$x = \frac{5 - y}{2}$$

$$\text{So } f^{-1}(x) = \frac{5-x}{2}$$

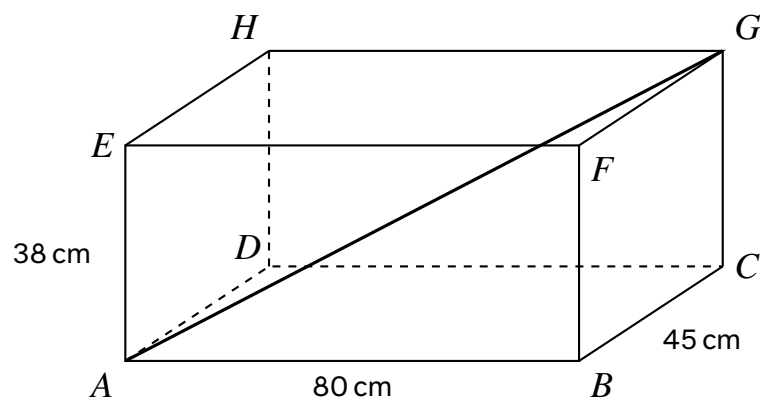
Answer: $\frac{5-x}{2}$

Tip: Check: $f(1) = 3$, so $f^{-1}(3)$ should be 1: $\frac{5-3}{2} = 1$.

Marks: 1 mark for a correct first step in rearranging $y = 5 - 2x$ (or $x = 5 - 2y$). 1 mark for $\frac{5-x}{2}$.

21 (4 marks)

A storage box is a cuboid $ABCDEFGH$ with a horizontal base $ABCD$.
A thin straight rod fits exactly inside the box from corner A to corner G .



Not drawn accurately

Work out the angle between the rod and the base of the box.
Give your answer to 1 decimal place.

The angle between AG and the base is angle GAC , in the right-angled triangle ACG .

First find AC , the diagonal of the base, using Pythagoras in triangle ABC :

$$AC^2 = 80^2 + 45^2 = 8425, \text{ so } AC = 91.787 \dots \text{ cm}$$

$CG = 38 \text{ cm}$ (the height of the box) is opposite the angle, and AC is adjacent.

$$\tan GAC = \frac{38}{91.787 \dots}$$

Angle $GAC = 22.489 \dots^\circ = 22.5^\circ$ (1 d.p.)

Answer: 22.5°

Tip: The angle with the base is between the rod and the diagonal of the base (AC), not an edge such as AB .

Marks: 1 mark for $80^2 + 45^2$. 1 mark for $AC = 91.78 \dots$ 1 mark for $\tan$ of the angle $= 38 \div AC$. 1 mark for 22.5.

22 (5 marks)

A school spends £180 on some calculators, all at the same price. A year later the price of each calculator has gone up by £1.50. The school spends £180 again and gets 4 fewer calculators. Work out the price of one calculator before the price went up.

Let the price before the rise be £ x . The new price is £ $(x + 1.5)$.

Number of calculators before $= \frac{180}{x}$. Number after $= \frac{180}{x + 1.5}$

There are 4 fewer after the rise: $\frac{180}{x} - \frac{180}{x + 1.5} = 4$

Multiply every term by $x(x + 1.5)$: $180(x + 1.5) - 180x = 4x(x + 1.5)$

$$180x + 270 - 180x = 4x^2 + 6x$$

$$270 = 4x^2 + 6x, \text{ so } 4x^2 + 6x - 270 = 0$$

$$\text{Divide by 2: } 2x^2 + 3x - 135 = 0$$

$$\text{Factorise: } (2x - 15)(x + 9) = 0, \text{ so } x = 7.5 \text{ or } x = -9$$

A price cannot be negative, so $x = 7.5$

Check: $180 \div 7.5 = 24$ calculators, $180 \div 9 = 20$ calculators, $24 - 20 = 4$ ✓

The price before the rise was £7.50

Answer: £7.50

Tip: Use a letter for the unknown price and write the number of calculators each time as $180 \div \text{price}$. Then use the fact that the difference is 4. Reject the negative solution: a price cannot be negative.

Marks: 1 mark for an expression for the number of calculators, such as $\frac{180}{x}$. 1 mark for the equation $\frac{180}{x} - \frac{180}{x+1.5} = 4$. 1 mark for clearing the fractions to get a quadratic such as $4x^2 + 6x - 270 = 0$. 1 mark for solving it. 1 mark for £7.50.

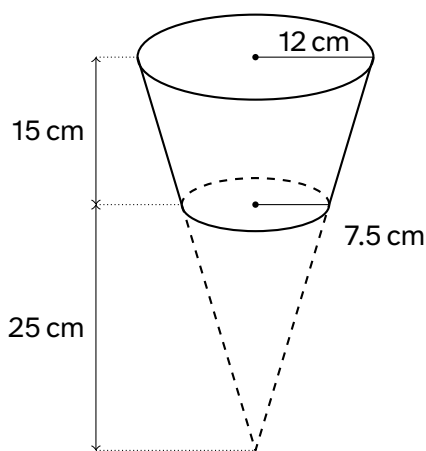
23 (4 marks)

The diagram shows a plant pot.

The pot is a frustum. It is made by removing a small cone from a large cone.

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the perpendicular height



Not drawn accurately

Compost is sold in 50-litre bags.

Faisal fills some of these plant pots with compost from one bag.

How many pots can he fill completely?

You must show your working.

Height of the large cone = $25 + 15 = 40$ cm

Volume of the large cone: $\frac{1}{3}\pi \times 12^2 \times 40 = 1920\pi = 6031.857 \dots \text{cm}^3$

Volume of the small cone: $\frac{1}{3}\pi \times 7.5^2 \times 25 = 468.75\pi = 1472.621\dots \text{cm}^3$

Volume of one pot = $6031.857\dots - 1472.621\dots = 4559.236\dots \text{cm}^3$

1 litre = 1000 cm^3 , so one pot holds $4.559\dots$ litres.

$50 \div 4.559\dots = 10.96\dots$

He can fill 10 pots completely (there is not enough for an 11th).

Answer: 10 pots

Tip: The large cone's height is $25 + 15 = 40 \text{ cm}$, not 15 cm . At the end round **down:** 10.96 means only 10 pots can be filled completely.

Marks: 1 mark for the volume of either cone. 1 mark for subtracting to get the volume of the pot. 1 mark for changing to litres and dividing into 50. 1 mark for 10.

24 (5 marks)

A bag contains only red counters and blue counters.

There are 6 blue counters in the bag.

Jamal takes two counters at random from the bag, without replacement.

The probability that the two counters are different colours is $\frac{8}{15}$

Work out the number of red counters in the bag.

You must show your working.

Let n be the number of red counters. There are $n + 6$ counters altogether.

$$P(\text{red then blue}) = \frac{n}{n+6} \times \frac{6}{n+5} \quad P(\text{blue then red}) = \frac{6}{n+6} \times \frac{n}{n+5}$$

$$P(\text{different colours}) = \frac{12n}{(n+6)(n+5)} = \frac{8}{15}$$

$$\text{Cross-multiply: } 180n = 8(n+6)(n+5) = 8(n^2 + 11n + 30)$$

$$180n = 8n^2 + 88n + 240$$

$$0 = 8n^2 - 92n + 240, \text{ and dividing by 4: } 2n^2 - 23n + 60 = 0$$

$$\text{Factorise: } (2n - 15)(n - 4) = 0$$

$$n = 7.5 \text{ or } n = 4$$

The number of counters must be a whole number, so $n = 4$.

Check: $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{48}{90} = \frac{8}{15}$ ✓

Answer: 4 red counters

Tip: “Different colours” can happen two ways: red then blue, **or** blue then red. Without replacement, the second denominator is one less than the first.

Marks: 1 mark for one correct product such as $\frac{n}{n+6} \times \frac{6}{n+5}$. 1 mark for adding both orders and setting equal to $\frac{8}{15}$. 1 mark for the quadratic $2n^2 - 23n + 60 = 0$. 1 mark for solving it. 1 mark for 4 (rejecting 7.5).
