

Mock Set A – Paper 1

Solutions

For AQA GCSE Mathematics (8300)

Higher tier • Paper 1 (Non-Calculator)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Each ticket sold for a concert is a standing ticket or a seated ticket.

number of standing tickets : number of seated tickets = 7:4

What fraction of the tickets sold are seated tickets?

Circle your answer.

Options: $\frac{4}{7}$, $\frac{4}{11}$, $\frac{7}{11}$, $\frac{7}{4}$

Total number of parts: $7 + 4 = 11$

Seated tickets are 4 of the 11 parts.

Answer: $\frac{4}{11}$

Tip: $\frac{4}{7}$ compares seated tickets with standing tickets. A fraction of all the tickets must be out of the total, $7 + 4$.

2 (3 marks)

240 people use a climbing centre on one day.

- $\frac{5}{8}$ of the people are adults. The rest are juniors.
- 40% of the adults reach the top of the hardest wall.
- 20% of the juniors reach the top of the hardest wall.

Complete the frequency tree.

Adults: $\frac{5}{8}$ of 240: $240 \div 8 = 30$, $30 \times 5 = 150$

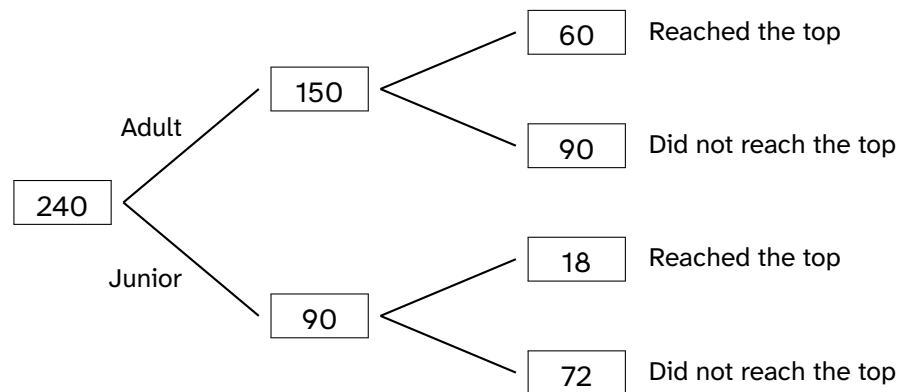
Juniors: $240 - 150 = 90$

Adults who reached the top: 40% of 150: 10% is 15, so 40% is $4 \times 15 = 60$

Adults who did not: $150 - 60 = 90$

Juniors who reached the top: 20% of 90: 10% is 9, so 20% is 18

Juniors who did not: $90 - 18 = 72$



Answer: Adult 150 (reached the top 60, did not 90); Junior 90 (reached the top 18, did not 72)

Tip: Each percentage is of a different group: 40% of the **adults** and 20% of the **juniors**, not of all 240 people.

Marks: 1 mark for 150 and 90. 1 mark for 60 and 90 on the adult branches. 1 mark for 18 and 72 on the junior branches.

3 (2 marks)

Each interior angle of a regular polygon is 160°

(a) Work out the size of each exterior angle of the polygon.

Interior angle + exterior angle = 180°

$$180 - 160 = 20$$

Answer: 20°

Tip: The interior and exterior angles at each vertex lie on a straight line, so they add to 180° .

(b) Work out the number of sides of the polygon.

The exterior angles of any polygon add up to 360° .

$$360 \div 20 = 18$$

Answer: 18

Tip: Divide 360 by the **exterior** angle, not by the interior angle.

4 (3 marks)

Work out $2\frac{2}{3} \div 1\frac{3}{5} + \frac{3}{4}$

Give your answer as a mixed number in its simplest form.

Divide first (order of operations).

Write as improper fractions: $2\frac{2}{3} = \frac{8}{3}$, $1\frac{3}{5} = \frac{8}{5}$

$$\frac{8}{3} \div \frac{8}{5} = \frac{8}{3} \times \frac{5}{8} = \frac{40}{24} = \frac{5}{3}$$

$$\text{Add: } \frac{5}{3} + \frac{3}{4} = \frac{20}{12} + \frac{9}{12} = \frac{29}{12}$$

$$\frac{29}{12} = 2\frac{5}{12}$$

Answer: $2\frac{5}{12}$

Tip: Do the division before the addition. Turn the second fraction upside down and multiply; do not look for a common denominator to divide.

Marks: 1 mark for dividing correctly. 1 mark for adding with a common denominator. 1 mark for the answer.

5 (3 marks)

- (a) Write 0.000 058 in standard form.

Move the decimal point to get a number from 1 to less than 10: 5.8

The point moves 5 places to the right, so the power is -5 .

Answer: 5.8×10^{-5}

Tip: Small numbers (less than 1) have a **negative** power of 10.

- (b) Work out $(4 \times 10^7) \times (3.5 \times 10^{-2})$

Give your answer in standard form.

Multiply the numbers: $4 \times 3.5 = 14$

Add the powers: $10^7 \times 10^{-2} = 10^5$

14×10^5 is not in standard form.

$14 = 1.4 \times 10$, so $14 \times 10^5 = 1.4 \times 10^6$

Answer: 1.4×10^6

Tip: Check the first number is at least 1 and less than 10. 14×10^5 is the right value but not in standard form.

Marks: 1 mark for 14×10^5 or 1 400 000. 1 mark for 1.4×10^6 .

6 (4 marks)

A mosaic for a community centre is made from blue tiles, white tiles and red tiles only.

- number of blue tiles : number of white tiles = 3:5
- number of white tiles : number of red tiles = 2:3
- There are 108 more red tiles than blue tiles.

Work out the total number of tiles in the mosaic.

White is in both ratios. Make the white parts the same.

Blue : white = 3:5 = 6:10 (multiply by 2)

White : red = 2:3 = 10:15 (multiply by 5)

So blue : white : red = 6:10:15

Red – blue = $15 - 6 = 9$ parts, and this is 108 tiles.

One part = $108 \div 9 = 12$ tiles

Total parts = $6 + 10 + 15 = 31$

Total tiles = $31 \times 12 = 372$

Answer: 372

Tip: You cannot just join the ratios as 3:5:3 or add them. The two numbers for white must match first.

Marks: 1 mark for making the white parts equal (6 : 10 : 15). 1 mark for $108 \div 9$. 1 mark for multiplying one part by 31. 1 mark for 372.

7 (3 marks)

Solve $\frac{3x+2}{4} - \frac{x-5}{3} = 3$

Multiply every term by 12 (the lowest common multiple of 4 and 3).

$$3(3x+2) - 4(x-5) = 36$$

Expand: $9x + 6 - 4x + 20 = 36$

Simplify: $5x + 26 = 36$

$$5x = 10$$

$$x = 2$$

Answer: $x = 2$

Tip: Put brackets round each numerator before multiplying. $-4(x-5)$ gives $-4x + 20$, **plus** 20.

Marks: 1 mark for multiplying through by 12 correctly. 1 mark for expanding correctly to $5x + 26 = 36$ or equivalent. 1 mark for $x = 2$.

8 (2 marks)

Work out the smallest integer value of x that satisfies $4 - 3x < 2x + 23$

Solve the inequality first.

Collect the x terms: $4 - 23 < 2x + 3x$

$-19 < 5x$

Divide by 5: $-3.8 < x$, so $x > -3.8$

The integers greater than -3.8 are $-3, -2, -1, \dots$

The smallest is -3 .

Answer: -3

Tip: -4 is **less** than -3.8 , so it does not satisfy the inequality. If you divide by a negative number (e.g. $-5x < 19$), reverse the inequality sign.

Marks: 1 mark for collecting terms correctly, e.g. $-19 < 5x$ or $x > -3.8$. 1 mark for -3 .

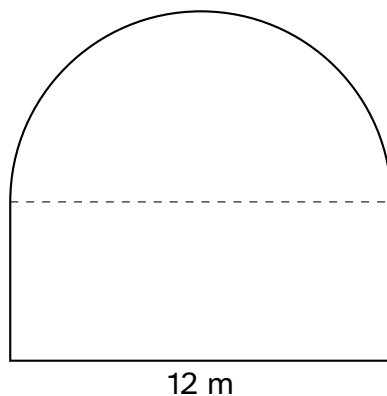
9 (3 marks)

The diagram shows the outline of a garden pond.

It is made from a rectangle and a semicircle.

The rectangle is 12 m long and has area 60 m^2 .

The diameter of the semicircle is one of the long sides of the rectangle.



Not drawn accurately

Work out the perimeter of the pond.

Give your answer in the form $a + b\pi$

Width of the rectangle: $60 \div 12 = 5 \text{ m}$

The semicircle has diameter 12 m.

Curved edge = $\frac{1}{2} \times \pi \times 12 = 6\pi \text{ m}$

The dashed side is inside the pond, so it is not part of the perimeter.

Straight edges: $12 + 5 + 5 = 22 \text{ m}$

Perimeter = $22 + 6\pi \text{ m}$

Answer: $22 + 6\pi \text{ m}$

Tip: Only count the outside edges. The side of the rectangle that touches the semicircle is not part of the perimeter.

Marks: 1 mark for the width 5 m. 1 mark for the curved edge 6π . 1 mark for $22 + 6\pi$.

10 (4 marks)

$\xi = \{\text{students in Year 10}\}$

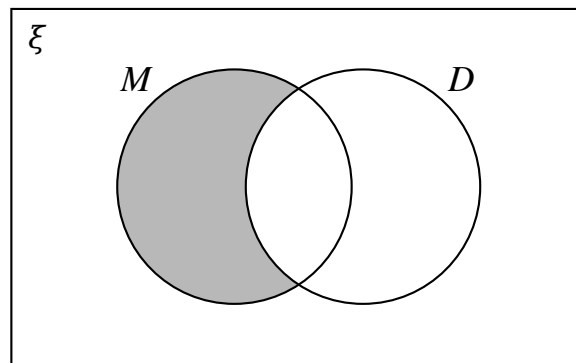
$M = \{\text{students in the music club}\}$

$D = \{\text{students in the drama club}\}$

- (a) On a Venn diagram, shade the region $M \cap D'$

$M \cap D'$ means in M **and** not in D .

This is the part of circle M outside circle D .



Answer: The part of M that is not in D is shaded

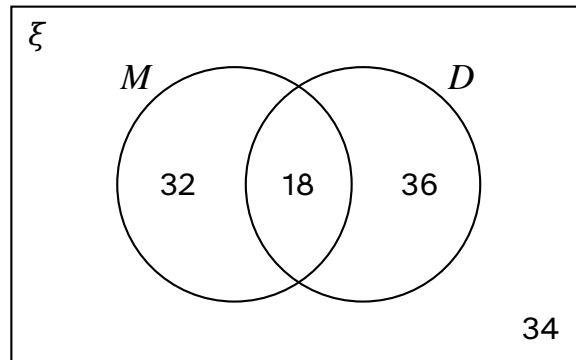
Tip: $\cap$ means “and” and D' means “not in D ”.

- (b) There are 120 students in Year 10.
- 50 students are in the music club.
 - 34 students are in neither club.
 - Twice as many students are in the drama club only as are in both clubs.
- A student in the drama club is chosen at random. Work out the probability that this student is also in the music club.

Drama only: $120 - 50 - 34 = 36$

Both clubs: $36 \div 2 = 18$

Music only: $50 - 18 = 32$



Students in the drama club: $36 + 18 = 54$

Of these, 18 are also in the music club.

$$\text{Probability} = \frac{18}{54} = \frac{1}{3}$$

Answer: $\frac{18}{54} = \frac{1}{3}$

Tip: “Chosen from the drama club” means the denominator is the number in D (54), not 120.

Marks: 1 mark for 36 in drama only. 1 mark for 18 in both. 1 mark for $\frac{18}{54}$.

11 (3 marks)

Prove algebraically that $0.5\dot{6} = \frac{17}{30}$

Let $x = 0.5666 \dots$

$10x = 5.666 \dots$

$100x = 56.666 \dots$

Subtract: $100x - 10x = 56.666 \dots - 5.666 \dots$

$90x = 51$

$$x = \frac{51}{90} = \frac{17}{30} \quad (\text{divide the top and bottom by 3})$$

Answer: $100x - 10x = 51$, so $x = \frac{51}{90} = \frac{17}{30}$

Tip: Write out enough digits to show the decimal recurs (5.666 ..., not 5.6). Pick two multiples whose decimal parts are the same, so they cancel when you subtract.

Marks: 1 mark for two multiples of x with the same recurring part (e.g. $10x$ and $100x$). 1 mark for subtracting to get $90x = 51$. 1 mark for showing $\frac{51}{90} = \frac{17}{30}$.

12 (4 marks)

The table shows the times taken by 60 runners to complete a 5 km fun run.

Time (t minutes)	Frequency	Cumulative frequency
$16 < t \leq 20$	4	4
$20 < t \leq 24$	11	15
$24 < t \leq 28$	19	34
$28 < t \leq 32$	14	48
$32 < t \leq 36$	8	56
$36 < t \leq 40$	4	60

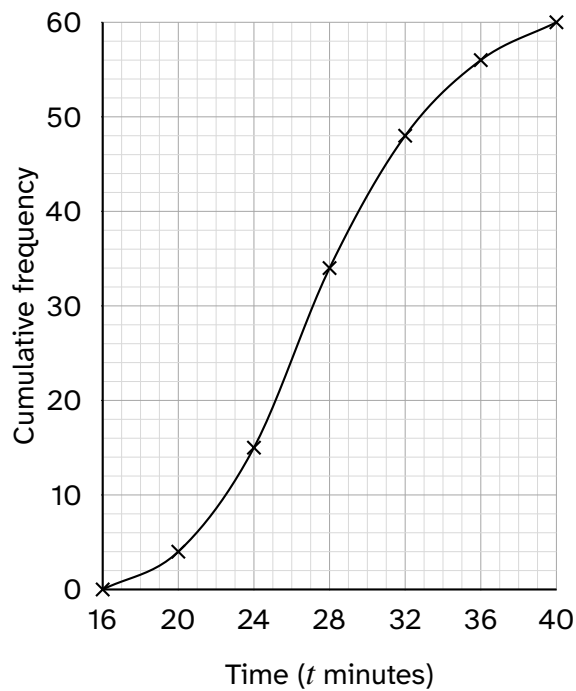
(a) On the grid, draw a cumulative frequency graph for the data.

Cumulative frequencies (running totals): 4, 15, 34, 48, 56, 60

Plot each one at the **end** of its class: (20, 4), (24, 15), (28, 34), (32, 48), (36, 56), (40, 60)

No one took 16 minutes or less, so also plot (16, 0).

Join the points with a smooth curve (or straight lines).



Answer: Points (16, 0), (20, 4), (24, 15), (28, 34), (32, 48), (36, 56), (40, 60) joined with a smooth curve

Tip: Plot at the upper end of each class (e.g. 20 for $16 < t \leq 20$), not the middle. The graph starts at (16, 0) here, not at (0, 0).

Marks: 1 mark for the correct cumulative frequencies or at least 4 points plotted correctly.
1 mark for all points plotted and joined.

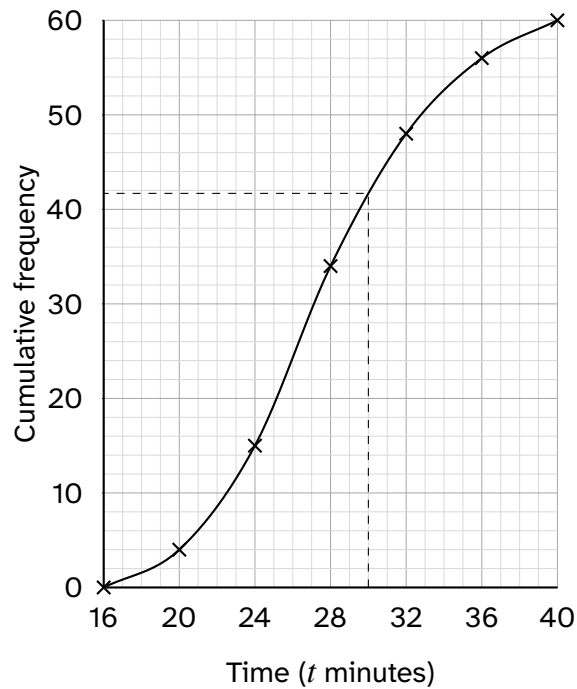
(b) The organisers say,

“More than a quarter of the runners took longer than 30 minutes.”

Use your graph to decide whether the organisers are correct.

Tick a box. Show how you decide.

Read up from 30 minutes on the time axis to the curve, then across.



About 42 runners took 30 minutes or less.

Longer than 30 minutes: $60 - 42 = 18$ runners

A quarter of the runners: $60 \div 4 = 15$

$18 > 15$, so the organisers are correct.

Answer: Yes: about 18 runners took longer than 30 minutes, which is more than $\frac{1}{4} \times 60 = 15$

Tip: The graph gives the number who took 30 minutes **or less**. Subtract from 60 to get “longer than”, then compare with a quarter of 60.

Marks: 1 mark for reading the graph at 30 minutes. 1 mark for Yes with the number who took longer than 30 minutes compared with 15.

13 (3 marks)

A garden centre sells pots of herbs and trays of seedlings.

Maya buys 3 pots of herbs and 4 trays of seedlings.

She pays £23.30

Leo buys 5 pots of herbs and 2 trays of seedlings.

He pays £21.10

Work out the cost of one pot of herbs and the cost of one tray of seedlings.

You must show your working.

Let h = cost of a pot of herbs and s = cost of a tray of seedlings, in pounds.

$$3h + 4s = 23.30 \quad (1)$$

$$5h + 2s = 21.10 \quad (2)$$

$$\text{Multiply (2) by 2: } 10h + 4s = 42.20 \quad (3)$$

$$(3) - (1): 7h = 18.90$$

$$h = 18.90 \div 7 = 2.70$$

$$\text{Substitute into (2): } 5 \times 2.70 + 2s = 21.10$$

$$13.50 + 2s = 21.10, \text{ so } 2s = 7.60 \text{ and } s = 3.80$$

$$\text{Check in (1): } 3 \times 2.70 + 4 \times 3.80 = 8.10 + 15.20 = 23.30 \quad \checkmark$$

Answer: Pot of herbs £2.70, tray of seedlings £3.80

Tip: Write money to 2 decimal places: £2.70, not £2.7. Check your answers in the equation you did not use to find the second value.

Marks: 1 mark for a correct method to eliminate one letter. 1 mark for one correct value. 1 mark for both values.

14 (4 marks)

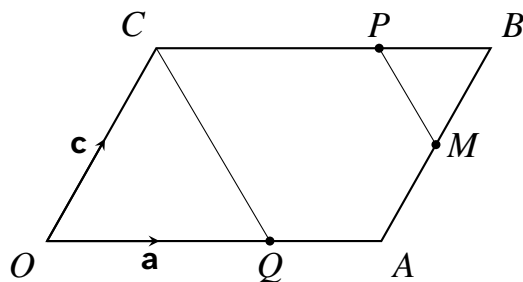
$OABC$ is a parallelogram.

$$\overrightarrow{OA} = \mathbf{a} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{c}$$

M is the midpoint of AB .

P is the point on CB such that $CP:PB = 2:1$

Q is the point on OA such that $OQ:QA = 2:1$



Not drawn accurately

Prove that MP is parallel to QC .

$OABC$ is a parallelogram, so $\overrightarrow{AB} = \mathbf{c}$ and $\overrightarrow{CB} = \mathbf{a}$.

M is the midpoint of AB , so $\overrightarrow{MB} = \frac{1}{2}\mathbf{c}$

$CP:PB = 2:1$, so PB is $\frac{1}{3}$ of CB : $\overrightarrow{BP} = -\frac{1}{3}\mathbf{a}$

$$\overrightarrow{MP} = \overrightarrow{MB} + \overrightarrow{BP} = \frac{1}{2}\mathbf{c} - \frac{1}{3}\mathbf{a}$$

$OQ:QA = 2:1$, so $\overrightarrow{OQ} = \frac{2}{3}\mathbf{a}$ and $\overrightarrow{QO} = -\frac{2}{3}\mathbf{a}$

$$\overrightarrow{QC} = \overrightarrow{QO} + \overrightarrow{OC} = -\frac{2}{3}\mathbf{a} + \mathbf{c} = \mathbf{c} - \frac{2}{3}\mathbf{a}$$

$$\mathbf{c} - \frac{2}{3}\mathbf{a} = 2\left(\frac{1}{2}\mathbf{c} - \frac{1}{3}\mathbf{a}\right), \text{ so } \overrightarrow{QC} = 2\overrightarrow{MP}$$

$\overrightarrow{QC}$ is a multiple of $\overrightarrow{MP}$, so MP is parallel to QC .

Answer: $\overrightarrow{MP} = \frac{1}{2}\mathbf{c} - \frac{1}{3}\mathbf{a}$ and $\overrightarrow{QC} = \mathbf{c} - \frac{2}{3}\mathbf{a} = 2\overrightarrow{MP}$, so MP is parallel to QC

Tip: To prove two lines are parallel, write both vectors in terms of $\mathbf{a}$ and $\mathbf{c}$ and show that one is a multiple of the other. $CP:PB = 2:1$ means PB is $\frac{1}{3}$ of CB , not $\frac{1}{2}$.

Marks: 1 mark for $\overrightarrow{MB} = \frac{1}{2}\mathbf{c}$ and $\overrightarrow{BP} = -\frac{1}{3}\mathbf{a}$ (or another correct route). 1 mark for $\overrightarrow{MP} = \frac{1}{2}\mathbf{c} - \frac{1}{3}\mathbf{a}$. 1 mark for $\overrightarrow{QC} = \mathbf{c} - \frac{2}{3}\mathbf{a}$. 1 mark for showing one is a multiple of the other and saying they are parallel.

15 (3 marks)

The brightness, B units, of a lamp at a distance of d metres from the lamp is inversely proportional to d^2

When $d = 4$, $B = 9$

Ana stands 8 metres from the lamp.

She walks directly towards the lamp until the brightness is 16 units.

How far does Ana walk?

$$B = \frac{k}{d^2}$$

Substitute $d = 4$ and $B = 9$: $9 = \frac{k}{16}$, so $k = 144$ and $B = \frac{144}{d^2}$

When $B = 16$: $16 = \frac{144}{d^2}$, so $d^2 = 144 \div 16 = 9$

$d = 3$ (a distance is positive)

Ana ends 3 m from the lamp, so she walks $8 - 3 = 5$ m.

Answer: 5 m

Tip: The question asks how far she **walks**, not how far from the lamp she ends up. Check it makes sense: the brightness goes up, so she must get closer to the lamp.

Marks: 1 mark for $k = 144$ (or $B = \frac{144}{d^2}$). 1 mark for $d = 3$. 1 mark for 5 m.

16 (3 marks)

Make x the subject of the formula $y = \frac{2x+7}{x-4}$

Multiply both sides by $(x-4)$: $y(x-4) = 2x+7$

Expand: $xy - 4y = 2x + 7$

Collect the x terms on one side: $xy - 2x = 4y + 7$

Factorise: $x(y-2) = 4y+7$

Divide by $(y-2)$: $x = \frac{4y+7}{y-2}$

Answer: $x = \frac{4y+7}{y-2}$

Tip: Once x appears in two terms, collect them on one side and factorise to get a single x .

Marks: 1 mark for multiplying by $(x-4)$ and expanding. 1 mark for collecting the x terms and factorising. 1 mark for the answer.

17 (4 marks)

(a) Work out the value of $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

The negative power means “turn the fraction upside down”: $\left(\frac{125}{8}\right)^{\frac{2}{3}}$

The $\frac{1}{3}$ means cube root: $\sqrt[3]{\frac{125}{8}} = \frac{5}{2}$

The 2 means square: $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

Answer: $\frac{25}{4}$ (or $6\frac{1}{4}$ or 6.25)

Tip: Deal with the three parts of the power one at a time: the minus (reciprocal), the denominator 3 (cube root) and the numerator 2 (square).

Marks: 1 mark for one correct step (reciprocal, cube root or square). 1 mark for 25/4.

(b) Write $\frac{4^3 \times \sqrt{2}}{8^3}$ as a power of 2

$$4 = 2^2, \text{ so } 4^3 = 2^6$$

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$8 = 2^3, \text{ so } 8^3 = 2^9$$

$$\frac{2^6 \times 2^{\frac{1}{2}}}{2^9} = 2^{6+\frac{1}{2}-9} = 2^{-\frac{5}{2}}$$

Answer: $2^{-\frac{5}{2}}$

Tip: 8^3 is $(2^3)^3 = 2^9$: multiply the powers, do not add them.

Marks: 1 mark for writing two of the three parts as powers of 2. 1 mark for the answer.

18 (3 marks)

In 2025 the price of a season ticket at a football club increased by 40%

In 2026 the price of the season ticket decreased by 25%

Nina says,

“Over the two years the price has increased by 15%”

Is Nina correct?

Tick a box.

Show working to support your answer.

An increase of 40% is a multiplier of 1.4

A decrease of 25% is a multiplier of 0.75

Overall multiplier: $1.4 \times 0.75 = 1.05$

1.05 means an increase of 5%, not 15%.

So Nina is not correct.

Answer: No, $1.4 \times 0.75 = 1.05$, which is an overall increase of 5%

Tip: Percentage changes do not just add or subtract, because the 25% is taken from the new, higher price. Multiply the multipliers.

Marks: 1 mark for the multipliers 1.4 and 0.75 (or an example price). 1 mark for 1.05 or the final price. 1 mark for No with 5%.

19 (4 marks)

- (a) Write $2x^2 - 20x + 41$ in the form $a(x + b)^2 + c$ where a , b and c are integers.

Take out the factor 2 from the x terms: $2(x^2 - 10x) + 41$

Complete the square inside: $x^2 - 10x = (x - 5)^2 - 25$

So $2((x - 5)^2 - 25) + 41 = 2(x - 5)^2 - 50 + 41$
 $= 2(x - 5)^2 - 9$

Answer: $2(x - 5)^2 - 9$

Tip: When you multiply back in, the -25 is also multiplied by 2, giving -50 .

Marks: 1 mark for taking out 2. 1 mark for $2(x - 5)^2$. 1 mark for -9 .

- (b) Hence, write down the coordinates of the turning point of the curve
 $y = 2x^2 - 20x + 41$

$2(x - 5)^2$ is smallest (zero) when $x = 5$.

Then $y = -9$.

Answer: $(5, -9)$

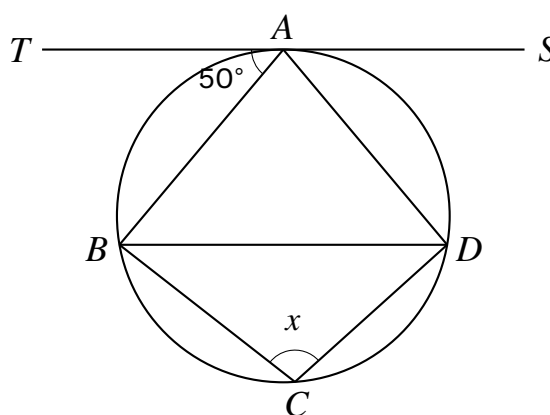
Tip: The x -coordinate has the opposite sign to the number in the bracket: $(x - 5)$ gives $x = 5$.

20 (3 marks)

A, B, C and D are points on a circle.

TAS is a tangent to the circle at A .

$AB = AD$



Not drawn accurately

Work out the size of angle x .

You must show your working.

Alternate segment theorem: the angle between the tangent and the chord AB equals the angle in the alternate segment.

So angle $ADB = 50^\circ$

$AB = AD$, so triangle ABD is isosceles and angle $ABD = 50^\circ$

Angles in a triangle add to 180° : angle $BAD = 180 - 50 - 50 = 80^\circ$

Opposite angles of a cyclic quadrilateral add to 180° : $x = 180 - 80 = 100^\circ$

Answer: $x = 100^\circ$

Tip: In the alternate segment theorem the equal angle is on the other side of the chord from the tangent angle. Write each angle with three letters, e.g. angle ADB .

Marks: 1 mark for angle ADB (or ABD) = 50° . 1 mark for angle $BAD = 80^\circ$. 1 mark for 100° .

21 (4 marks)

Prove that the sum of the squares of any three consecutive odd numbers is always 1 less than a multiple of 12

Let the three consecutive odd numbers be $2n - 1$, $2n + 1$ and $2n + 3$, where n is an integer.

$$\begin{aligned} & (2n - 1)^2 + (2n + 1)^2 + (2n + 3)^2 \\ &= (4n^2 - 4n + 1) + (4n^2 + 4n + 1) + (4n^2 + 12n + 9) \\ &= 12n^2 + 12n + 11 \\ &= 12n^2 + 12n + 12 - 1 \\ &= 12(n^2 + n + 1) - 1 \end{aligned}$$

$n^2 + n + 1$ is an integer, so $12(n^2 + n + 1)$ is a multiple of 12.

So the sum is always 1 less than a multiple of 12.

Answer: $12n^2 + 12n + 11 = 12(n^2 + n + 1) - 1$, which is 1 less than a multiple of 12

Tip: Odd numbers are $2n + 1$ (or $2n - 1$), and consecutive odd numbers go up in 2s. Using n , $n + 1$, $n + 2$ gives consecutive integers, not consecutive odd numbers.

Marks: 1 mark for three consecutive odd numbers in terms of n . 1 mark for squaring correctly. 1 mark for $12n^2 + 12n + 11$. 1 mark for writing it as $12(\dots) - 1$ with a conclusion.

22 (2 marks)

Show that $\frac{20}{\sqrt{5}} + \sqrt{45}$ can be written in the form $m\sqrt{5}$, where m is an integer.

Rationalise: $\frac{20}{\sqrt{5}} = \frac{20 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$

Simplify using the square factor 9: $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$4\sqrt{5} + 3\sqrt{5} = 7\sqrt{5}$, so $m = 7$

Answer: $4\sqrt{5} + 3\sqrt{5} = 7\sqrt{5}$

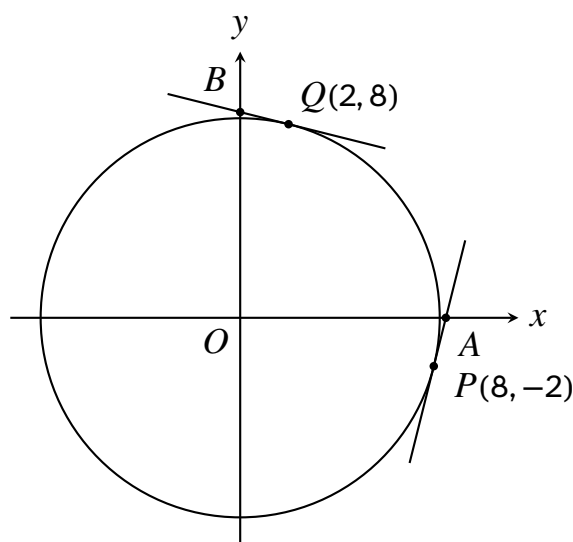
Tip: Multiply the top and bottom by $\sqrt{5}$, and remember $\sqrt{5} \times \sqrt{5} = 5$. For $\sqrt{45}$, look for the largest square factor (9).

Marks: 1 mark for $4\sqrt{5}$ or $3\sqrt{5}$. 1 mark for $7\sqrt{5}$ with both seen.

23 (6 marks)

A circle has centre $O(0, 0)$.

The points $P(8, -2)$ and $Q(2, 8)$ are on the circle.



Not drawn accurately

(a) Write down the equation of the circle.

A circle with centre $(0, 0)$ and radius r has equation $x^2 + y^2 = r^2$.

P is on the circle: $r^2 = 8^2 + (-2)^2 = 64 + 4 = 68$

Answer: $x^2 + y^2 = 68$

Tip: The number on the right is r^2 , not r . There is no need to find $r = \sqrt{68}$.

- (b) The tangent to the circle at P meets the x -axis at A .

The tangent to the circle at Q meets the y -axis at B .

Work out the area of triangle OAB .

You must show your working.

Gradient of the radius OP : $\frac{-2 - 0}{8 - 0} = -\frac{1}{4}$

The tangent at P is perpendicular to OP , so its gradient is the negative reciprocal: 4

$y = 4x + c$ passes through $(8, -2)$: $-2 = 32 + c$, so $c = -34$. Tangent at P :

$$y = 4x - 34$$

At A , $y = 0$: $4x - 34 = 0$, so $x = 8.5$ and A is $(8.5, 0)$

Gradient of the radius OQ : $\frac{8}{2} = 4$, so the tangent at Q has gradient $-\frac{1}{4}$

$y = -\frac{1}{4}x + c$ passes through $(2, 8)$: $8 = -\frac{1}{2} + c$, so $c = 8.5$. Tangent at Q :

$$y = -\frac{1}{4}x + 8.5$$

At B , $x = 0$: $y = 8.5$, so B is $(0, 8.5)$

$OA = 8.5$ and $OB = 8.5$, and angle $AOB = 90^\circ$ (the axes are perpendicular).

Area of triangle $OAB = \frac{1}{2} \times 8.5 \times 8.5 = 36.125$

Answer: 36.125 (or $36\frac{1}{8}$)

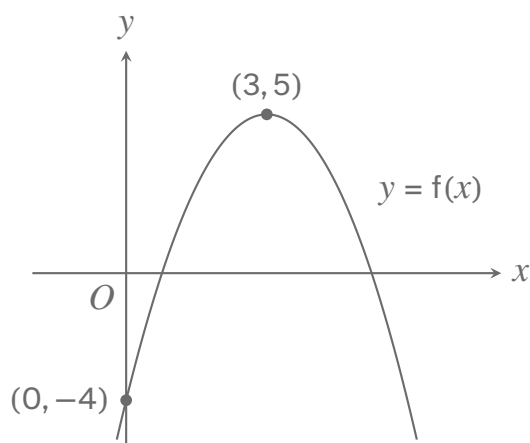
Tip: Each tangent is perpendicular to the radius at that point. Find each tangent's equation, then put $y = 0$ (for A) or $x = 0$ (for B). Do not mix them up: A is on the tangent at P , and B is on the tangent at Q .

Marks: 1 mark for a radius gradient ($-\frac{1}{4}$ or 4). 1 mark for a tangent gradient using the negative reciprocal. 1 mark for the equation of one tangent, or for A or B . 1 mark for $\frac{1}{2} \times OA \times OB$ using both A and B . 1 mark for 36.125.

24 (2 marks)

Here is a sketch of the curve $y = f(x)$

The curve has a maximum point at $(3, 5)$ and crosses the y -axis at $(0, -4)$.



(a) Ravi says,

“The curve $y = f(x + 4)$ has a maximum point at $(7, 5)$ ”

What mistake has Ravi made?

$f(x + 4)$ is a translation of 4 units to the **left**, not to the right.

Ravi has moved the curve 4 units to the right: $3 + 4 = 7$

The maximum point is really at $(3 - 4, 5) = (-1, 5)$.

Answer: He has translated the curve 4 to the right instead of 4 to the left (the maximum is at $(-1, 5)$)

Tip: A change inside the bracket moves the graph the opposite way to the sign: $+4$ moves it 4 to the left.

- (b) The curve $y = f(x) + k$ passes through the origin.
Work out the value of k .

$y = f(x) + k$ is a translation of the curve by k units up.

The point $(0, -4)$ moves to $(0, -4 + k)$.

This is the origin, so $-4 + k = 0$ and $k = 4$.

Answer: $k = 4$

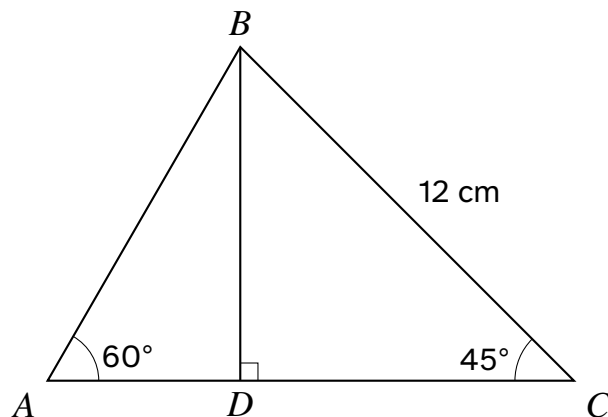
Tip: Use the point where the curve crosses the y -axis: only that point can move to the origin in a vertical translation.

25 (4 marks)

ABC is a triangle.

D is the point on AC such that BD is perpendicular to AC .

$BC = 12$ cm angle $BAD = 60^\circ$ angle $BCD = 45^\circ$



Not drawn accurately

Show that $AB = 4\sqrt{6}$ cm

In triangle BCD : $\sin 45^\circ = \frac{BD}{12}$, so $BD = 12 \times \frac{\sqrt{2}}{2} = 6\sqrt{2}$

In triangle ABD : $\sin 60^\circ = \frac{BD}{AB}$, so $AB = \frac{BD}{\sin 60^\circ}$

$$AB = 6\sqrt{2} \div \frac{\sqrt{3}}{2} = \frac{12\sqrt{2}}{\sqrt{3}}$$

$$\text{Rationalise: } \frac{12\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{6}}{3} = 4\sqrt{6} \text{ cm}$$

Answer: $BD = 6\sqrt{2}$, $AB = \frac{6\sqrt{2}}{\sin 60^\circ} = \frac{12\sqrt{2}}{\sqrt{3}} = \frac{12\sqrt{6}}{3} = 4\sqrt{6}$ cm

Tip: You need $\sin 45^\circ = \frac{\sqrt{2}}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$ without a calculator. In a “show that”, write every step, including the rationalising.

Marks: 1 mark for $BD = 6\sqrt{2}$. 1 mark for a correct method for AB using $\sin 60^\circ$. 1 mark for $\frac{12\sqrt{2}}{\sqrt{3}}$ or equivalent. 1 mark for rationalising to $4\sqrt{6}$.