

Circles and Semicircles

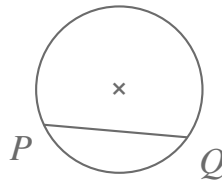
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

- (a) The centre of this circle is marked with a cross (×).



Write down the mathematical name of the straight line PQ .

PQ is a straight line joining two points on the circle.

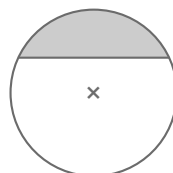
It does not go through the centre, so it is not a diameter.

It is a chord.

Answer: Chord

Tip: A chord that goes through the centre is called a diameter. PQ misses the centre, so it is just a chord.

- (b) Here is another circle. On the diagram, draw a segment of the circle. Shade the segment.



Draw a chord: a straight line joining two points on the circle.

A segment is the region between the chord and the arc.

Shade one side of the chord, as in the diagram.

Answer: A region between a chord and the arc, shaded

Tip: A segment is cut off by a straight line (a chord). A slice that goes in to the centre, between two radii, is a sector, not a segment.

2 (4 marks)

A paper fan opens out to make a semicircle of radius 20 cm.

- (a) A strip of lace is sewn along the curved edge of the fan.
Work out the length of the lace.
Give your answer correct to the nearest centimetre.

The curved edge of a semicircle is half of the circumference of the circle.

Circumference of the whole circle = $2 \times \pi \times 20 = 125.66 \dots$ cm

Curved edge = $125.66 \dots \div 2 = 62.83 \dots$ cm

To the nearest centimetre: 63 cm

Answer: 63 cm

Tip: The lace goes along the curved edge only, so do not add the straight edge. Halve the circumference of the whole circle.

Marks: 1 mark for $2 \times \pi \times 20 \div 2$ (or $\pi \times 20$). 1 mark for 63.

- (b) Work out the area of the fan.
Give your answer in terms of π .

Area of the whole circle = $\pi \times 20^2 = 400\pi$

The fan is half of the circle.

Area of the fan = $400\pi \div 2$

$$= 200\pi$$

Answer: $200\pi \text{ cm}^2$

Tip: Work out $\pi \times 20^2$ first, then halve it. Leave π in your answer: do not change it to a decimal.

Marks: 1 mark for $\frac{1}{2} \times \pi \times 20^2$. 1 mark for 200π .

3 (3 marks)

The diagram shows a sector OAB of a circle with centre O .
Angle $AOB = 90^\circ$ and $OA = OB = 7.5 \text{ cm}$.

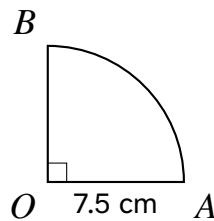


Diagram **NOT** accurately drawn

Work out the perimeter of the sector.

Give your answer correct to 1 decimal place.

Angle $AOB = 90^\circ$, so the sector is a quarter circle.

The perimeter is the arc AB plus the two straight sides OA and OB .

$$\text{Arc } AB = \frac{1}{4} \times 2 \times \pi \times 7.5$$

$$= 11.78 \dots \text{ cm}$$

$$OA + OB = 7.5 + 7.5 = 15 \text{ cm}$$

$$\text{Perimeter} = 11.78 \dots + 15 = 26.78 \dots$$

To 1 decimal place: 26.8 cm

Answer: 26.8 cm

Tip: The perimeter goes all the way round, so it includes the two straight sides as well as the arc. 11.8 is only the curved edge.

Marks: 1 mark for $\frac{1}{4} \times 2 \times \pi \times 7.5$ (or 11.7...). 1 mark for adding 15 (the two radii). 1 mark for 26.8.

4 (3 marks)

Table top A is a quarter circle with radius 1.2 m.

Table top B is a circle.

Table top A and table top B have the same area.

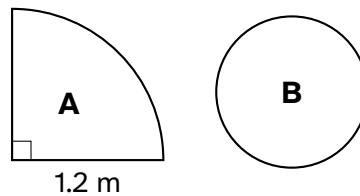


Diagram **NOT** accurately drawn

Show that the radius of table top B is 0.6 m.

$$\text{Area of A} = \frac{1}{4} \times \pi \times 1.2^2$$

$$= \frac{1}{4} \times 1.44\pi = 0.36\pi \text{ m}^2 \text{ (or } 1.130 \dots \text{ m}^2\text{)}$$

A circle with radius 0.6 m has area $\pi \times 0.6^2 = 0.36\pi \text{ m}^2$ (or 1.130 ... m²)

The two areas are the same, so the radius of B is 0.6 m.

Answer: Area of A = $0.36\pi \text{ m}^2$ = area of a circle with radius 0.6 m, so the radius of B is 0.6 m

Tip: Keep π in both areas, or use the full calculator values, so you can see that they are equal. Finish with a sentence saying what you have shown.

Marks: 1 mark for the area of A or of B, eg $\frac{1}{4} \times \pi \times 1.2^2$ or $\pi \times 0.6^2$. 1 mark for both areas. 1 mark for a conclusion that compares them.

5 (3 marks)

A flower bed is in the shape of a semicircle.

The diameter of the flower bed is 8 m.

Ali puts fertiliser on the flower bed.

One box of fertiliser covers 5 m^2

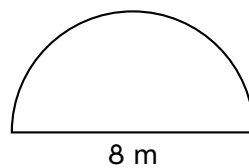


Diagram **NOT** accurately drawn

How many boxes of fertiliser does Ali need?

The radius is half of the diameter: $8 \div 2 = 4 \text{ m}$.

$$\text{Area of the flower bed} = \frac{1}{2} \times \pi \times 4^2$$

$$= 25.13 \dots \text{ m}^2$$

Each box covers 5 m^2 , so divide by 5:

$$25.13 \dots \div 5 = 5.02 \dots$$

5 boxes would not be enough, so Ali needs 6 boxes.

Answer: 6

Tip: Use the radius, 4 m, not the diameter. Then round the number of boxes **up**: 5 boxes cover only 25 m².

Marks: 1 mark for the area of the semicircle, $\frac{1}{2} \times \pi \times 4^2$ (or 25.1...). 1 mark for dividing the area by 5. 1 mark for 6.

6 (4 marks)

The diagram shows a quarter circle OAB , centre O and radius 12 cm.
A semicircle is drawn with OA as its diameter.

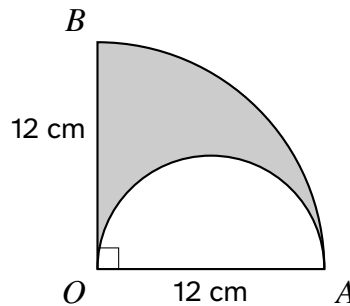


Diagram **NOT** accurately drawn

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

Shaded region = quarter circle – semicircle.

$$\text{Area of quarter circle} = \frac{90}{360} \times \pi \times 12^2$$

$$= 113.09 \dots \text{ cm}^2$$

The semicircle has diameter $OA = 12$ cm, so its radius is 6 cm.

$$\text{Area of semicircle} = \frac{1}{2} \times \pi \times 6^2 = 56.54 \dots \text{ cm}^2$$

$$\text{Shaded area} = 113.09 \dots - 56.54 \dots = 56.54 \dots$$

Round to 3 significant figures: 56.5

Answer: 56.5 cm²

Tip: The semicircle's radius is half of OA , so it is 6 cm, not 12 cm.

Marks: 1 mark for the area of the quarter circle. 1 mark for the area of the semicircle. 1 mark for subtracting. 1 mark for 56.5.

7 (3 marks)

The diagram shows a square tile $ABCD$ with sides of length 10 cm.

BED is an arc of a circle with centre A and BD is a diagonal of the square.

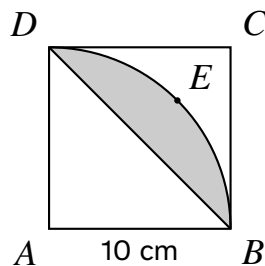


Diagram **NOT** accurately drawn

Work out the area of the shaded segment.

Give your answer correct to 3 significant figures.

Shaded segment = quarter circle $ABED$ – triangle ABD .

The radius of the arc is $AB = 10$ cm.

$$\text{Area of quarter circle} = \frac{1}{4} \times \pi \times 10^2 = 78.53 \dots \text{ cm}^2$$

$$\text{Triangle } ABD \text{ is half of the square: area} = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

$$\text{Segment} = 78.53 \dots - 50 = 28.53 \dots$$

To 3 significant figures: 28.5

Answer: 28.5 cm²

Tip: The shaded part is not the whole quarter circle. Take away the triangle ABD , which is half of the square.

Marks: 1 mark for the area of the quarter circle, $\frac{1}{4} \times \pi \times 10^2$ (78.5...). 1 mark for subtracting the area of triangle ABD , $\frac{1}{2} \times 10 \times 10$. 1 mark for 28.5.

8 (5 marks)

The diagram shows the plan of Ravi's garden.

$ABCD$ is a rectangle with $AB = 18$ m and $BC = 10$ m.

The pond is a quarter circle with centre D and radius 7 m.

The rest of the garden is lawn.

Ravi is going to spray the lawn with weedkiller.

One bottle of weedkiller covers 35 m² of lawn.

Each bottle costs £6.79

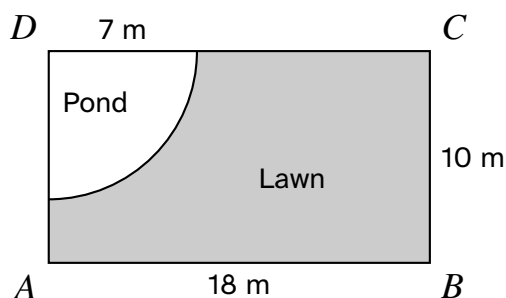


Diagram **NOT** accurately drawn

Work out how much it will cost Ravi to buy all the bottles of weedkiller he needs.

Lawn = rectangle – quarter circle (the pond).

$$\text{Area of rectangle} = 18 \times 10 = 180 \text{ m}^2$$

$$\text{Area of pond} = \frac{1}{4} \times \pi \times 7^2 = 38.48 \dots \text{ m}^2$$

$$\text{Area of lawn} = 180 - 38.48 \dots = 141.51 \dots \text{ m}^2$$

$$\text{Number of bottles: } 141.51 \dots \div 35 = 4.04 \dots$$

4 bottles would not be enough, so Ravi needs 5 bottles.

$$\text{Cost} = 5 \times \text{£}6.79 = \text{£}33.95$$

Answer: £33.95

Tip: Take the pond away from the rectangle: the weedkiller is for the lawn only. Then round the number of bottles **up**.

Marks: 1 mark for the area of the rectangle (180). 1 mark for the area of the quarter circle (38.4...). 1 mark for the area of the lawn and dividing by 35. 1 mark for 5 bottles. 1 mark for £33.95.
